ENGINEERING TRIPOS   PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS   PART II

Friday 25 April 2003 2.30 to 4.00

Module 4C8

APPLICATIONS OF DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The car moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients of C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v}+u\Omega)+\left(C_f+C_r\right)\frac{v}{u}+\left(aC_f-bC_r\right)\frac{\Omega}{u}=C_f\delta$$

$$I\dot{\Omega}+\left(aC_f-bC_r\right)\frac{v}{u}+\left(a^2C_f+b^2C_r\right)\frac{\Omega}{u}=aC_f\delta$$

State your assumptions. [40%]

(b) An automatic rapid transit vehicle is designed to follow a straight path at speed u. A control system steers the front wheels in response to angular deviations of the vehicle from the direction of motion θ, so that $\delta = -K\theta$.

(i) Derive a characteristic equation for small deviations in trajectory from a straight path. [30%]

(ii) Explain how you would use the Routh-Hurwitz criteria to find the range of K for which the motion of the vehicle is stable. [10%]

(iii) Comment briefly on the effectiveness of this control strategy. [20%]

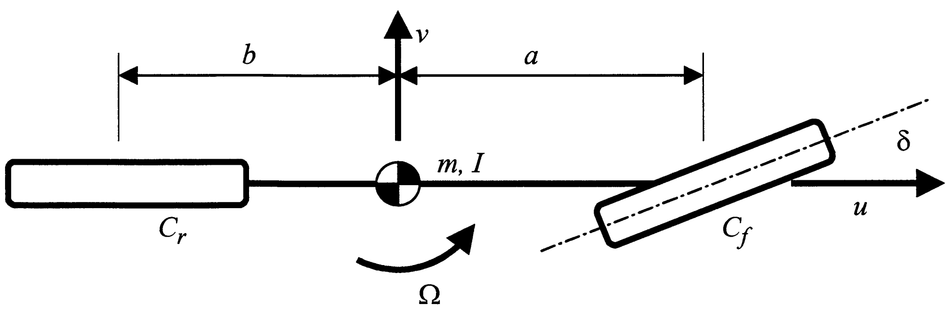

Fig. 1

2 A 'coned' railway wheelset with effective conicity ε, average wheel radius r and track gauge $2d$ is moving along a straight track at steady speed u. It has small lateral tracking error y and small yaw angle θ. The coefficients of *both* lateral and longitudinal creep of the wheels are C.

(a) Show that the net lateral force Y and net moment N acting on the wheelset due to the creep forces are given by:

$$Y = 2C\left(\theta + \frac{\dot{y}}{u}\right)$$

$$N = 2dC\left(\frac{\varepsilon y}{r} - \frac{d\dot{\theta}}{u}\right)$$

and indicate their directions on a sketch of the wheelset. State your assumptions. [50%]

Figure 2 shows a special railway vehicle with a central wheelset that measures the roughness profile of the track. The central wheelset, which may be assumed massless, is attached to the vehicle body through a suspension system which has lateral stiffness k (restoring force per unit displacement) and yaw stiffness ka^2, (restoring moment per unit angle), where a is a constant. The front and rear wheelsets may be assumed to track perfectly, with zero lateral tracking error and zero yaw angle.

(b) Derive an equation for the motion of the central wheelset.

[30%]

(c) Find an expression for the wavelength of the hunting motion. Compare it with the hunting wavelength of a free wheelset.

[10%]

(d) Find an expression for the damping ratio of the hunting motion and sketch a graph of its variation with k. [10%]

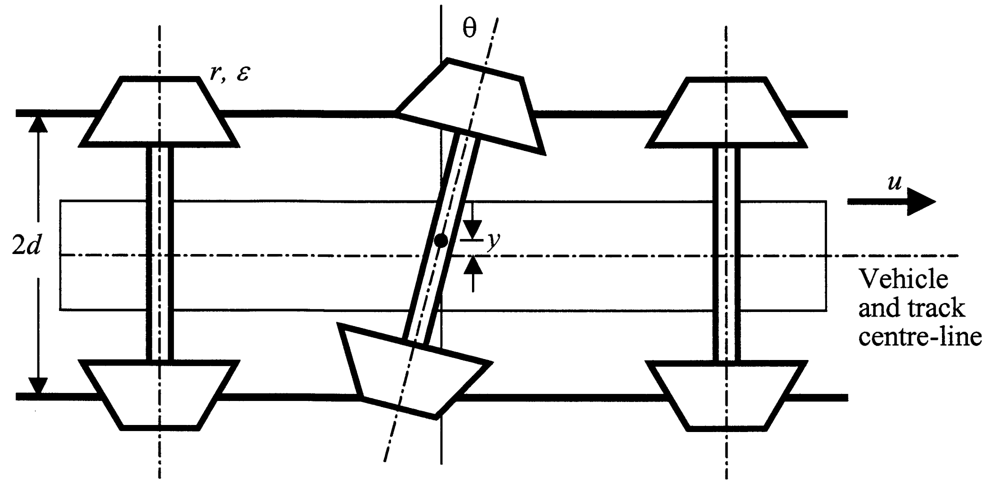

Fig. 2

3 Figure 3 shows a quarter-car model for predicting vehicle vibration. It has sprung mass m_s, unsprung mass m_u, tyre stiffness k_t, suspension stiffness k and suspension damping c. Mean square responses to a white noise velocity input at the tyre/road contact are given by:

$$E\left[\ddot{z}_s^2\right]=\frac{\pi S_0\left\{(m_s+m_u)k^2+k_t c^2\right\}}{m_s^2 c}$$

$$E\left[(k_t(z_r-z_u))^2\right]=\frac{\pi S_0\left\{(m_s+m_u)^3 k^2+(m_s+m_u)^2 k_t c^2-2(m_s+m_u)m_u m_s k k_t+m_u m_s^2 k_t^2\right\}}{m_s^2 c}$$

(a) State one other response that would normally be of interest. State the practical importance of each of these three responses. Explain the meaning of the variable S_0. [30%]

(b) Show that the value of suspension damping c that minimises mean square sprung mass acceleration is given by

$$c=k\sqrt{\frac{m_s+m_u}{k_t}}$$ [15%]

(c) Derive an expression for the value of c that minimises the mean square dynamic tyre force. [25%]

(d) Find a relationship between stiffness ratio (k_t/k) and mass ratio (m_u/m_s) that allows the minimum values of sprung mass acceleration and dynamic tyre force to be achieved simultaneously. Hence propose guidelines for determining which vehicles could benefit from a damper that can be adjusted to suit varying performance requirements. [30%]

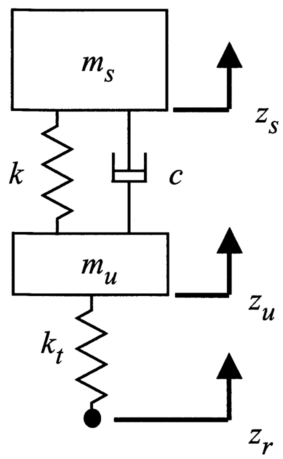

Fig. 3

4 Figure 4 shows a pitch-plane model of a vehicle. The sprung mass is m and the pitch inertia is I. The vehicle is symmetrical about its centreline, having wheelbase $2a$, suspension stiffness k and damping c at each axle. The vehicle travels with velocity U over a road with random roughness.

(a) Explain, with the aid of sketches, the mechanism of 'wheelbase filtering' in the vehicle. Your explanation should include mention of the effect of vehicle speed on the pitch and bounce responses of the sprung mass. [50%]

(b) It has been suggested that to improve ride comfort, the excitation of the pitch mode of the vehicle can be minimised by varying the suspension stiffness according to vehicle speed. By considering excitation of the undamped vehicle, show that a suitable design would require

$$k \propto U^2$$

and state the constant of proportionality in terms of the vehicle parameters. [25%]

(c) It is further suggested that the excitation of the bounce mode could be minimised at the same time as minimising the pitch excitation. Find the constraints on the vehicle parameters needed for this to happen and comment on whether it is practical to satisfy the constraints. [25%]

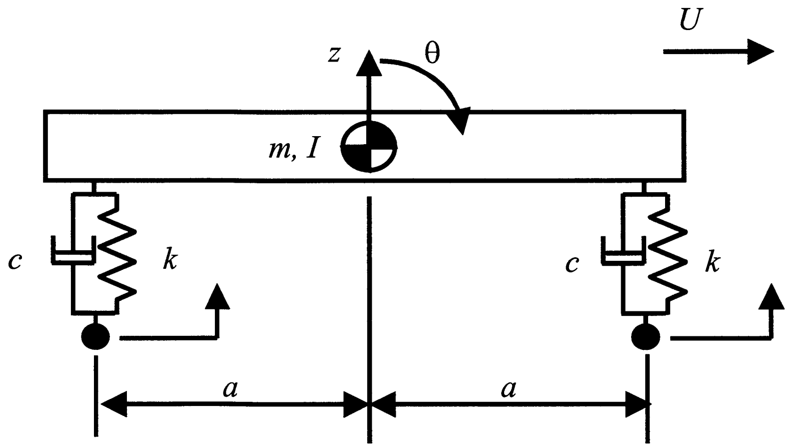

Fig. 4

END OF PAPER

ENGINEERING TRIPOS PART IIB

Module 4C8 Examination, 2003

Answers

1. (b)(i)

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0,$$

with

$$a_3 = mI$$

$$a_2 = \frac{(a^2 C_f + b^2 C_r)m + (C_f + C_r)I}{u}$$

$$a_1 = \frac{(C_f + C_r)(a^2 C_f + b^2 C_r)}{u^2} - \frac{(aC_f - bC_r)^2}{u^2} - (aC_f - bC_r)m + maC_f K$$

$$a_0 = \frac{(C_f + C_r)aC_f K}{u} - \frac{(aC_f - bC_r)C_f K}{u}$$

2. (b) $\ddot{y} + \left[\frac{uk(d^2 + a^2)}{2d^2 C}\right]\dot{y} + \left[\frac{u^2}{d}\left(\frac{\varepsilon}{r} + \frac{a^2 k^2}{4dC^2}\right)\right]y = 0$

(c) $\lambda = \dfrac{2\pi}{\sqrt{\dfrac{\varepsilon}{dr} + \dfrac{a^2 k^2}{4d^2 C^2}}}$

(d) $\varsigma = \dfrac{1 + a^2/d^2}{4C}\left[\dfrac{k}{\sqrt{\dfrac{\varepsilon}{dr} + \dfrac{a^2 k^2}{4d^2 C^2}}}\right]$

3. (c) $c = \sqrt{\dfrac{(m_s + m_u)^3 k^2 - 2kk_t m_u m_s (m_s + m_u) + m_u m_s^2 k_t^2}{(m_s + m_u)^2 k_t}}$

(d) $\dfrac{1}{2}\dfrac{k_t}{k} = 1 + \dfrac{m_u}{m_s}$

4. (b) $k = U^2 \dfrac{2I\pi^2}{a^2 L^2}$

(c) $I = ma^2\left(n_2 + \dfrac{1}{2}\right)^2$