ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Tuesday 22 April 2003 9.00-10.30

Module 4A8
ENVIRONMENTAL FLUID MECHANICS

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 a) Discuss the relative merits of Direct Numerical Simulation, Large Eddy Simulation and RANS/turbulence modelling for computing turbulent flows. [10%]

b) A turbulent boundary layer has been computed with DNS and has a Reynolds number of $\frac{U_\tau \delta}{\nu} = 600$. The computation took 100 hours on a particular supercomputer. It has been suggested that the same technique be used in a neutrally-stratified atmospheric boundary layer with a Reynolds number of $\frac{U_\tau \delta}{\nu} = 6 \times 10^6$. Estimate the time (in centuries) required to do this on the same supercomputer. [40%]

c) Estimate the ratio of the size of the smallest eddy at $z/\delta = 0.01$ to δ in the atmospheric boundary layer of part (b). [20%]

d) Explain qualitatively how stable density stratification would change the estimate obtained in part c). [20%]

e) Explain the meaning and importance of the Richardson number and the Monin-Obukhov length in atmospheric turbulent boundary layers. [10%]

All symbols have their usual meaning.

2 Carbon dioxide from a chemical plant flows down a slope and into a village as shown in Fig. 1. There is concern as to the danger this will pose to the villagers.

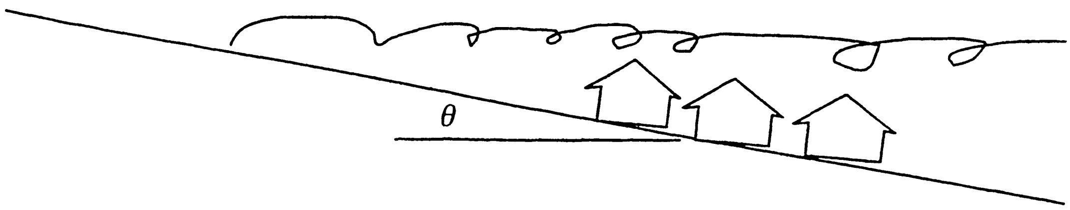

Fig. 1

Assume that the flow is constrained laterally so that the problem is one-dimensional and set up equations and solve them in order to:

a) estimate the thickness of this layer when it encounters the village; [40%]

b) estimate the concentration by mass of CO_2 in the layer when it encounters the village,500m from the source; [20%]

c) a small amount of a highly toxic gas enters the CO_2 layer at the source. Estimate the time it will take to reach the village; [40%]

At a distance of 10 m from the source the average velocity is 5 ms^{-1}, the layer thickness is 1 m and the density of the layer is equal to the density of CO_2. The entrainment coefficient is $\alpha = 0.1$.

The density of CO_2 is 1.8 kgm^{-3} and the density of air is 1.2 kgm^{-3}.

(TURN OVER

3 a) By considering the balance for the mass concentration ϕ of a pollutant (ϕ has units kg m^{-3}) in a box model for a city of length L and width W, show that, for the case that the concentration and the emission rate $q(x)$ (in kg m^{-2} s^{-1}) are allowed to vary in the downwind direction x,

$$U\frac{d\phi}{dx} = \frac{q(x)}{H} + \dot{w}$$

where H is the mixing height, U the wind velocity and $\dot{w}$ the pollutant generation rate (in kg m^{-3} s^{-1}). Hence, show that the concentration of an inert pollutant at the downwind edge of the city does not depend on the actual distribution of $q(x)$, but only on the total emission of pollutant. [30%]

b) During a smog episode, the air above a city is stagnant (U=0) but well-mixed. The emission rate and the photochemical reaction rate of a pollutant vary with time according to

$$q(t) = Q\left[1 + \sin(\omega\, t)\right] \quad \text{and} \quad \dot{w} = -K\left[1 + \sin(\omega\, t)\right]\phi$$

where Q (in kg m^{-2} s^{-1}) and K (in s^{-1}) are constant and $\omega = \pi/24$ h^{-1}. Find an expression for the time evolution of the concentration ϕ, if $\phi = 0$ at $t = 0$. Also find an expression for the maximum possible concentration that can be reached. [40%]

c) Discuss qualitatively why and how the mixing height changes during the day and what are the implications of these changes for pollution dispersion. [30%]

4 a) Assume that the mean concentration of an inert pollutant $\overline{\phi}$ obeys the one-dimensional Gaussian plume equation in the absence of solid boundaries, with the dispersion coefficient varying linearly with streamwise (downwind) distance x according to $\sigma = Cx$, with $C << 1$. Write down the balance equation for the variance g of ϕ, supplemented with the usual models for the scalar dissipation and turbulent diffusion. Hence show that the production and diffusion terms in the x-direction are negligible compared to the corresponding terms in the cross-wind direction. If all the remaining terms are of the same order of magnitude, what can you say about the order of magnitude of g? [50%]

b) A motorway can be taken to be at ground level (z=0) and is oriented at 90 degrees to the wind that has speed 5 ms^{-1}. The dispersion coefficient may be taken as $0.1x$, where x is the downwind distance from the motorway. If there is one car per second passing at a speed of 100 kmh^{-1}, each car releasing 100 g of NO per km travelled, find the NO ground concentration 2 km from the motorway. Take NO to be chemically inert. [50%]

END OF PAPER