ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Wednesday 23 April 2003 2.30 to 4.00

Module 4C9

CONTINUUM MECHANICS

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Candidates may bring their notebooks to the examination.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 The following analytic function in (ξ,η) can closely approximate the shape of a square with sides of length 2 as shown in Fig. 1:

$$\xi^2 + d\eta^2 + \frac{1}{5}\left(\xi^4 - 6\xi^2\eta^2 + \eta^4\right) = b$$

where b and d are constants.

(a) Obtain values for b and d such that this equation approximates the square. [10%]

(b) For a uniform elastic shaft with shear modulus G, obtain a Prandtl stress function $\phi(x,y)$ for a cross-section that closely approximates a square section with sides of width $2a$. Test whether your stress function satisfies equilibrium. [25%]

(c) For twist α applied to a square section shaft with sides of width $2a$, evaluate and sketch the distribution of shear stress on a line passing through both the centre of the square and a point of maximum shear stress. [30%]

(d) For torque T applied about the axis of the shaft in (c), calculate the maximum shear stress and compare your answer with the exact solution, $0.622T/a^3$. Explain why your calculated value is either an upper or lower bound. [35%]

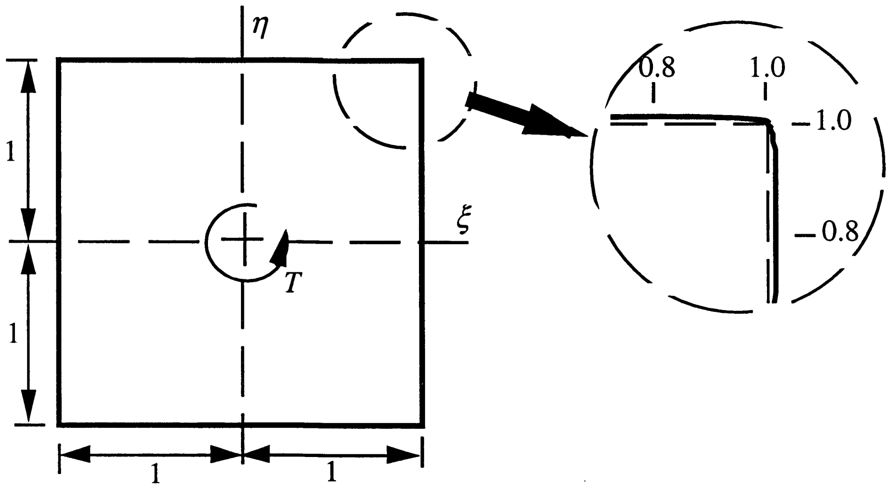

Fig. 1

2 (a) What assumptions are needed for Drucker's postulates to be valid? Why are the postulates important for the development of modern plasticity theory? [20%]

(b) For a metallic material satisfying the von Mises yield criterion, show that

$$\dot{J}_2 = n_{ij}\dot{\sigma}_{ij} \quad \text{and} \quad n_{ij} = s_{ij}$$

where σ_{ij} is the stress tensor, $J_2 = \frac{1}{2} s_{ij} s_{ij}$ is the second invariant of the deviatoric stress s_{ij}, and n_{ij} is normal to the yield surface. [25%]

(c) A metallic solid has a Young's modulus E, an initial yield stress σ_y, a tangent modulus E_t and obeys the following bilinear uniaxial stress-strain relation,

$$\varepsilon = \begin{cases} \sigma / E & \text{for} \quad 0 < \sigma < \sigma_y \\ \sigma_y / E + \left((\sigma - \sigma_y) / E_t \right) & \text{for} \quad \sigma \geq \sigma_y \end{cases}$$

This material is subjected to the $(\sigma_{11}, \sigma_{12})$ loading history shown in Fig. 2. All other stress components equal zero.

Derive an explicit formula for ε_{11} as a function of σ_{11} on the basis of J_2 flow theory. (Hint: $\int (1 + x^2)^{-1} dx = \tan^{-1} x$.) [45%]

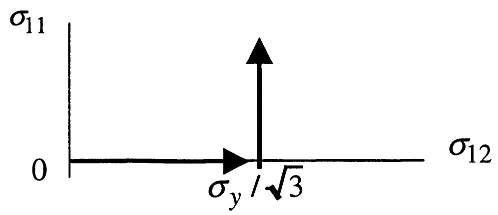

Fig. 2

(d) Explain why your answer to (c) will change if J_2 deformation theory is used. [10%]

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3 (a) A prismatic wedge subtends an angle β, with the lateral faces defined by $y = 0$ and $y = x \tan\beta$ in the Cartesian co-ordinates (x, y). Stresses in the wedge are given by

$$\sigma_{xx} = \alpha(-x + y\cot\beta), \quad \sigma_{xy} = \sigma_{yy} = 0 \ .$$

Find the body force b_i in the wedge and the traction t_i on each lateral surface. [25%]

(b) The deviatoric stress s_{ij} and deviatoric strain e_{ij} are defined by

$$s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}, \quad e_{ij} = \varepsilon_{ij} - \frac{1}{3} e \delta_{ij}$$

where σ_{ij} is the stress, $\sigma_m = \sigma_{ii} / 3$ is the mean stress, ε_{ij} is the strain and $e = \varepsilon_{ii}$ is the dilatation. For an elastic solid with Young's modulus E and Poisson's ratio ν, the shear modulus is $G = E / 2(1+\nu)$ and the bulk modulus is $K = E / 3(1-2\nu)$. Given that $e = \sigma_m / K$, prove that Hooke's law gives $s_{ij} = 2Ge_{ij}$. [25%]

(c) Figure 3 shows a square plate containing a central hole of radius a; the side length and thickness of the plate are $2b$ and t, respectively. The plate is made from an incompressible, rigid perfectly-plastic solid, and yields in accordance with the von Mises criterion, with shear yield stress τ_y. A uniform pressure p applied to the surface of the hole is gradually increased until the stress state reaches the collapse pressure p_c.

For the choice $b/a = 2$, determine an upper bound to p_c by assuming a kinematically admissible radial displacement field $u = A/r$, where A is a constant, and r is the radial co-ordinate from the centre.

(Hint: $\int_0^{\pi/4} \ln\cos\theta \, d\theta = -(\pi / 4)\ln 2 + 0.458$) [50%]

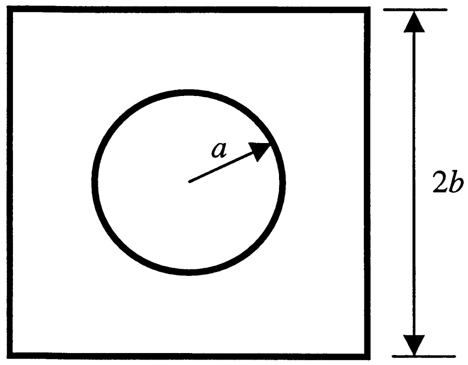

Fig. 3

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