ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Wednesday 23 April 2003 2.30 to 4.00

Paper 4F1

CONTROL SYSTEMS DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 A system $G(s)$ is to be stabilised with a compensator $K(s)$ in a standard negative feedback system.

(a) Suppose $G(s)$ can be written as $G(s) = G_0(s) + \triangle(s)$ where $G_0(s)$ is fixed and known and $\triangle(s)$ is known only to be stable and satisfy $|\triangle(j\omega)| \leq h(\omega)$ for all ω, for some positive continuous function $h(\omega)$. Find a necessary and sufficient condition for $K(s)$ to stabilise all such $G(s)$, stating clearly any results you assume. [20%]

(b) Let $G_0(s) = \dfrac{2(s+1)}{s(s-1)}$ and $G(s) = G_0(s) + \triangle(s)$.

(i) Sketch the complete Nyquist diagram of $G_0(s)$ and use it to determine the number of right half plane poles of the closed loop system for a compensator $K(s) = k$ for all values of k, positive or negative. [40%]

(ii) Suppose $\triangle(s) = \dfrac{\varepsilon e^{-s}}{s+1}$ and $K(s) = 1$. Find the largest ε for which the result of part (a) guarantees stability. [25%]

(iii) What can be said about closed-loop stability when $K(s) = 1$, $\triangle(s) = \dfrac{\varepsilon e^{-s}}{s-2}$ and ε is very small? [15%]

2 (a) Let $G(s) = \dfrac{3(s-2)}{(s+1)^2}$. Suppose a stabilising controller is required to achieve the following specifications:

$$\begin{aligned} A &: |S(j\omega)| \leq 1.5 \ \text{ for all } \ \omega, \\ B &: |S(j\omega)| \leq \varepsilon \ \text{ for } \ 0 \leq \omega \leq 1. \end{aligned}$$

Find a positive lower bound for ε. [30%]

(b) (i) Briefly explain when pole-zero cancellations between plant and controller are satisfactory and when they are not, giving reasons. [20%]

(ii) Let $G(s) = \dfrac{s}{s^2-1}$. Design a stabilising control scheme with the property that the transfer function from reference input to plant output is equal to $\dfrac{s}{(s+1)^2}$. [50%]

3 Figure 1 is the Bode diagram of a system $G(s)$ for which a feedback compensator $K(s)$ is to be designed. It may be assumed that $G(s)$ is a real-rational transfer function.

(a) (i) Sketch on a copy of Fig. 1 the expected phase of $G(j\omega)$ if $G(s)$ had no poles with $\text{Re}(s) > 0$ and was minimum phase; [10%]

(ii) Determine whether G(s) has any right half plane poles or right half plane zeros (it doesn't have both); [10%]

(iii) Comment on any limitations that this might impose on the achievable crossover frequency. [10%]

(b) A feedback compensator $K(s)$ is required to simultaneously satisfy the following specifications:

A : internal stability of the closed-loop,
B : a velocity error constant equal to 10,
C : $|G(j\omega)K(j\omega)| = 1$ at $\omega = 1$,
D : a phase margin of at least 30^o.

(i) Explain why these specifications cannot be met with a lag compensator alone; [15%]

(ii) Explain why these specifications cannot be met with a lead compensator alone; [15%]

(iii) Find a controller consisting of a lead compensator and a lag compensator together to achieve the specifications. [Hint: you may find it helpful to design the lead compensator first to comfortably achieve the phase required in D at $\omega = 1$.] [40%]

Two copies of Fig. 1 are provided on separate sheets. These should be handed in with your answers.

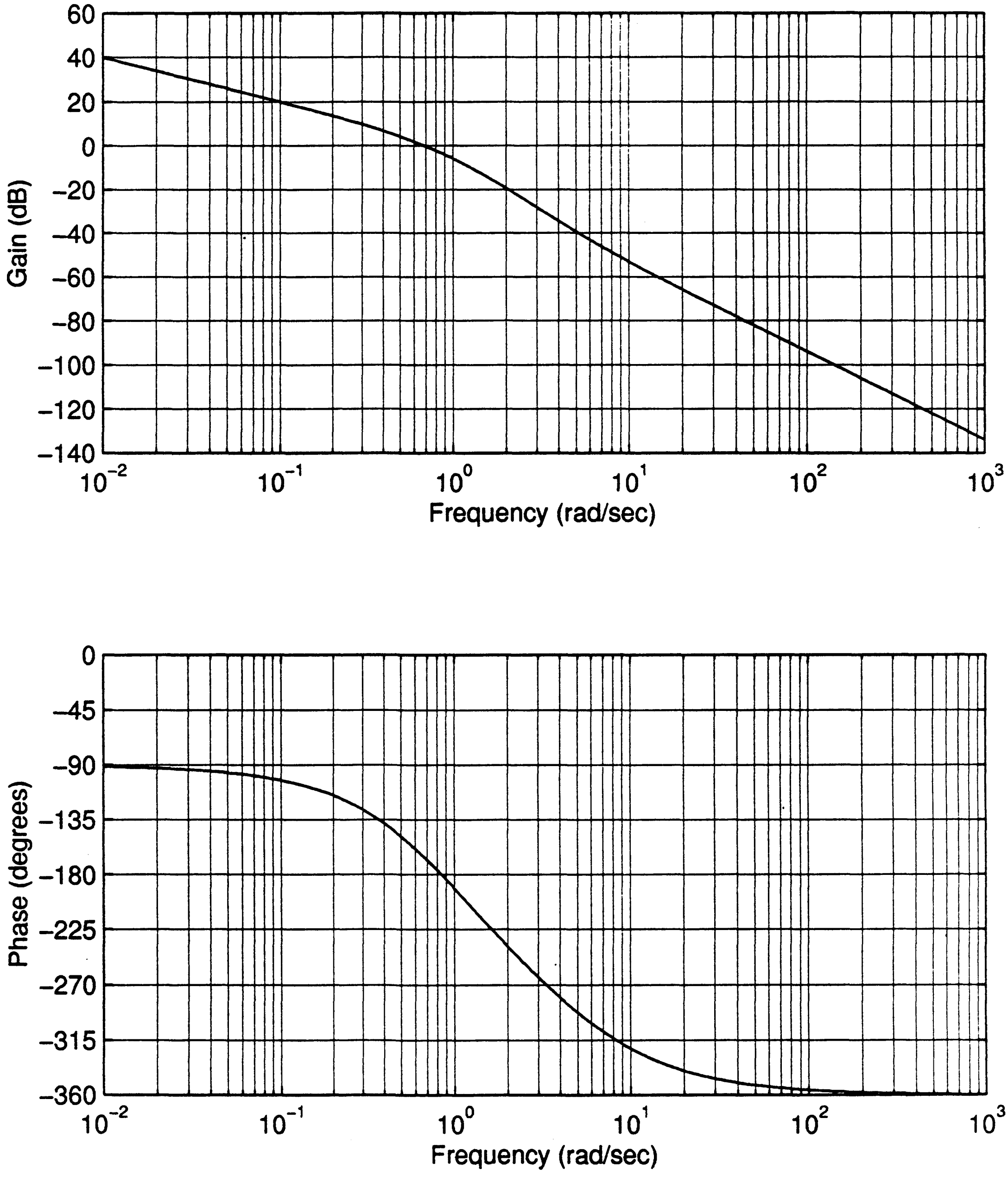

Fig. 1: Bode diagram of $G(s)$ for Question 3.

END OF PAPER

ENGINEERING TRIPOS PART IIB
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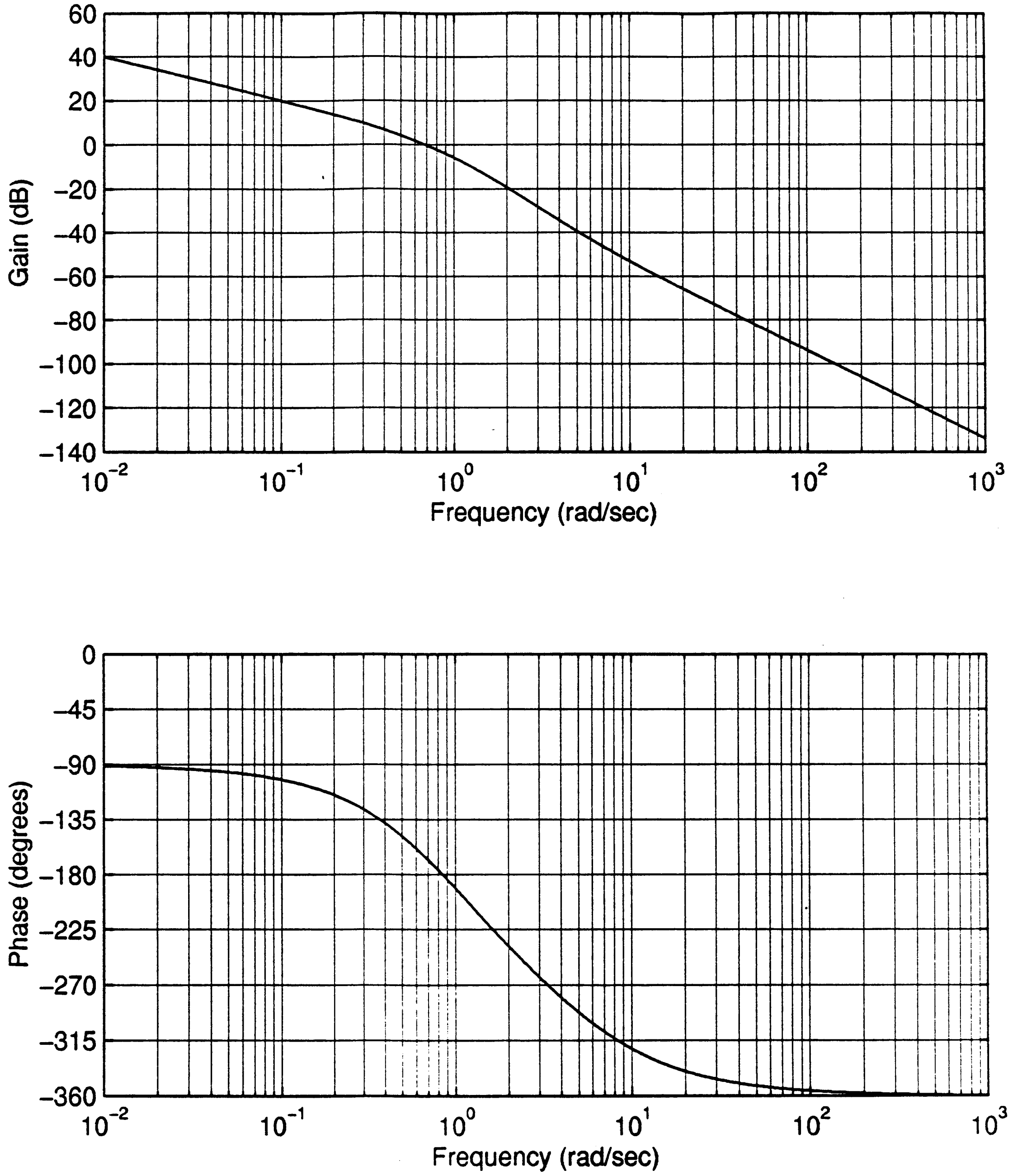

Extra Copy of Fig. 1: Bode diagram of $G(s)$ for Question 3.