ENGINEERING TRIPOS PART IIB

ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Tuesday 22 April 2003 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Candidates may bring their notebooks to the examination.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 An impulse hammer of mass 1 kg is to be used to conduct a modal test on a large uniform cantilever floor beam. The beam is clamped rigidly at one end as shown in Fig. 1 and an accelerometer is fixed to the other end of the blade at A, near one corner. During the test both flexural and torsional modes are to be excited by the hammer. The first three flexural modes are thought to have frequencies around 2 Hz, 13 Hz and 35 Hz and the frequencies of the first three torsional modes are thought to be 10 Hz, 30 Hz and 50 Hz.

(a) Show that a hammer pulse of 10 ms duration will enable the first three torsional and flexural modes to be investigated without exciting too many higher modes. What value for the stiffness of the hammer tip will give a suitable pulse? [20%]

(b) The force transducer in the hammer has a sensitivity of 5 pC/N and the hammer strikes the beam with a velocity of around 3 ms^{-1}. Design a simple charge amplifier to produce an output signal suitable for a data logger with an input range of ± 5 V. [20%]

(c) Select a suitable sampling rate for the data logger so that the use of an anti-aliasing filter is not required. [20%]

(d) *Sketch* the mode shapes for
(i) the three flexural modes;
(ii) the three torsional modes [20%]

(e) *Sketch* the transfer function which might be measured between
(i) points A and B;
(ii) points A and C [20%]

Detailed calculation is not required for parts (d) and (e).

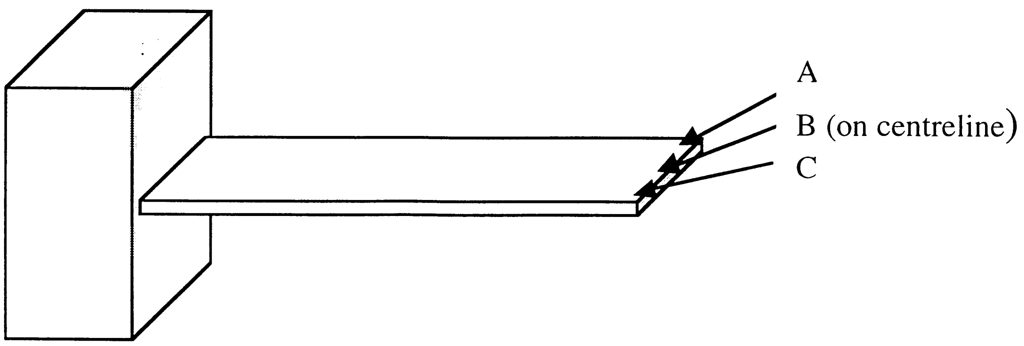

Fig. 1

2 A damped vibrating system with two degrees of freedom has mass, stiffness and damping matrices as follows:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.3 \end{bmatrix}.$$

(a) Find the modes and natural frequencies of the undamped system (i.e. ignoring C). [30%]

(b) Calculate the 4×4 matrix equation which determines the modes and complex natural frequencies of the damped system. Verify that the real mode shapes $\mathbf{u}_n$ found in part (a) for the undamped system give rise to vectors $\begin{bmatrix} \mathbf{u}_n \\ \lambda_n \mathbf{u}_n \end{bmatrix}$ which satisfy this equation, where λ_n is the corresponding complex eigenvalue. Hence find the modal damping factors for this system. [20%]

(c) Verify that for this system,

$$K = \alpha C^2 \qquad (1)$$

where α is a constant. [10%]

(d) A viscously-damped vibrating system with N degrees of freedom has a mass matrix which is the unit matrix ($M = I$) and its stiffness and damping matrices satisfy equation (1). Show that for any such system the modes of the damped system are identical to those of the undamped system, and that all modes have the same damping factor.

[Hint: Consider the eigenvectors of C, and show that they are also eigenvectors of K.] [40%]

3 The Helmholtz resonator shown in Fig. 2 consists of a vessel of volume V with a neck of cross-sectional area S and effective length L. The bottom of the vessel consists of a sprung piston with mass M, surface area A, and spring stiffness K.

(a) Show that the equations of motion that govern the displacement u of the neck air-mass and the displacement x of the piston have the form

$$\begin{bmatrix} \rho SL & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{x} \end{bmatrix} + \frac{1}{V}\begin{bmatrix} \rho c^2 S^2 & -\rho c^2 AS \\ -\rho c^2 AS & KV + \rho c^2 A^2 \end{bmatrix}\begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (2)$$

where c is the speed of sound in air, and ρ is the air density. Explain any assumptions made. [30%]

(b) Write down an expression for the natural frequency ω_p of the piston when the vessel is capped ($u = 0$). Also write down an expression for the Helmholtz frequency ω_h of the vessel that is obtained when the piston is restrained from motion ($x = 0$). [20%]

(c) Show that ω_p and ω_h lie between the natural frequencies of the coupled system that are obtained from equation (2). Could this fact have been deduced from the interlacing theorem? [30%]

(d) With reference to the interlacing theorem, discuss how the coupled natural frequencies change if K is increased and becomes very large. Also discuss how the natural frequencies change if, alternatively, M is increased and becomes very large. [20%]

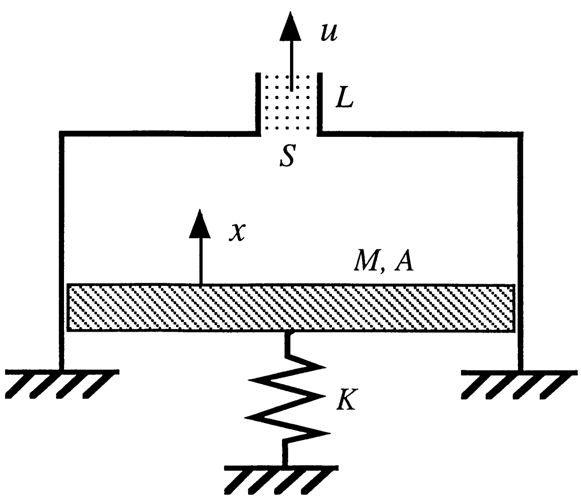

Fig. 2

4 A circular membrane of radius a has tension T and mass per unit area m. The boundary conditions are arranged so that the outer circumference is free to move, and no force is applied normal to the surface of the membrane.

(a) Write down the differential equation of motion that governs out-of-plane harmonic vibration of the membrane. Also write down the general solution to this equation, defining carefully any symbols you introduce. [20%]

(b) Explain why the boundary condition at the edge of the membrane is $\partial w/\partial r = 0$. Hence derive an equation that governs the natural frequencies of the membrane. Use Fig. 3 to estimate the first seven non-zero natural frequencies for the case $T = 500\ \mathrm{N\,m^{-1}}$, $m = 0.05\ \mathrm{kg\ m^{-2}}$, and $a = 0.15$ m. Sketch the associated mode shapes. A separate copy of Fig. 3 is supplied, and should be annotated and handed in with your script. [50%]

(c) Explain why the third and fourth non-zero natural frequencies are identical. A small lumped mass is added to the membrane in an effort to cause as much separation as possible between these natural frequencies. Explain where you would place the mass and calculate the maximum possible resulting frequency spacing. [30%]

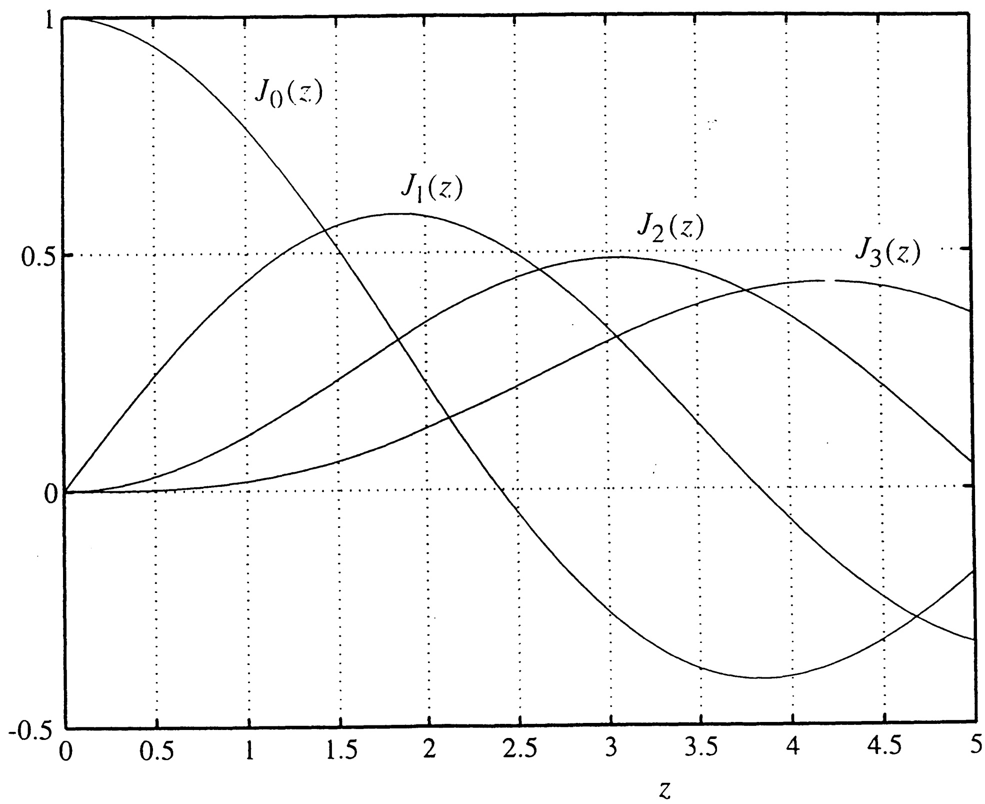

Fig. 3 The Bessel function $J_n(z)$ for various n

END OF PAPER

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Question 4

This copy of Figure 3 may be annotated as part of your answer, and should attached to your script when it is handed in.