ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Wednesday 23rd April 2003 9.00-10.30

Module 4A10

FLOW INSTABILITY

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 A liquid jet with velocity U, density ρ and the initial radius a is surrounded by the air of negligible density. The surface tension of the liquid-air interface is σ.

a) Explain how U can be eliminated by a change of reference frame. [10%]

b) Consider a disturbance to the surface of the liquid in which its radial position is moved to $r = \alpha + \beta \cos n\theta \cos kx$, where r, θ, x are polar coordinates, β is very small and n is an integer. Show that α and β are related by

$$\alpha \approx a\left(1 - \frac{1}{8}\frac{\beta^2}{a^2}\right) \quad \text{for } n \neq 0$$

[30%]

$$\alpha \approx a\left(1 - \frac{1}{4}\frac{\beta^2}{a^2}\right) \quad \text{for } n = 0$$

Use an energy argument to show that the liquid surface is only unstable to long wavelength axisymmetric disturbances with $ka < 1$. [45%]

c) A detailed analysis shows that $ka = 0.7$ for the most rapidly growing disturbance. What size of droplets would you expect? [15%]

Note that: For a cylinder, whose radius r is a function of axial position x and polar angle θ,

Volume in a length $L = \dfrac{1}{2}\displaystyle\int_0^L \int_0^{2\pi} r^2 \, d\theta dx$

Surface area of a length $L = \displaystyle\int_0^L \int_0^{2\pi} \left[1 + \left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{1}{r}\frac{\partial r}{\partial \theta}\right)^2\right]^{1/2} r d\theta \, dx$.

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2 For each of the flows (i) – (iv):

a) explain the physical mechanism that can lead to instability;

b) state the stabilising influences;

c) derive an appropriate non-dimensional number to describe the susceptibility to instability;

and d) describe the flow pattern **just** after the onset of the *first* instability.

i) fluid between two cylinders, with the inner cylinder rotating and the outer cylinder fixed; [25%]

ii) boundary layer flow over a flat plate; [25%]

iii) temperature gradients in the atmosphere; [25%]

iv) a very thin layer of liquid with a free upper surface and heated from below. [25%]

3 Briefly describe ten examples of flow-structure interaction from engineering or nature explaining the fluid mechanics of the interaction. Bonus marks will be given for any examples not discussed in the lectures. [100%]

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4 Work is being undertaken at depth on an offshore oil rig. The workers are inside a spherical capsule whose average density (including the occupants) is less than that of the surrounding water. In the case of an accident the capsule is designed to rise to the surface as a result of its own lower density.

a) Develop an expression for its terminal rise velocity, through still water, based on a constant drag coefficient, C_D. [20%]

b) There is concern that in the case of an accident, such as a well blowout, the water will be highly turbulent. For preliminary design calculations it is assumed that there is no mean flow due to the accident, that the turbulence is the same everywhere and we need only consider the turbulence in the vertical direction.

What information about the turbulence is necessary to determine the effect of the turbulence on the rising capsule? [30%]

c) Derive an expression for root-mean-square fluctuating force on the capsule due to the turbulence. [30%]

d) Suggest how you would determine information about the acceleration, velocity and displacement of the capsule due to the turbulence. [20%]

END OF PAPER