ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

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Saturday 26 April 2003 9 to 10.30

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Module 4F9

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

*This paper consists of three sections.*

*Answer not more than* **one** *question from each section.*

*Answers to questions in each section should be tied together and handed in separately.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

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SECTION A *Medical Image Acquisition*

*Answer not more than **one** question from this section*

1 Figure 1 shows an ultrasound pressure wave $p_i = A_1 \exp(i(\omega t - k_1 x))$ travelling at speed $c_1$ through a homogeneous, non-dissipative medium of density $\rho_1$. The wave is normally incident on a boundary with a material, in which the speed of sound is $c_2$ and the density is $\rho_2$. The resulting reflected wave is $p_r = B_1 \exp(i(\omega t + k_1 x))$, and the transmitted wave is $p_t = A_2 \exp(i(\omega t - k_2 x))$.

The linear inviscid force equation is

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

where $u$ is the particle velocity. You may assume that the ultrasound energy is either reflected or transmitted, but not absorbed at the boundary.

(a) Write down the relationship between $k_1$ and $c_1$. [10%]

(b) Write down an equation in terms of $A_1$, $A_2$ and $B_1$ which must be satisfied for the pressure on both sides of the boundary to be equal. [10%]

(c) Write down an equation in terms of $A_1$, $A_2$, $B_1$, $\rho_1$, $\rho_2$, $c_1$ and $c_2$ which must be satisfied for the particle velocities, normal to the interface, to be equal at the boundary. [30%]

(d) Write down expressions for the *ultrasound intensity reflection coefficient*, $R$, and the *ultrasound intensity transmission coefficient*, $T$, in terms of $A_1$, $A_2$, $B_1$, $\rho_1$, $\rho_2$, $c_1$ and $c_2$. [20%]

(e) Derive expressions for $R$ and $T$ in terms of $\rho_1$, $\rho_2$, $c_1$ and $c_2$ alone. [30%]

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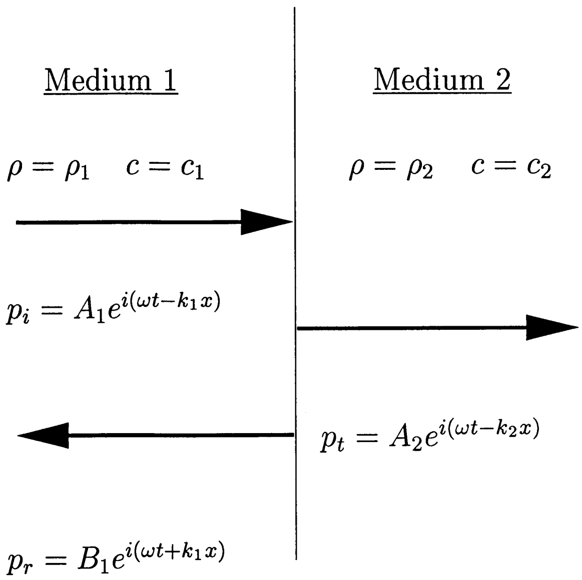


Fig. 1

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2 (a) Define the radon transform and the X-ray transform in three dimensions. [20%]

(b) Calculate the X-ray transform of the three dimensional function:

$$f(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^2}$$

[40%]

(c) Calculate the radon transform of the same function. [40%]

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SECTION B *Curves, Surfaces and Interpolation*

*Answer not more than **one** question from this section*

3 (a) Figure 2(a) shows a cubic grid, with each side of length one unit, that is to be used to construct a surface around a single voxel centre using the marching cubes algorithm. The black circle indicates a point inside the object, with a data value of one. Empty circles indicate points outside the object, with data values of zero. Calculate the volume inside the surface that the marching cubes algorithm produces. (Note: the volume of a tetrahedra is one third the base area times the height.) [25%]

(b) Figure 2(b) shows a tetrahedral grid that is to be used to construct a surface around a single voxel centre using the marching tetrahedra algorithm. The circles have the same meaning as in Fig. 2(a). All the edges shown as solid lines are of length $2/\sqrt{3}$ units. All the edges shown as dashed lines are the same length. Calculate this length. [30%]

(c) How many tetrahedra contribute to the volume constructed by the marching tetrahedra algorithm based on Fig. 2(b)? [15%]

(d) Calculate the volume inside the surface constructed by the marching tetrahedra algorithm based on Fig. 2(b). [30%]

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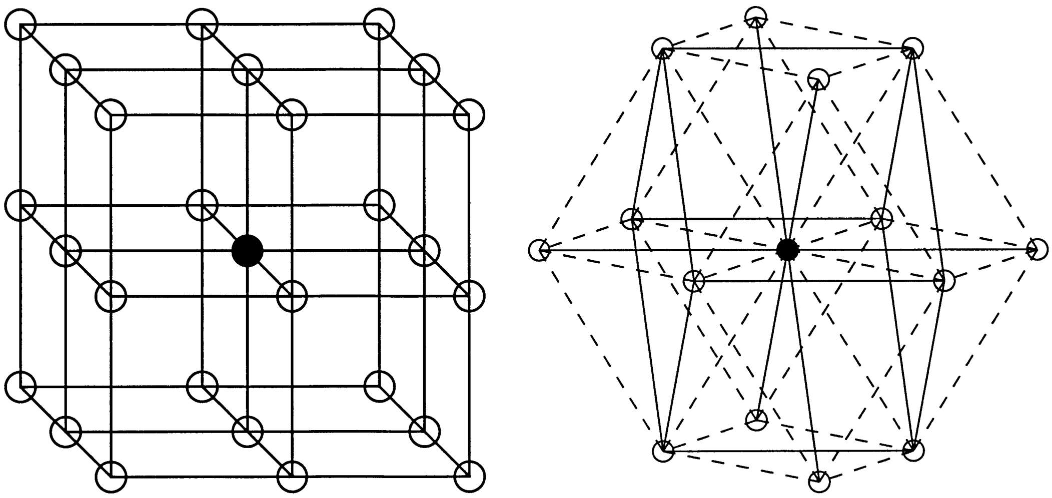

(a) (b)

Fig. 2

4 Figure 3 shows a segment of a two dimensional grid derived from CT data. Because the distance between the CT slices is greater than the resolution within each slice, the CT numbers in the centre of the grid are unknown. A CT number of 5 or above indicates that the square is inside the object of interest.

(a) Copy Fig. 3 onto graph paper and calculate the position of the object boundaries that would be found using a linear interpolation algorithm. [25%]

(b) Copy Fig. 3 onto graph paper and calculate the position of the object boundaries that would be found using a shape-based interpolation algorithm. [35%]

(c) Figure 4 shows two slices though a three dimensional voxel grid. There are three planes of voxels between the slices shown in Fig. 4. (Fig. 3 shows the orthogonal slice through the centre of the 3D block.)

Using shape based interpolation based on *city block* distances and not using *chamfer* distances, construct on graph paper the shape of the object in the middle slice, between the two slices shown in Fig. 4. [40%]

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 8 | 7 | 8 | 10 | 8 | 6 | 4 | 1 | First CT slice |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| 2 | 3 | 4 | 3 | 4 | 6 | 8 | 8 | 6 | 3 | Second CT slice |

Fig. 3

(cont.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 1 | 1 | 2 | 3 | 2 | 1 | 1 | 2 |
| 3 | 3 | 4 | 6 | 6 | 7 | 5 | 4 | 2 | 1 |
| 2 | 5 | 8 | 7 | 8 | 10 | 8 | 6 | 4 | 1 |
| 3 | 4 | 5 | 6 | 6 | 7 | 4 | 2 | 2 | 2 |
| 3 | 3 | 2 | 2 | 4 | 3 | 2 | 1 | 1 | 2 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 2 | 3 | 2 | 2 | 1 | 2 | 1 |
| 1 | 2 | 3 | 4 | 5 | 5 | 6 | 6 | 5 | 4 |
| 2 | 3 | 4 | 3 | 4 | 6 | 8 | 8 | 6 | 3 |
| 3 | 2 | 3 | 4 | 4 | 5 | 5 | 5 | 4 | 3 |
| 4 | 3 | 3 | 2 | 1 | 1 | 3 | 2 | 1 | 3 |

Fig. 4

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SECTION C *3D Graphical Rendering*

*Answer not more than **one** question from this section*

5 (a) Describe what is meant by *volume rendering.* Discuss its advantages and disadvantages compared to *surface rendering.* [25%]

(b) Consider a volume rendering of a 3D ultrasound data set. Explain how it is possible to obtain renderings that show

(i) vasculature [10%]

(ii) cartilage [10%]

(c) As a preliminary to volume rendering, a regular rectangular voxel array is to be constructed from a freehand 3D ultrasound scan acquired in a uniform curved trajectory. There are 91 rectangular B-scan slices; the outlines of eight of these are shown in Fig. 5. The B-scans are 30 mm by 40 mm, comprising $300 \times 400$ square pixels and the top edge of the B-scan is always parallel to its initial position. Calculate the minimum dimensions, in voxels, of the regular voxel array required if no information is to be lost from the original scan. [30%]

(d) If one byte is used to store the value of both a pixel and a voxel, calculate the ratio of the storage required for the data in the regular voxel array, divided by the storage required for the images making up the raw freehand data. (Ignore the storage required for the position information associated with the raw freehand data.) [10%]

(e) Suggest how the values of voxels that do not directly intersect with B-scan planes can be determined. [15%]

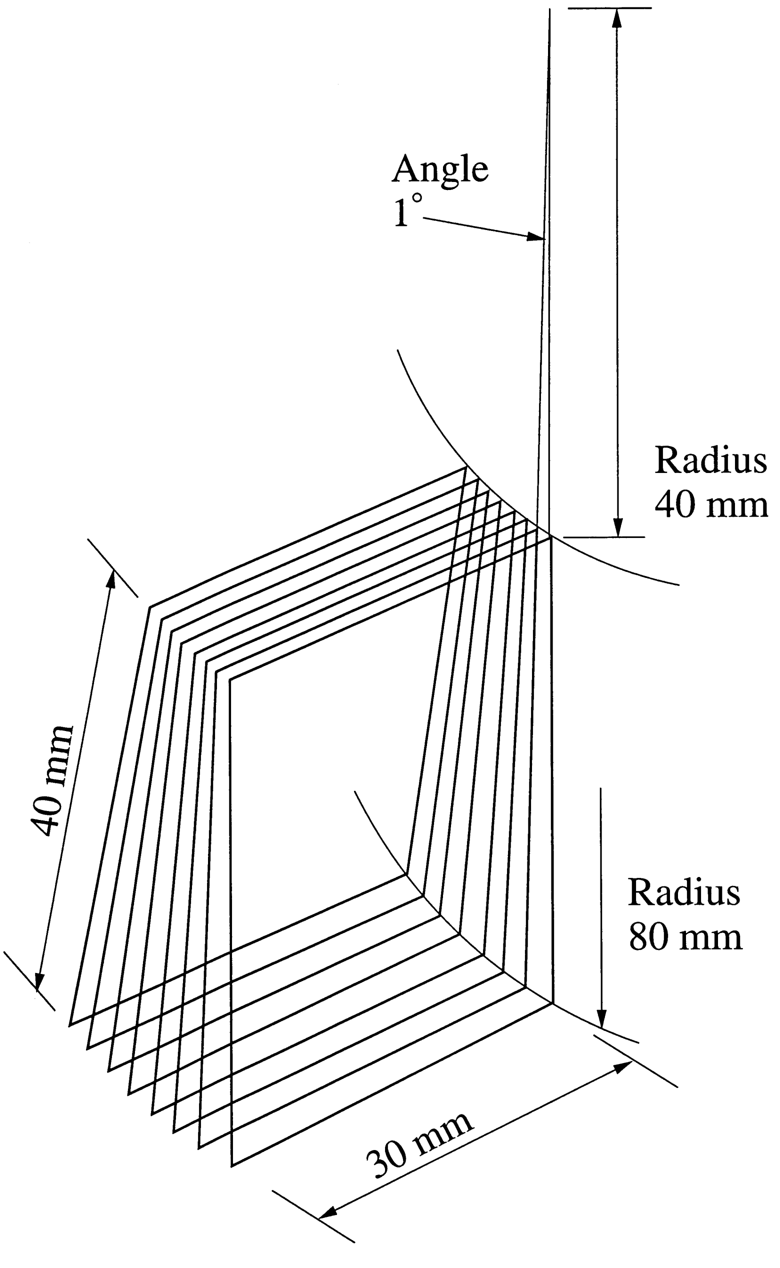


Fig. 5

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6 (a) A graphics system includes facilities for: pre-multiplying the homogeneous coordinates of a point by a projection matrix, pre-multiplying the homogeneous coordinates of a point by a modelview matrix, and performing clipping. Explain the purpose of these three operations, and specify the order in which they are performed. [20%]

(b) A user desires to construct a scene comprising a sphere of 1 cm radius, resting in the middle of a plane, 20 cm $\times$ 20 cm, aligned with the $x$ and $y$ axes, as shown in Fig. 6. The view point is located immediately above the sphere, at $(10, 10, 6)$, and the view window is arranged so as to exactly contain the 20 cm square plane. The near clipping plane is located at $z = 3$ cm, and the far clipping plane is located at $z = -2$ cm.

If the view window is $256 \times 256$ pixels, how many pixels width does the sphere occupy? [20%]

(c) An inexperienced user decides to add a shadow to the scene from a light source located at $(-40, 60, 31)$, see Fig. 6, using the shadow z-buffer algorithm. The view point and clipping planes for the shadow z-buffer are configured appropriately. The other viewing parameters, including focal length and view window dimensions, are identical to those used for the main image buffer.

(i) How many pixels width does the sphere occupy in the shadow z-buffer? [15%]

(ii) Sketch the resulting scene, including the shadow. [15%]

(iii) Discuss what would be the result if only an 8-bit shadow z-buffer was employed. [30%]

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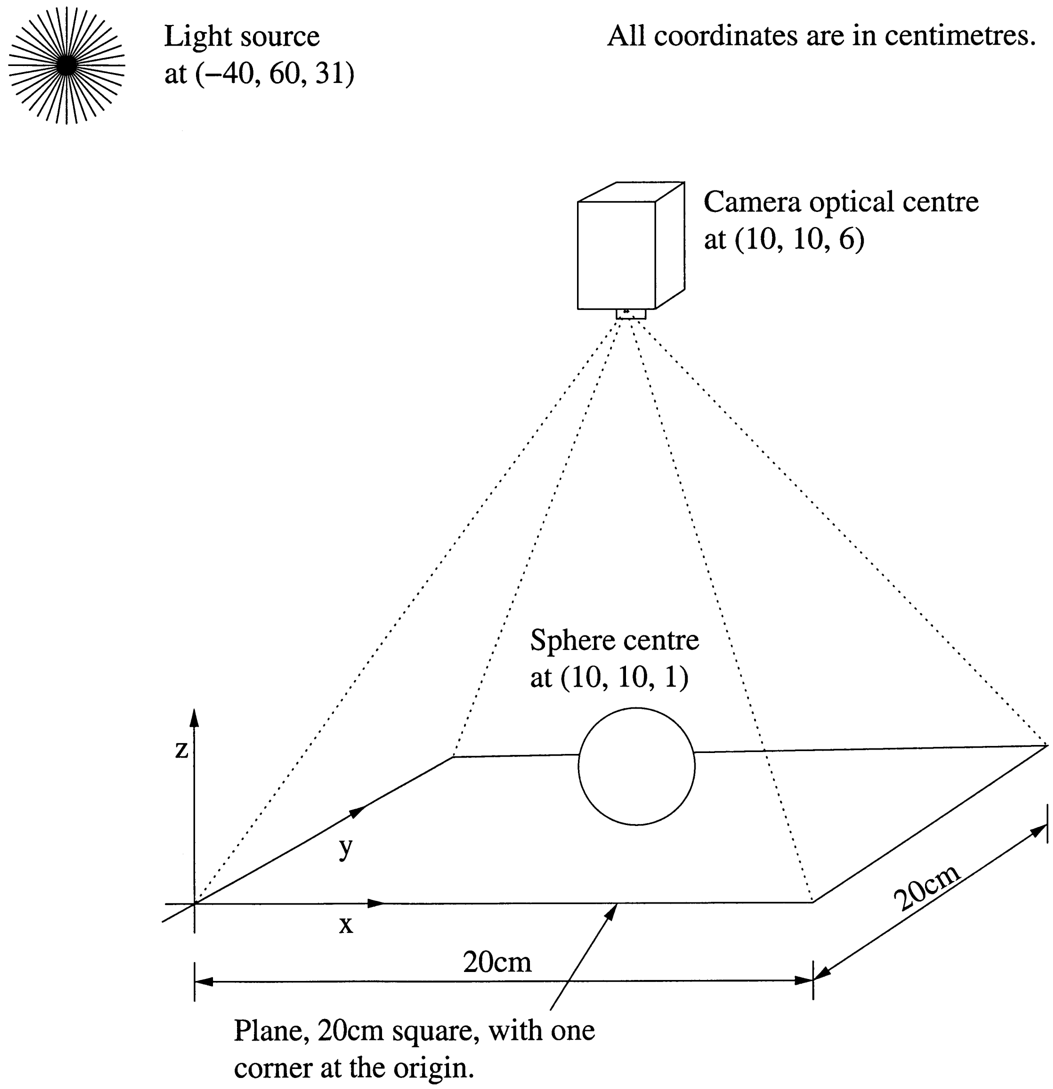


Fig. 6

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