1. (a) α - the release of a helium-4 nucleus, usually accompanied by γ-rays (EM radiation). Mass number falls by 4, atomic number by 2.

β - the emission of a -vely charged particle equivalent to an electron by the transformation of a neutron to a proton, usually accompanied by γ-rays and a neutrino. Mass number is unchanged, atomic number increases by 1.

γ - the emission of EM radiation. If alone, there is no change in mass number or atomic number. [20%]

(b) Exposure is minimized by minimizing the time of exposure, maximizing distance from the source, and the use of shielding where appropriate.

α's are highly charged and heavy and are absorbed by the outer layers of the skin (dead tissue), so they are only dangerous when ingested or inhaled. To work with α's no shielding is needed but work should be carried out in glove boxes to ensure the material cannot enter the body.

β's do require some shielding and, again, precautions must be taken to ensure that they do not enter the body. It must be remembered that when β's are attenuated by shielding they give rise to secondary radiation (bremsstrahlung, a form of EM radiation) that may require secondary shielding.

γ's are much more difficult to attenuate and can cause serious damage to living tissue even if the source does not enter the body. Significant shielding using materials such as lead is required. [30%]

(e) Activity $A = \lambda N$

N = number of atoms $= \dfrac{m N_A}{M}$

m = mass (in kg)

N_A = Avogadro's no. (per kmol)

M = mass number

$$= \frac{10^{-3}}{60} \times 6.022\times10^{26}$$

$$= 1.004\times10^{22}$$

λ = decay constant $= \ln 2 / t_{1/2}$

$$= \ln 2 \div (5.272\times365\times24\times60\times60)$$

$$= 4.169\times10^{-9}\ s^{-1}$$

$$\therefore A = \lambda N = 4.169\times10^{-9} \times 1.004\times10^{22} = \underline{\underline{4.18\times10^{13}\ Bq}}$$

Unshielded flux $\phi_0 = \underbrace{2}_{2\ \gamma\text{'s per decay}}A/(4\pi R^2)$

$\therefore$ At $R = 0.25$ m

$$\phi_0 = \frac{2\times4.18\times10^{13}}{4\pi\times0.25^2} = 1.064\times10^{14}\ m^{-2}s^{-1}$$

Shielded flux $\phi = \phi_0 e^{-\mu x}$ (here $x = R$)

$$\therefore \phi = 1.064\times10^{14} \exp(-0.046\times250)$$

$$= 1.078\times10^{9}\ m^{-2}s^{-1}$$

E (average γ energy) $= \frac{1}{2}(1.17 + 1.33) = 1.25$ MeV

Dose rate $\dot{D} = 1.6\times10^{-13} \times \dfrac{E\Sigma\phi}{\rho}$

$$= 1.6\times10^{-13} \times \frac{1.25 \times 3 \times 1.078\times10^{9}}{10^{3}}$$

$$= 6.468\times10^{-7}\ \text{Gy s}^{-1}$$

$$\equiv 2.33\ \text{mGy hr}^{-1}$$

The γ weighting factor is 1

$\therefore$ Effective dose rate $= \underline{\underline{2.33\ \text{mSv hr}^{-1}}}$

This figure is rather large (about 8 hours exposure would give annual dose limit) so the thickness of shielding should be increased.

[50%]

2. (a) $$\frac{dn}{dt} = -\nabla.\underline{j} + (\eta - 1)\Sigma_a \phi + S$$

Steady-state $\Rightarrow$ $\frac{dn}{dt} = 0$

Source-free $\Rightarrow$ $S = 0$

Fick's Law $\underline{j} = -D\nabla\phi$

$$\therefore \quad 0 = -\nabla.(-D\nabla\phi) + (\eta-1)\Sigma_a\phi$$

If D is constant

$$\therefore \quad D\nabla^2\phi + (\eta-1)\Sigma_a\phi = 0$$

$$\therefore \quad \nabla^2\phi + B_m^2\phi = 0 \qquad \text{where } B_m^2 = \frac{(\eta-1)\Sigma_a}{D}$$

The material buckling B_m^2 is a function of the material properties of the core constituents, while the geometric buckling B_g^2 is a function of the geometry of the core, e.g. for a cylindrical core $B_g^2 = \left(\frac{2\cdot405}{R_0}\right)^2 + \left(\frac{\pi}{H_0}\right)^2$. For criticality $B_g^2 = B_m^2$.

[25%]

(b) The minimum volume bare core will be <u>spherical.</u>

$$\phi = \frac{A}{r}\sin(B_g r)$$

<u>Neglecting the extrapolation distance</u>, i.e. assuming $R_0 \approx R$, the boundary condition is

$\phi = 0$ at $r = R$

$\therefore \quad B_g R = \pi$

For criticality $B_g^2 = B_m^2$

$\therefore \quad R = \pi / B_m = \pi / \sqrt{25 \times 10^{-6}} = 628 \text{ cm} = 6.28 \text{ m}$

$\therefore \quad V = \frac{4}{3}\pi R^3 = \underline{\underline{1037 \text{ m}^3}}$

[30%]

(c) Minimum volume parallelepiped is cubic.

Without reflector $\phi \propto \cos(B_g x/\sqrt{3})$

Boundary condition $\phi = 0$ at $x = X$

$$\therefore \quad \frac{B_g X}{\sqrt{3}} = \frac{\pi}{2}$$

$$\therefore \quad X = \frac{\sqrt{3}\,\pi}{2 B_g} = \frac{\sqrt{3}\,\pi}{2\sqrt{25 \times 10^{-6}}} = 544 \text{ cm} = 5.44 \text{ m}$$

From the 4A1 Data Sheet L for graphite $= 0.54$ m

$\therefore$ for reflected reactor $X' = 5.44 - 0.54 = 4.90$ m

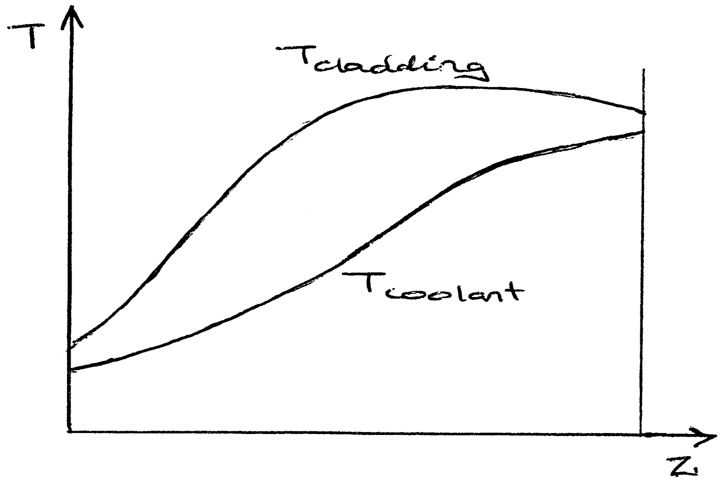
$\therefore \quad V = 8X'^3 = \underline{\underline{941 \text{ m}^3}}$

[30%]

(d) With a graphite reflector (and therefore probably moderator as well) a gaseous coolant would be chosen, e.g. CO_2 (AGR) or He (PBMR). A gaseous coolant allows high coolant temperatures to be achieved (therefore good power plant thermal efficiencies) without having to cope with a coolant phase change (as in a BWR) which makes control more difficult.

[15%]

3. (a)



Key features:

$\frac{dT}{dz} \neq 0$ at ends as power is not zero there (chopped cosine)

$T_{coolant}$ symmetric about channel centre

$T_{cladding} > T_{coolant}$ throughout

Difference $T_{cladding} - T_{coolant}$ is a "chopped cosine"

[20%]

(b) Ginn's equation (4A1 Data Sheet)

$$\Theta = \sin\left(\frac{\pi z}{2L'}\right) + Q\cos\left(\frac{\pi z}{2L'}\right)$$ [1]

Θ is a maximum when $\frac{d\Theta}{dz} = 0$

$$\therefore \frac{\pi}{2L'}\cos\left(\frac{\pi z}{2L'}\right) - \frac{\pi Q}{2L'}\sin\left(\frac{\pi z}{2L'}\right) = 0$$

$$\therefore z = \frac{2L'}{\pi}\tan^{-1}\frac{1}{Q}$$

Here $L' = 5$ m

$$Q = \frac{\pi \dot{m} c_p}{U 4\pi r_o L} \times \frac{L}{L'} = \frac{\dot{m} c_p}{U 4 r_o L'}$$

Need $\dot{m}c_p$. This can be found from the overall channel power $\dot{m}c_p(T_{out} - T_{in}) = P$ [2]

$$\therefore \dot{m}c_p = \frac{P}{T_{out} - T_{in}} = \frac{10\times10^6}{635 - 335} = 33.3\times10^3\ \mathrm{WK^{-1}}$$

$$\therefore Q = \frac{33.3\times10^3}{5\times10^3\times4\times0.4\times5} = 0.8333$$

$$\therefore Z = \frac{2L'}{\pi}\tan^{-1}\frac{1}{Q} = \frac{2\times5}{\pi}\tan^{-1}\frac{1}{0.8333} = 2.789\ \mathrm{m}$$

i.e. 2.789 m past the channel centre

$\therefore$ Using [1]

$$\Theta_{max} = \sin\frac{2.789\pi}{2\times5} + 0.8333\cos\frac{2.789\pi}{2\times5} = 1.302$$

$$\Theta = \frac{T - T_{1/2}}{T_{out} - T_{1/2}}\sin\frac{\pi L}{2L'}$$

$$\therefore T = T_{1/2} + \frac{\Theta}{\sin\frac{\pi L}{2L'}}\left(T_{out} - T_{1/2}\right)$$

Here $T_{1/2} = 485°\mathrm{C}$ (average of T_{in} and T_{out})

$$\therefore T_{max} = 485 + \frac{1.302}{\sin\frac{4\pi}{2\times5}}(635 - 485)$$

$$= \underline{\underline{690\ °\mathrm{C}}}$$

[60%]

It is also acceptable to find Θ_{max} using the relation $\Theta_{max}^2 = 1 + Q^2$ (if remembered)

(c) (i) From [2] for same T_{out} we need $P/\dot{m}c_p$ to be constant. As c_p is unchanged if P is halved we need to halve (50% reduction) $\dot{m}$.

(ii) If T_{in} and T_{out} are the same, T_{max} depends only on Θ_{max} (and therefore Q). As $\dot{m}c_p$ is halved, Q is halved $\Rightarrow$ Θ_{max} is lower $\Rightarrow$ T_{max} is lower.

[20%]

4. (a)

I	Iodine-135 concentration
X	Xenon-135 concentration
γ_i	Iodine-135 yield fraction from fission
Σ_f	macroscopic fission cross-section
ϕ	neutron flux
λ_i	Iodine-135 decay constant
λ_x	Xenon-135 decay constant
σ	Xenon-135 microscopic capture cross-section
t	time

[15%]

(b) In steady state $\frac{dI}{dt} = 0$, so the steady-state Iodine concentration

$$I_o = \frac{\gamma_i \Sigma_f \phi}{\lambda_i}$$

and for the Xenon

$$X_o = \frac{\lambda_i I_o}{\lambda_x + \sigma\phi} = \frac{\gamma_i \Sigma_f \phi}{\lambda_x + \sigma\phi}$$

For high power (high flux) $\phi \to \infty$

$$\therefore \quad X_o \to \frac{\gamma_i \Sigma_f}{\sigma}$$

The reactivity loss due to Xenon is given by

$$\rho_x = -\frac{\sigma X_o}{\nu \Sigma_f}$$

$$\therefore \quad \rho_o \to -\frac{\gamma_i}{\nu}$$

[15%]

(c) At shutdown $\phi = 0$

$$\therefore \quad \frac{dI}{dt} = -\lambda_i I$$

$$\therefore \quad I = I_o e^{-\lambda_i t}$$ by inspection

$$\frac{dX}{dt} + \lambda_x X = \lambda_i I = \lambda_i I_0 e^{-\lambda_i t}$$

PI $\quad X_p = A e^{-\lambda_i t}$ by inspection

$\therefore \quad A = \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i}$

CF $\quad X_c = B e^{-\lambda_x t}$ by inspection

$\therefore$ GS $\quad X = \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i} e^{-\lambda_i t} + B e^{-\lambda_x t}$

BC $\quad X = X_0$ at $t = 0 \quad \Rightarrow \quad B = X_0 - \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i}$

$$\therefore \quad X = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} e^{-\lambda_i t} + \left(X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \right) e^{-\lambda_x t}$$

Substituting for I_0 and X_0

$$X = \frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i} e^{-\lambda_i t} + \left(\overbrace{\frac{\gamma_i \Sigma_f \phi}{\lambda_x + \sigma\phi}}^{C} - \overbrace{\frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i}}^{D} \right) e^{-\lambda_x t}$$

If $|\lambda_x + \sigma\phi| \gg |\lambda_i - \lambda_x| \quad C \ll D$, so

$$X \approx \frac{\gamma_i \Sigma_f \phi}{\lambda_i - \lambda_x} \left\{ e^{-\lambda_x t} - e^{-\lambda_i t} \right\}$$

[40%]

(d) X is maximised when $\dfrac{dX}{dt} = 0$

$$\therefore \quad \lambda_x e^{-\lambda_x t} = \lambda_i e^{-\lambda_i t}$$

$$\therefore \quad \ln \lambda_x - \lambda_x t = \ln \lambda_i - \lambda_i t$$

$$\therefore \quad t = \frac{\ln(\lambda_i / \lambda_x)}{\lambda_i - \lambda_x}$$

$$\therefore \quad \rho_{max} = -\frac{\sigma X_{max}}{\nu \Sigma_f}$$

$$= -\frac{\sigma \gamma_i \phi}{\nu(\lambda_i - \lambda_x)} \left\{ e^{-\frac{\lambda_x}{\lambda_i - \lambda_x} \ln \frac{\lambda_i}{\lambda_x}} - e^{-\frac{\lambda_i}{\lambda_i - \lambda_x} \ln \frac{\lambda_i}{\lambda_x}} \right\}$$

$$\therefore \quad \frac{\ell_{max}}{\ell_0} = \frac{\sigma\phi}{\lambda_i - \lambda_x} \left\{ \left(\frac{\lambda_x}{\lambda_i}\right)^{\frac{\lambda_x}{\lambda_i - \lambda_x}} - \left(\frac{\lambda_x}{\lambda_i}\right)^{\frac{\lambda_i}{\lambda_i - \lambda_x}} \right\}$$

$$= \frac{\sigma\phi}{\lambda_i - \lambda_x} \left(\frac{\lambda_x}{\lambda_i}\right)^{\frac{\lambda_x}{\lambda_i - \lambda_x}} \left[1 - \frac{\lambda_x}{\lambda_i} \right]$$

$$= \underline{\underline{\frac{\sigma\phi}{\lambda_i} \left(\frac{\lambda_x}{\lambda_i}\right)^n}}$$

where $n = \lambda_x / (\lambda_i - \lambda_x)$ [40%]