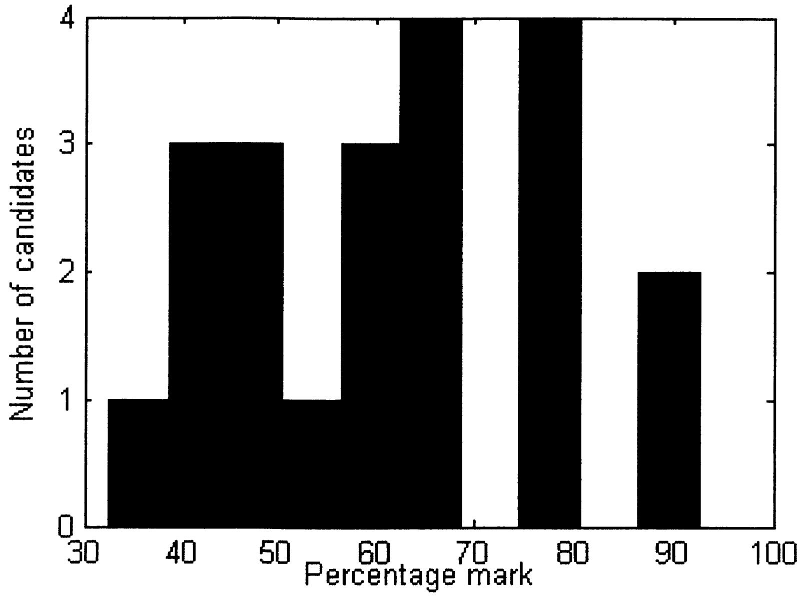
4A3 2004 SOLUTIONS

The course had 26 candidate (21 undergraduate and 5 graduate). The course is split 75% exam 25% course work. The mean mark for the exam was 61.1% with a standard deviation of 15.7%. The course work was well undertaken with candidates obtaining an average mark of 64.1% and a standard deviation of 11.8%. The distribution of undergraduate marks in the exam is given below.

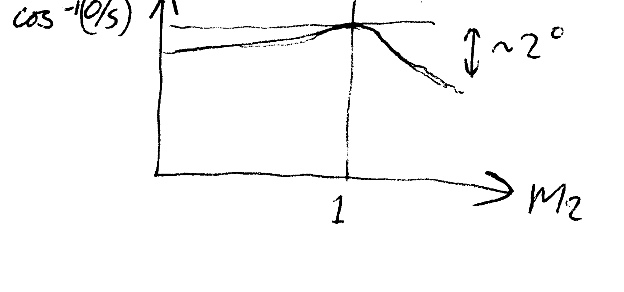


Question 1 (MARK OUT OF 20)

Only 7 candidates attempted this question. The mean mark was 52.5%. The candidates seemed to have been put off this question by the statement in part (c) 'Note a trial and error or a graphical solution is expected'. The question was very similar to work that had been given in detail in this years 4A3 notes. Part (b) most candidates completed this section in full. Part (c) was not well answered with a significant number of candidates missing out the ratio of total pressures (throat to trailing edge) from the calculation. No candidate got a correct solution to part (d).

1 A $\quad Y_P = \dfrac{P_{o1} - P_{o2}}{P_{o2} - P_2}$ [1]

Y_P RISES AS MACH → 1 THIS INCREASE IN LOSS IS A STRONG FUCTION OF TRAILING EDGE THICKNESS. THE LOSS OCCURS BOTH DUE TO COMPLEX FLOW STRUCTURES AT BLADE TE AND THE TRAILING EDGE SHOCK [1]



$\Pi_{m_{TH}} = F(1) \qquad \Pi_{m_{EXIT}} = F(M_2)$

SAME $\dot{m}$, C_P, T_o AT BOTH PLANES

SO α_2 CHANGED BY M_2 AND P_{o2} [1]

Q1/

a)

$Y_P = 0.05, \quad M_2 = 1.0$

$P_2 / P_{02} = .52828$

$$\frac{P_{01} - P_{02}}{P_{02} - P_2} = .05 = \frac{P_{01} - P_{02}}{P_{02}(1 - .52828)}$$

$\rightarrow \; .023586 \, P_{02} = P_{01} - P_{02}$

$\rightarrow \; P_{02} = .97696 \; P_{01}$ [1]

$$\frac{P_2}{P_{01}} = .52828 \times .97696 = .51611$$

$$\left(\frac{T_2}{T_{01}}\right)_{IS} = (.51611)^{\frac{\gamma-1}{\gamma}} = .82781$$

$$\frac{T_2}{T_{01}} = (.52828)^{\frac{\gamma-1}{\gamma}} = .83334$$ [1]

$$\xi = \frac{(T_{01} - T_{2s}) - (T_{01} - T_2)}{(T_{01} - T_{2s})}$$

$$= \frac{(1 - .82781) - (1 - .83334)}{(1 - .82781)}$$

$$\xi = \underline{\underline{.0321}}$$ [1]

b)

At throat $\dot{m}\sqrt{C_p T_0}/(O \cdot P_{0t}) = F(1)$

Downstream $\dot{m}\sqrt{C_p T_0}/(s \cos\alpha_2 P_{02}) = F(M_2)$

Q1 (b) CONT

At $M_2 = 1.0$, $\cos\alpha_2 = \frac{o}{s}\frac{P_{0t}}{P_{02}}$ [1]

$P_{02} = 0.97696\, P_{01}$

$P_{0t} = P_{01} - \frac{1}{3}(P_{01} - P_{02}) = .99232\, P_{01}$ [2]

$\cos\alpha_2 = 0.3 \times \frac{.99232}{.97696}$

$\rightarrow \alpha_2 = \underline{\underline{72.25^\circ}}$ [1]

(1)(c) LIMIT LOAD IS WHEN AXIAL MACH No $= 1$ AT THIS POINT THE BLADE BACK PRESSURE IS INDEPENDANT OF EXIT CONDITIONS. [1]

UPSTREAM OF THE THROAT IS NOT EFFECTED BY EXIT CONDITIONS. THE VELOCITY DOWNSTREAM OF THE THROAT CLOSE TO THE SUCTION SURFACE RISES. PROFILE LOSS $\propto V^3$ SO LOSS ON LATE SUCTION SURFACE RISES. [1]

c/ At the limit load $M_2 \cos\alpha_2 = 1.0$

$M_2 = \frac{1}{\cos\alpha_2}$

and $\frac{\dot{m}\sqrt{C_p T_0}}{s\cos\alpha_2\, P_{02}} = F(M_2)$

$\frac{\dot{m}\sqrt{C_p T_0}}{o \cdot P_{0t}} = F(1)$. [1]

$\rightarrow \cos\alpha_2 = \frac{o}{s}\frac{P_{0t}}{P_{02}} \frac{F(1)}{F(1/\cos\alpha_2)}$

① ⓒ CONT

P_{0t} = same as before = .99232 P_{01}

$P_{02} = P_{0t} - 2 \times (P_{0t} - P_{02})_{at\ M=1}$

→ $P_{02} = 0.9616\ P_{01}$ [1]

→ $\cos\alpha_2 = 0.3 \times \frac{.99232}{.9616} \times 1.281 \times \frac{1}{F\left(\frac{1}{\cos\alpha_2}\right)}$

→ $\cos\alpha_2 = 0.3966 \frac{1}{F(1/\cos\alpha_2)}$ [1]

α_2	$\cos\alpha_2$	$\frac{.3966}{F(1/\cos\alpha_2)}$
65°	.4226	.744
60°	0.5	.5225
59°	.515	.497
→ 59.5°	.5075	.509

[2]

V. Closely $\alpha_2 = \underline{\underline{59.5°}}$

$M_2 = \frac{1}{\cos\alpha_2} = \underline{\underline{1.98}}$ [1]

d) For the stator $\Delta S = -R \ln \frac{P_{02}}{P_{01}}$ [1]

$\Delta S = -287.5 . \ln(.97696) = 6.7$ J/Kg.K.

For the turbine $\Delta h_0 = 0.3 \times C_p T_{01} \Leftarrow \frac{C_p}{T_{01}}(1-0.$

$T_{02} \Delta S_{stat} = 0.7 T_{01} \times 6.7$

$\Delta\eta_{stator} = \frac{0.7 \times 6.7}{0.3 \times 1005} = \underline{1.55\%}$ [1]

Question 2

21 candidates attempted this question. The mean mark was 62.2%. All candidates completed part (a) and made a good attempt at part (b). Part (c) was very poorly answered. Half the candidates answered part (d) correctly. There were no complete answers to part (e) but a significant number of candidates outlined the correct method. Overall this was a well answered question with a good distribution of solutions.

(1) (A)

$$R_{TT} = \frac{h_{o2s} - h_{o1}}{h_{o2} - h_{o1}}$$ (1)

FOR A COMPRESSOR

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma R_p}}$$ [1]

ISENTROPIC EFFICIENCY IS A MEASURE OF THE RATIO OF REAL TO IDEAL WORK. POLYTROPIC EFFICIENCY IS CALCULATED BY SPLITTING THE STAGE UP INTO SMALL ΔP CHANGES AND USING THE LOCAL EXIT CONDITIONS FOR EACH STAGE.

THE DIFFERENCE BETWEEN THE TWO INCREASES WITH PRESSURE RATIO AND IS DUE TO THE REHEAT EFFECT [1]

[NB] ISENTROPIC EFFICIENCIES ARE WHAT DETERMINE THE OVERALL WORK INPUT OR OUTPUT OF A MACHINE.

Q2/ (A)

$$\frac{P_{02}}{P_{01}} = 4 \ , \quad \frac{T_{02IS}}{T_{01}} = 4^{\frac{\gamma-1}{\gamma}} = 1.486$$

$$\Delta T_{01S} = .486\, T_{01}$$

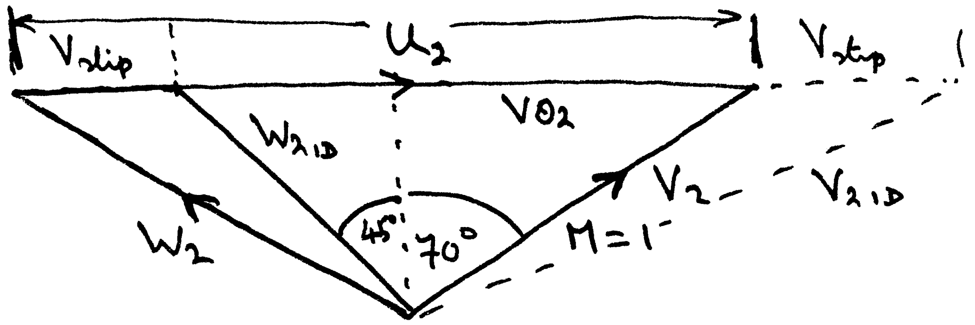
$$\Delta T_0 = \frac{.486}{.8} T_{01} = .6074\, T_{01}$$

$$\frac{T_{02}}{T_{01}} = 1.6074 = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_P}}$$

$$\rightarrow \eta_P = \underline{0.8345}$$

[1]

(B)



[2]

$$V_{\theta 2} = V_2 \sin 70^\circ$$

$$V_{\theta 2ID} = \frac{1}{\sigma} V_{\theta 2}$$

$$V_{slip} = V_{\theta 2ID} - V_{\theta 2} = V_2 \sin 70^\circ \left(\frac{1}{\sigma} - 1\right)$$

[2]

$$U_2 = V_2 \cos 70^\circ \left(\tan 70^\circ + \tan 45^\circ\right) + V_{slip}$$

$$U_2 = V_2 \cos 70^\circ \left(\tan 70^\circ + \tan 45^\circ + \frac{1}{\sigma} - 1\right)$$

$$=$$

377.5 + 137.4 + 66.6 = 581.5

$$M_2 = 1 \rightarrow V_2 = .577 \sqrt{C_p T_{02}}$$

$$= .577 \sqrt{1005 \times 1.6074 \times 300}$$

$$= 401.68 \text{ m/s.}$$

[1]

Q2

$U_2 = \Omega r_2 \rightarrow r_2 =$ 0.222m [1]

(c) At diffuser L.E. $M = 0.8$, $P/P_{0_{LE}} = 0.656$

$\Delta P_{is} = (1 - 0.656) P_{0LE}$

$\Delta P = 0.7 \times (1 - .656) P_{0LE} = .2408 P_{0LE}$. [2]

$\Delta P = .2408 / .656 P_{LE} = .367 P_{LE}$.

$\therefore 1.367 P_{LE} = 4$ bar.

$P_{LE} = 2.926$ bar. [2]

$T_{LE} = T_{0_2} \times (.656)^{\frac{\gamma-1}{\gamma}} = .8865 T_{02}$

(D) ṁ CONST + AREA INCREASES (Vr DROP) AND ANGULAR MOMENTUM CONSTANT (Vθ DROP) SO V DROPS

At impeller exit, $M_2 = 1.0 \rightarrow T_{TE} = .8333 T_{02}$

Between impeller exit and diffuser LE.

$$\frac{T_{LE}}{T_{TE}} = \left(\frac{P_{LE}}{P_{TE}}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_p}}$$ and $\eta_p = 0.7$ [2]

$\rightarrow P_{TE} = 0.859 P_{LE}$

$\rightarrow P_{TE} = \underline{2.515}$ bar.

$$\rho_{TE} = \frac{P_{TE}}{RT_{TE}} = 2.1802 \text{ Kg/m}^3$$

$\dot{m} = 2\pi r \rho V_r \times W = 10$ Kg/s. [2]

$V_r = V_2 \cos 70° = 137.38$ m/s.

$$\therefore W = \frac{10}{2\pi \times .2059 \times 3.498 \times 137.38} =$$ .00239m

Q2

(E) For the impeller alone

$P_{TE} = 2.515$ bar , $P/P_0 = .52828$

$P_{0_{TE}} = 4.761$ bar.

$T_{0_{TE,IS}} = 1.5617\ T_{01}$ [1]

$\Delta T_0 = .6074\ T_{01}$

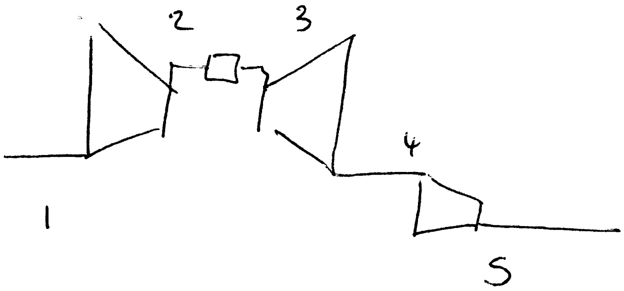
$\Delta T_{0IS} = .5617\ T_{01}$

$\eta_{TT} = 92.48\ \%$ [1]

Question 3

24 candidates attempted this question. The mean mark was 63.9%. Part (a) of the question only carried 10% of the mark but a significant number of students wasted time by giving a solution that was more detailed than necessary. Part (b) and (c) was extremely well answer with most candidates producing good solutions. Part (d) was badly answered with most candidates not showing how the locus of the operating line could be calculated. Full marks required the candidate to outline the construction method of the locus. Around a third of candidates answered part (e) correctly.

(3)(A)



$$\frac{\dot{m}\sqrt{C_{pe} T_{05}}}{P_{05} A_5} = \frac{\dot{m}\sqrt{C_{pe} T_{03}}}{P_{03} A_3}$$ [1]

CONSTANT BECAUSE BOTH CHOKE

$\dot{m} = \text{CONST}$

$$\frac{\sqrt{T_{03}}}{P_{03} A_3} = \frac{\sqrt{T_{05}}}{P_{05} A_5}$$

IF NO HEAT LOSS $T_{05} = T_{04}$

IF ISENTROPIC NOZZLE

$P_{05} = P_{04}$

$$\sqrt{\frac{T_{03}}{T_{04}}} = \left(\frac{A_3}{A_5}\right)\frac{P_{03}}{P_{04}}$$

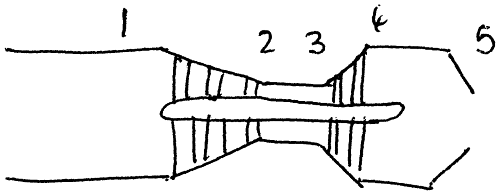
$$\frac{P_{03}}{P_{04}} = f\left(\frac{T_{03}}{T_{04}}, R\right)$$

FOR CONSTANT R [1]

$\frac{T_{03}}{T_{04}}$ FIXED BY $\frac{A_3}{A_5}$

(3) (B) TURBINE AND NOZZLE

$$1.389 = \frac{\dot{m}\sqrt{C_{pe} T_{05}}}{P_{05} A_5} = \frac{\dot{m}\sqrt{C_{pe} T_{03}}}{P_{03} A_3}$$



$$P_{04} = P_{05}$$

$$T_{04} = T_{05}$$

$$\frac{\sqrt{T_{04}}}{P_{04} A_5} = \frac{\sqrt{T_{03}}}{P_{03} A_3} \quad \therefore \left(\frac{P_{04}}{P_{03}}\right) = \left(\frac{A_3}{A_5}\right)^2 \times \left(\frac{P_{04}}{P_{03}}\right)^{R_p \frac{\gamma}{\gamma}}$$

$$\frac{P_{04}}{P_{03}} = \left(\frac{A_3}{A_5}\right)^{1.11} \text{ AND } \left(\frac{T_{04}}{T_{03}}\right) = \left(\frac{A_3}{A_5}\right)^{0.232}$$

NB $\frac{A_3}{A_5} = \frac{T_{04}}{T_{03}}^{\left(\frac{\gamma}{R(\gamma-1)}\right)}$

WORK OUT TURBINE = WORK IN COMPRESSOR

[3]

$$T_{03} - T_{04} = T_{03}\left(1 - \frac{T_{04}}{T_{03}}\right) = T_{03} \times k$$

$$k = 1 - \left(\frac{A_3}{A_5}\right)^{0.232}$$

$$C_p (T_{02} - T_{01}) = C_{pe} k T_{03}$$

[2]

$$T_{02} = 288 \times (8)^{\frac{0.4}{1.4 \times 0.9}} = 557.3\,k$$

$$k = \frac{C_p}{C_{pe}} \frac{(T_{02} - T_{01})}{T_{03}} = \frac{1005}{1224} \frac{(557.3 - 288)}{1500}$$

$$k = 0.1474 \qquad \frac{A_3}{A_5} = \underline{\underline{0.503}}$$

[2]

3 CONT

(c)

WORK COMP = WORK TURB

$$C_p(T_{02}-T_{01}) = C_{pe}\, k\, T_{03} \qquad \text{WHERE} \quad k = \left[1-\left(\frac{A_3}{A_S}\right)^{0.232}\right]$$

FROM PREVIOUS SECTION

EQU (1)
$$\frac{P_{02}}{P_{01}} = \left(1-\frac{T_{02}-T_{01}}{T_{01}}\right)^{R_p\frac{\gamma}{\gamma-1}} = \left(1+k\,\frac{C_{pe}}{C_p}\,\frac{T_{03}}{T_{01}}\right)^{R_p\frac{\gamma}{\gamma-1}} \quad (*)$$

[2]

EQU (2)
$$\frac{\dot m\sqrt{C_pT_{01}}}{P_{01}A_1} = \frac{\dot m\sqrt{C_{pe}T_{03}}}{P_{03}A_3}\times\sqrt{\frac{T_{01}}{T_{03}}}\times\frac{A_3}{A_1}\times\frac{P_{02}}{P_{01}}$$

[2]

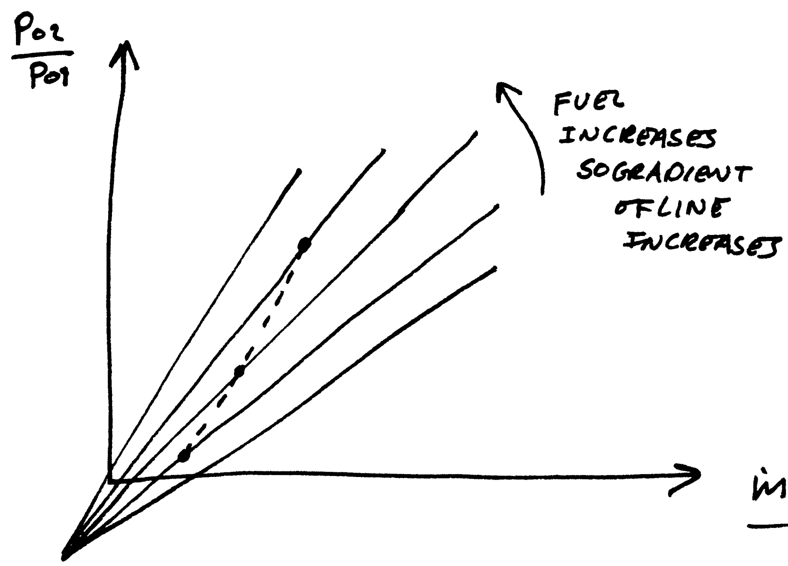
SUB IN (*)

EQU (3)
$$\frac{\dot m\sqrt{C_pT_{01}}}{P_{01}A_1} = \underbrace{1.389\times\frac{A_3}{A_1}}_{B}\times\sqrt{\frac{T_{01}}{T_{03}}}\times\left(1-\underbrace{\frac{C_{pe}k}{C_p}}_{C}\frac{T_{03}}{T_{01}}\right)^{R_p\frac{\gamma}{\gamma-1}}$$

[1]

$$B = 1.389\,\frac{A_3}{A_1} \qquad C = \frac{C_{pe}}{C_p}\left[1-\left(\frac{A_3}{A_S}\right)^{0.232}\right]$$

(d)



$$\frac{\dot m\sqrt{C_pT_{01}}}{P_{01}A_1}$$

[2]

EQU (2) SHOWS

$$\frac{\dot m\sqrt{C_pT_{01}}}{P_{01}A_1} \propto \sqrt{\frac{T_{01}}{T_{03}}}\times\frac{P_{02}}{P_{01}}$$

③ ⓓ FOR A SET T_{03} AND A LOWER COMPRESSOR EFFICIENCY

$$\frac{P_{02}}{P_{01}} = \left(1 + k\frac{C_{pe}}{C_p}\frac{T_{03}}{T_{01}}\right)^{R_p\frac{\gamma}{\gamma-1}}$$

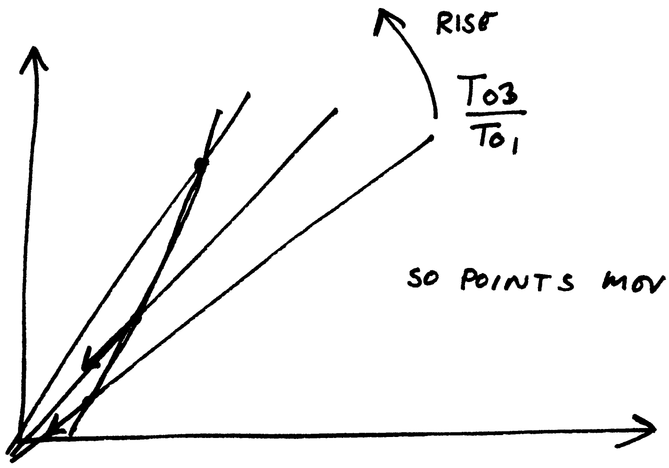
SHOWS $\frac{P_{02}}{P_{01}}$ DROPS

FROM EQUATION ②

$$\frac{\dot{m}\sqrt{C_p T_{01}}}{P_{01}A_1} \propto \sqrt{\frac{T_{01}}{T_{03}}} \times \frac{P_{02}}{P_{01}}$$

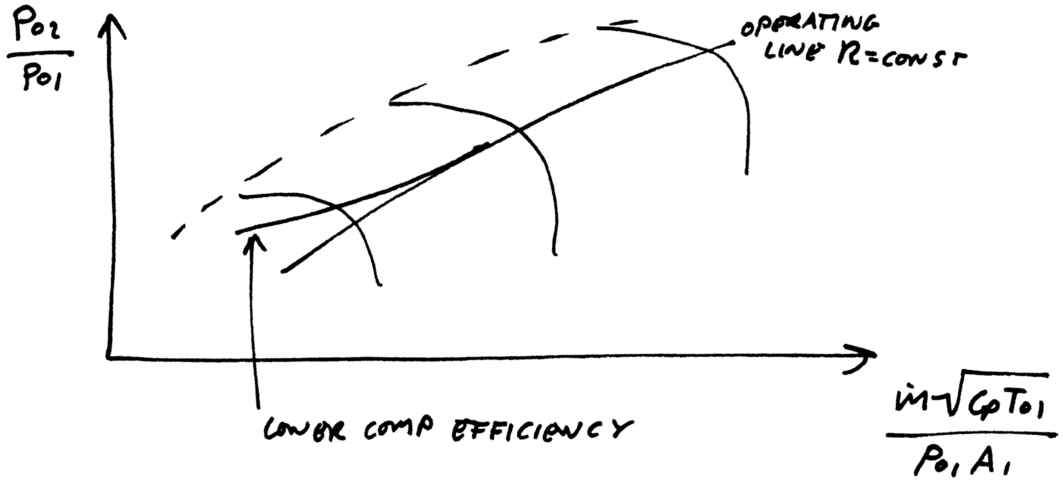
1

SO $\frac{\dot{m}\sqrt{C_p T_{01}}}{P_{01}A_1}$ DROPS BUT ALONG STRAIGHT LINE



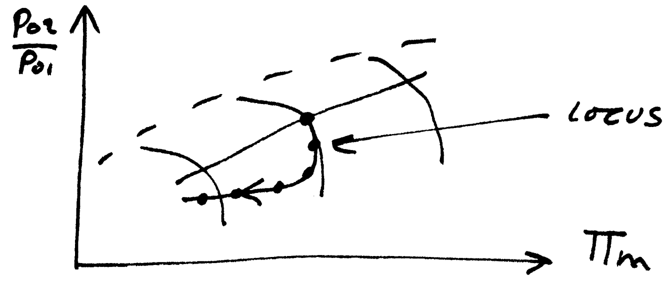
SO POINTS MOVE TO LEFT

1



ⓔ ENGINE CAN'T DECELERATE QUICKLY SO $N/\sqrt{T}$ = CONST WHILE $\frac{P_{02}}{P_{01}}$ DROPS.

MOVES AWAY FROM SURGE LINE SO SAFE.



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