ENGINEERING TRIPOS PART IIB

ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Friday 2 May 2003 9 to 10.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) For two-dimensional digital £lters, explain what is meant by a *zero-phase* £lter. Discuss the importance of phase in images indicating reasons for designing £lters that have the zero-phase property. [10%]

An image is sampled with spacings Δ_1 and Δ_2 in the x and y directions respectively. We wish to £lter this image with a particular form of ideal rectangular bandpass £lter whose frequency response is such that only frequencies between Ω_{L1} and Ω_{U1} in the ω_1 direction and between Ω_{L2} and Ω_{U2} in the ω_2 direction are allowed; this is shown in Fig. 1, where the shaded region takes value 1 and everywhere else takes value 0. Find the £lter impulse response, $h(n_1\Delta_1, n_2\Delta_2)$, of this ideal £lter. [40%]

Note that this impulse response has an in£nite region of support. Explain how we can create an FIR £lter from this via the method of windowing and indicate types of window function that might be suitable. [10%]

(b) Explain the concept of *histogram equalization* in images, outlining why it is often a useful operation. [10%]

An 8×8 image is shown below:

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	2	2	2	3	4
1	1	1	2	2	2	3	4
1	1	1	2	2	3	3	5
1	1	1	2	2	3	3	5
1	1	2	2	2	3	4	6
1	1	2	2	2	3	4	7

Determine the transformation which performs histogram equalization on this image, indicating the values onto which each of the levels 0 to 7 should be mapped. [30%]

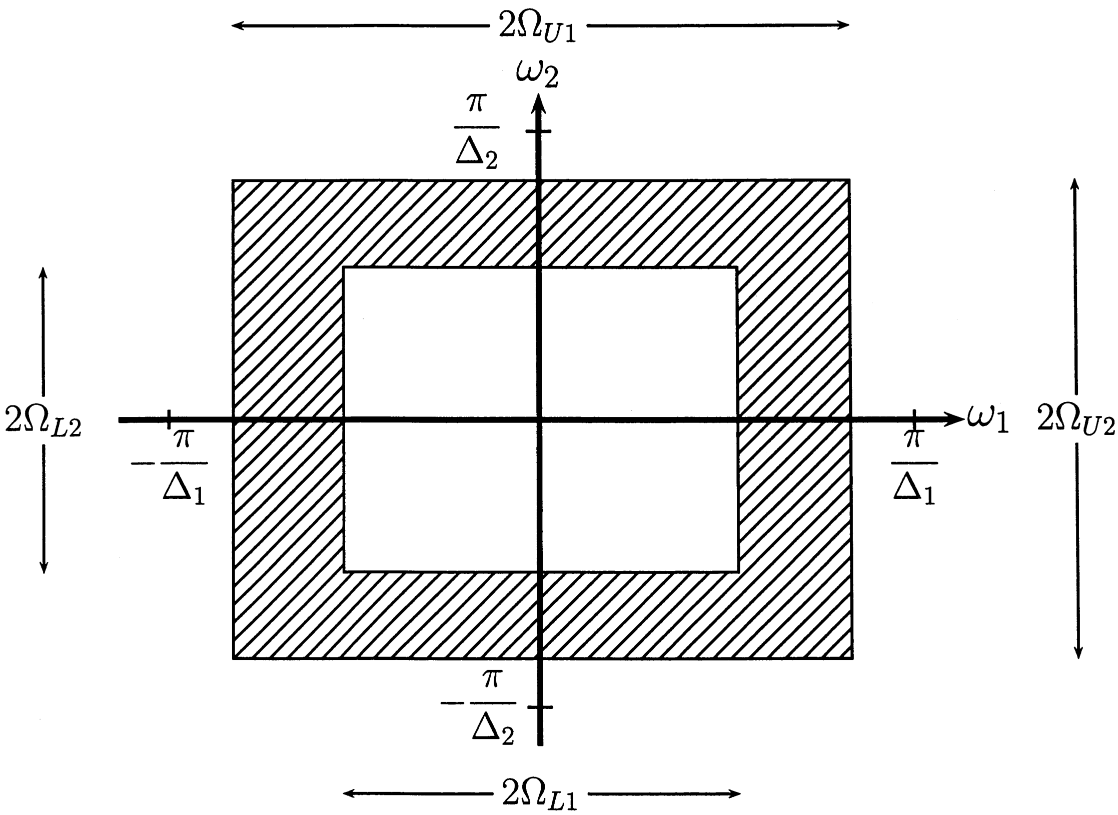

Fig. 1

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2 (a) By considering the spectrum of a sampled image in terms of the spectrum of the original image, discuss brie¤y the concept of *aliasing* and outline how to determine the sampling frequency in order to recover the original image from the sampled image without distortion. [20%]

(b) An image is described by the following function

$$f(x,y) = 2\cos(3x + 4y)$$

Form the Fourier transform, $F(\omega_1, \omega_2)$, of $f(x,y)$ and hence £nd ω_{1c} and ω_{2c} such that $F(\omega_1,\omega_2) = 0$ for $\{|\omega_1| > \omega_{1c},\ |\omega_2| > \omega_{2c}\}$, thereby verifying that f is bandlimited. Explain why the Nyquist sampling frequencies for this image are $\omega_1^{nyq} = 6$ and $\omega_2^{nyq} = 8$.

We now sample the image such that the sample spacings are $\Delta x = \Delta y = 0.4\pi$. Verify that it is then sampled at less than the Nyquist rates. [30%]

(c) Show that the spectrum of this sampled image, $F_s(\omega_1, \omega_2)$, is given by

$$F_s(\omega_1,\omega_2) = 25 \sum_{p_1=-\infty}^{\infty} \sum_{p_2=-\infty}^{\infty} \delta(\omega_1 - 3 - 5p_1, \omega_2 - 4 - 5p_2) + \delta(\omega_1 + 3 - 5p_1, \omega_2 + 4 - 5p_2)$$

The sampled image is now £ltered with the following low-pass £lter

$$H(\omega_1,\omega_2) = \begin{cases} \dfrac{4\pi^2}{25} & \text{if } \{|\omega_1| < 2.5,\ \ |\omega_2| < 2.5\} \\ 0 & \text{otherwise} \end{cases}$$

Show that the inverse Fourier transform, $\tilde{f}(x,y)$, of $H(\omega_1,\omega_2)F_s(\omega_1,\omega_2)$ takes the form

$$\tilde{f}(x,y) = 2\cos(\alpha x + \beta y)$$

and £nd the values of α and β. [50%]

3 (a) A discrete cosine transform (DCT) matrix T for one-dimensional n-point column vectors is de£ned by:

$$t_{1i} = \sqrt{\frac{1}{n}} \qquad \text{for } i = 1 \to n,$$
$$t_{ki} = \sqrt{\frac{2}{n}} \cos\left(\frac{\pi(2i-1)(k-1)}{2n}\right) \qquad \text{for } i = 1 \to n,\ k = 2 \to n$$

where t_{ki} is the element in row k and column i of T. Explain how T may be used to produce Y, the two-dimensional DCT of an $n \times n$ region X of an image, and how the inverse transform may be applied to Y to recover X. [20%]

(b) The matrix of coef£cients from the two-dimensional DCT of an 8×8 image region X is given by

$$Y = \begin{bmatrix} 30 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derive an expression for the elements x_{ki} of X. [30%]

(c) The JPEG image compression standard represents each matrix of DCT coef£cients as a list of values for *Run, Size* and *Additional bits*. Explain how these terms are generated and generate the list for the matrix Y of part (b). [30%]

(d) Explain why generation of the above list is a useful preprocessing step before Huffman coding is performed. In particular, discuss why it is advantageous to combine Run and Size values into a single code and why there is little point in applying Huffman coding to the Additional bits. [20%]

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4 (a) A set of wavelet coef£cients have a zero-mean Laplacian probability distribution with standard deviation σ and, when quantised with a stepsize of q, have the following discrete probability mass function (PMF):

$$p_k = \begin{cases} 1-\sqrt{r} & \text{for } k=0 \\ \alpha r^{|k|} & \text{for } k=\pm 1, \pm 2, \ldots \end{cases} \qquad \text{where } \alpha = \sinh\left(\frac{q}{\sqrt{2}\,\sigma}\right) \text{ and } r = e^{-\sqrt{2}\,q/\sigma}$$

Obtain an expression for the entropy H of a given wavelet subband, in terms of α and r (and hence q/σ) for that subband. [20%]

(b) Sketch the way that a 640 × 480 pixel image is decomposed into subbands by a 2-level 2-dimensional wavelet transform, giving the size of each subband, and brie¤y describe the types of image features that would be detected by each subband. [20%]

(c) A 2-dimensional wavelet transform employs orthonormal 1-dimensional £lters for rows and columns in turn, such that the transform preserves energy between the input and the outputs of each pair of £lters. With a typical image, at levels 1 and 2 the energy is found to split 95% to the lowpass and 5% to the highpass £lter output at each stage of £ltering. A 640 × 480 pixel typical image with zero mean and standard deviation 50 units is analysed by this 2-level wavelet transform and the wavelet coef£cients are quantised with a step-size of 20 units. Estimate the number of bits required to transmit the wavelet coef£cients for each subband of this image, assuming ideal entropy codes and Laplacian distributions; and hence obtain the mean bit rate in bits per pixel for transmission of the whole image. [40%]

(d) Brie¤y discuss (without detailed calculations) whether further levels of wavelet decomposition would be advantageous in this case. [20%]

END OF PAPER