ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Friday 25 April 2003 9 to 10.30

Module 4C10

FINITE ELEMENTS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Candidates may bring their textbook and notebooks to the examination.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 A two-node beam element of length L has displacement vector $\mathbf{d}^{(e)} = \{u_1\ \theta_1\ u_2\ \theta_2\}^T$, which represents transverse displacements $u_i^{(e)}$ and in-plane rotations $\theta_i^{(e)}$ at the two nodes. If the beam has elastic modulus E and second moment of cross-sectional area I, the stiffness matrix for this element can be expressed as

$$\mathbf{k}^{(e)} = \frac{2EI}{L^3}\begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix}$$

A propped cantilever of length $2L$ is shown in Fig. 1. End A is built-in while the midpoint B and end C are simply supported. The cantilever is uniformly loaded by pressure p across span BC. Represent the cantilever by two beam elements of equal length L.

(a) Determine the nodal forces and moments that are equivalent to the uniform pressure applied in section BC. [15%]

(b) Write an expression for the reduced stiffness matrix $\mathbf{K}_r$ of the propped cantilever, after assembling the global stiffness matrix and applying the necessary boundary conditions. [30%]

(c) Determine the rotations of the beam at B and C. [30%]

(d) Obtain expressions for the transverse displacements $u(x)$ experienced by the two elements. [25%]

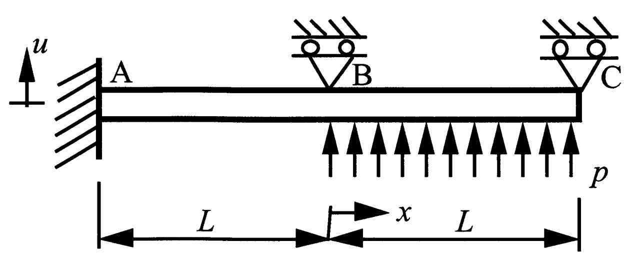

Fig. 1

2 A 4-node triangular parent element and a 4-node isoparametric triangular element are shown in Fig. 2. The parent element has the following shape functions:

$$N_1 = 1 - \xi - \eta - 3\xi\eta, \qquad N_2 = \xi - 3\xi\eta, \qquad N_3 = \eta - 3\xi\eta$$

(a) Write the shape function of node 4 that is located at the centroid of the parent element. [20%]

(b) Obtain an expression for the mapping from the parent to the global coordinates, $x(\xi, \eta)$ and $y(\xi, \eta)$. Use this to calculate the location in the global element of the midpoint of the hypotenuse of the parent element. [30%]

(c) Write the Jacobian matrix $\mathbf{J}$ for this specific transformation from parent element to global element. [20%]

(d) Calculate $|\mathbf{J}|$ for this specific transformation. Describe the property of the element that transforms in proportion to $|\mathbf{J}|$. [10%]

(e) For boundary conditions $u_1 = v_1 = u_3 = v_3 = 0$, derive an expression for the strain component ε_{xx} at the centroid of the global element in terms of nodal displacements u_2, v_2, u_4, v_4. [20%]

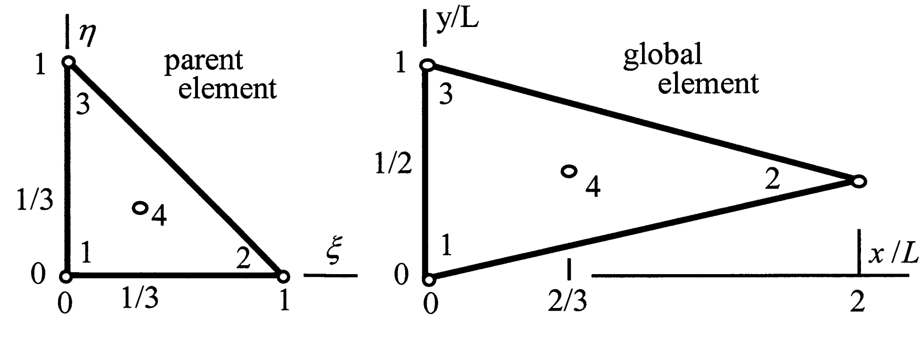

Fig. 2

3 (a) Consider a forming process in which hot metal is extruded through a rigid die. Describe briefly the three most important sources of nonlinearity. [20%]

(b) Discuss why a helicoidal spring subjected to compression has the load-deflection response diagram shown schematically in Fig. 3. [20%]

(c) Consider the 2-element beam assembly shown in Fig. 4. The two beams are connected rigidly at joint 2. A uniaxial compressive load P is applied horizontally at node 2. For both beams, let E, I, L denote the Young's modulus, second moment of the cross-section area, and the length, respectively. The beams can be assumed to be inextensional.

(i) Establish the nonlinear finite element equation for the assembly, including the effect of stress stiffening. [35%]

(ii) Calculate the linear buckling load for beam '*a*'. [25%]

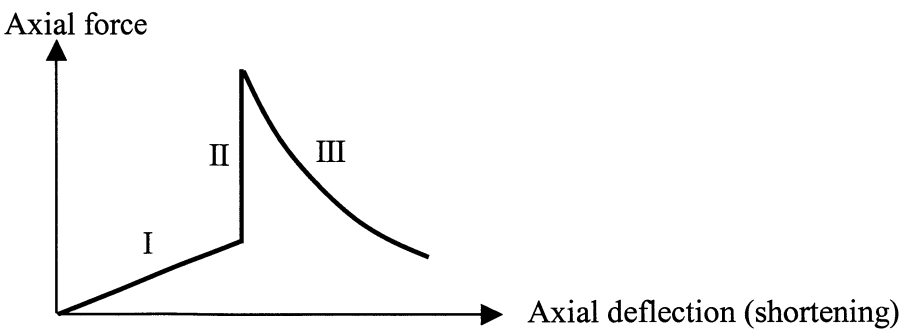

Fig. 3

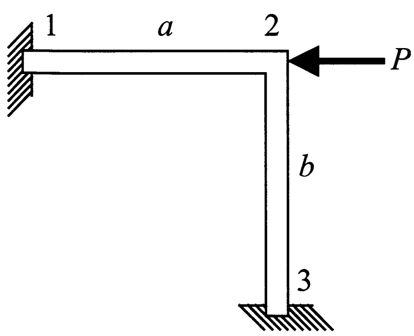

Fig. 4

4 The structure shown in Fig. 5 consists of a rod 1-2 of mass m and axial stiffness k, connected to a second rod 2-3 with axial stiffness k and *zero* mass. A lumped point mass m is fixed to the assembly at node 2. There are frictionless pin joints at nodes 1, 2 and 3. All motion is confined to the plane of the paper.

(a) Show that after constraints have been applied, the *reduced* structural stiffness matrix of the assembly is:

$$\mathbf{K} = \frac{k}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix} u_2 \\ v_2 \end{matrix}$$

where u_2 and v_2 are the horizontal and vertical displacements of joint 2. [30%]

(b) Calculate the structural mass matrix of the assembly. Note that in two-dimensions, the elemental mass matrix of a rod element of mass m is given by:

$$\mathbf{M} = \frac{m}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}\begin{matrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{matrix}$$

[20%]

(c) Determine the natural frequencies and natural mode shapes of vibration of the assembly. Sketch the mode shapes. [50%]

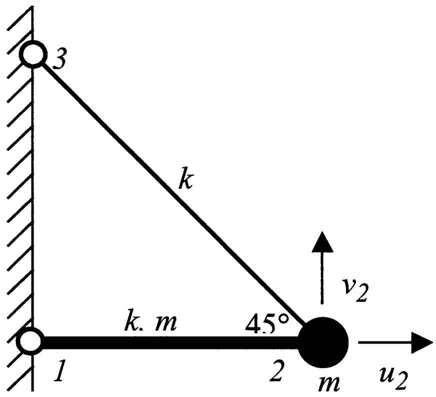

Fig. 5

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