ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Friday 2 May 2003 9:00 to 10:30

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator

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1 (a) Define the term "*Nanotechnology*". Explain using examples of two of the following topics why an understanding of basic quantum mechanics is essential in the field of nanotechnology:

(i) wave-particle duality;

(ii) tunnelling;

(iii) scanning-probe microscopy;

(iv) hot electron and resonant tunnelling devices.

[40%]

(b) Fig. 1 below is a sequence of simulations of a wave packet incident upon a narrow rectangular potential barrier, where the mean packet energy is slightly higher than the barrier height. By reference to the figure which shows the modulus squared of the wave-function as a function of position after successive time intervals, explain how the wave packet evolves before, during and after interaction with the barrier. [60%]

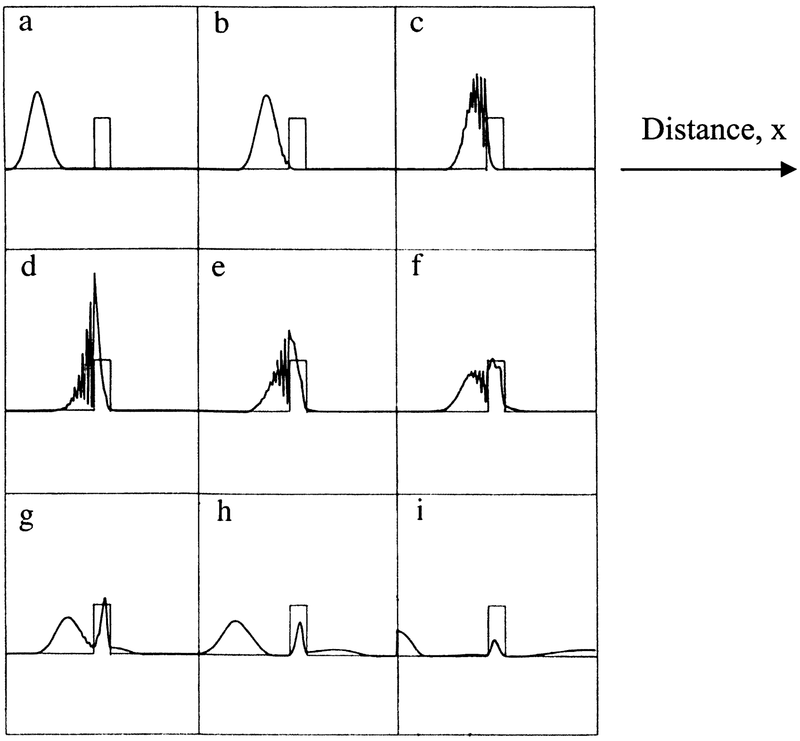

Fig. 1

2 (a) What is meant by the term *wave-particle duality*? Your answer should include a description of two phenomena which demonstrate it. [30%]

(b) With reference to their de Broglie wavelengths, which of the following systems can be described using quantum mechanics, and why:

(i) a dust particle of mass 10^{-13} g, which is travelling at a velocity of 1 nm.s^{-1}

(ii) a 1,500 kg car travelling at 160 km.hr^{-1}

(iii) a hydrogen molecule moving at 1 m.s^{-1}

[40%]

(c) Fig. 2 illustrates a particle of energy 0.25 eV incident on a potential step of height 1 eV. Determine the probability that the particle has of being reflected from the potential step. Describe quantitatively what would happen if the step were replaced by a 1 nm thick barrier of the same height. [30%]

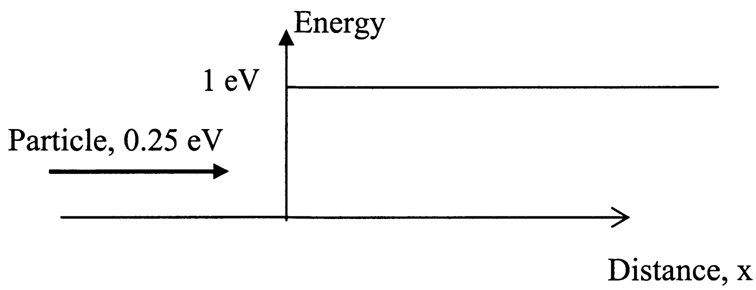

Fig. 2

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3 Fig. 3 illustrates the 1-dimensional case of an electron in an infinite potential well which extends from $x = 0$ to $x = L$.

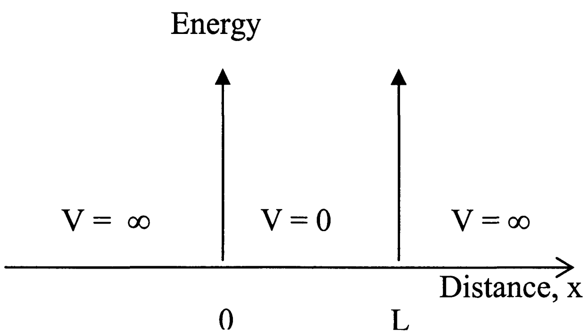

Fig. 3

(a) Solve Schrödinger's equation with the appropriate boundary conditions to find the general normalised wave-functions for this system. Derive an expression for the energy levels. Why are only certain values of energy allowed? [40%]

(b) By considering the symmetry of Schrödinger's equation, show that the wave-functions for *any* symmetric potential well have either even or odd parity. [20%]

(c) First-order perturbation theory tells us that when we take a quantum system for which we already know the original wave-functions and energies, and change the form of the potential *slightly*, we can determine the new wave-functions and energies in terms of the original ones. To first order, if the potential is perturbed by V, then the correction to the energy of the kth state of the perturbed system is V_{kk} where:

$$V_{kk} = \int_{-\infty}^{\infty} \psi_k^* V \psi_k$$

Here, ψ_k is the wave-function describing the *un*perturbed kth state.

(Cont.

Using this, determine the first order correction to the kth energy level in an infinite square well with a triangular potential at the bottom (as shown in fig.4). [30%]

(d) Consider a quantum-well laser whose operation depends on electrons in a potential well (in the conduction band) recombining with holes also in a potential well (in the valence band), as shown in fig. 5. How would the application of a voltage (i) parallel; and (ii) perpendicular to the well alter the lasing wavelength? [10%]

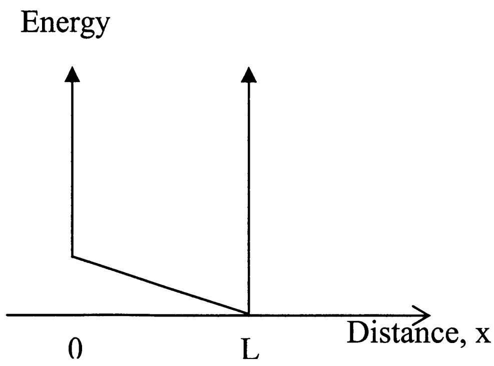

Fig. 4

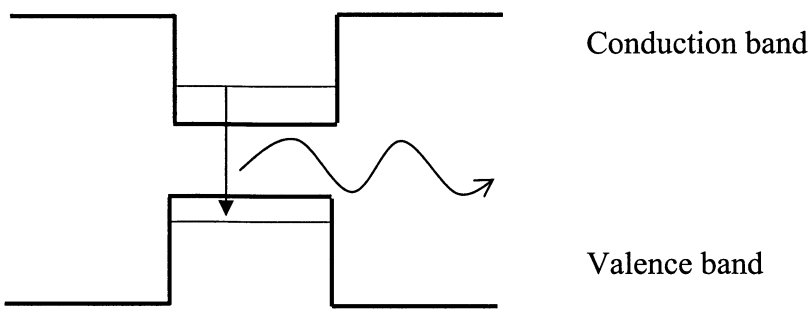

Fig. 5

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4 (a) State Moore's law. Discuss briefly the advances in technology over the last seven decades that have enabled the continued increase in the speed of operation of transistors. Why is there a desire to have smaller transistors? [30%]

(b) With particular reference to field-effect transistors, name and discuss with examples, one *quantum* and one *classical* phenomenon which will hinder further reduction in size of the transistor. How would you go about trying to overcome these? [30%]

(c) A number of alternative architectures to those of conventional transistors have been proposed and are currently being studied. Why this is deemed to be necessary? Describe the basic principles of fabrication and operation of devices based on one of the phenomena named below, with emphasis on size, speed and reproducibility: [40%]

(i) resonant tunnelling

(ii) hot-electrons

(iii) molecular electronics

5 (a) Briefly outline the assumptions made in the nearly-free electron model of electronic conduction. How do these differ from those of the free-electron model? [15%]

(b) By considering an electron in a weak periodic potential represented by $V(x) = 2V_1\cos(2\pi x/a)$, where the electronic wave-function is given by $\psi(x) = e^{ikx}[C_0 + C_1 e^{(i2\pi x/a)}]$, and the interatomic spacing is a show that in order for $\psi(x)$ to be an eigenfunction of Schrödinger's equation with energy eigenvalue E, the following conditions must be met: [45%]

$$\frac{C_1}{C_0} = \frac{\frac{\hbar^2 k^2}{2m} - E}{V_1} \qquad \text{and} \qquad \frac{C_1}{C_0} = \frac{V_1}{\frac{\hbar^2\left(k + \left(\frac{2\pi}{a}\right)\right)^2}{2m} - E}$$

(c) Sketch the dispersion relationship (E vs k) for the following cases:

(i) $V_1 = 0$; [20%]

(ii) V_1 is small but finite.

(d) How can the nearly-free electron model be used to explain the differences between metals, semiconductors and insulators? [20%]

END OF PAPER