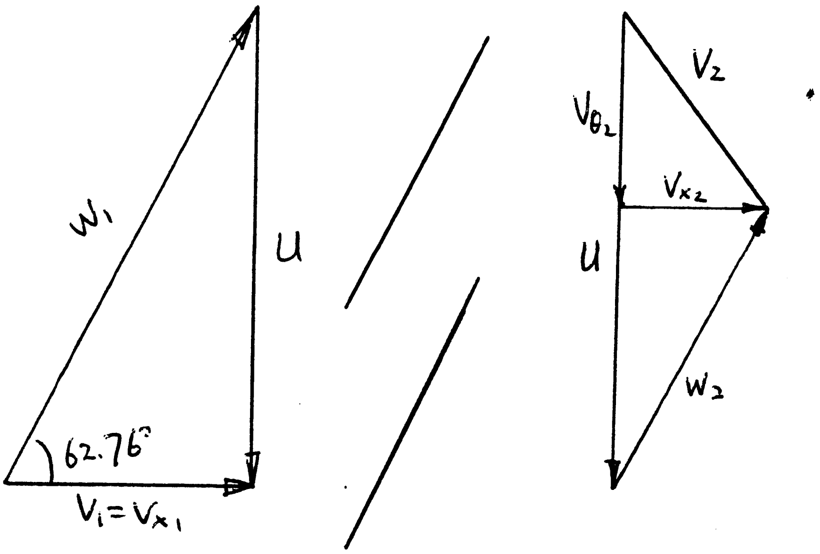
Solution:

1.

a).



b.) from table. $P_2/p_1 = 2.1858$ $\quad \sigma = 0.9531$

$P_1 = 0.7528 P_{01} = 76281.2$ Pa

$T_1 = 0.9221 T_{01} = 265.56$ K $\qquad a_1 = \sqrt{\gamma R T_1} = 326.65$ m/s

$P_{01}^{rel} = \frac{76281.2}{0.3055} = 249692.96$ Pa $\qquad U = 1.2625 \cdot 326.65 = 412.4$ m/s

$T_{01}^{rel} = 265.56/0.7126 = 372.27$ K

$T_{02}^{rel} = T_{01}^{rel} = 372.27$ K

$P_{s2} = 2.1858 \cdot 76281.2 = 166735.45$ Pa

$T_{s2} = 1.2676 \cdot 265.56 = 336.62$ K $\qquad a_2 = \sqrt{\gamma R T_2} = 367.8$ m/s

$V_{\theta 2} = U\left(1 - \frac{W_2}{W_1}\right) = U\left(1 - \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}\right) = 173.24$ m/s

$U V_{\theta 2} = 71445.0$ m²/s²

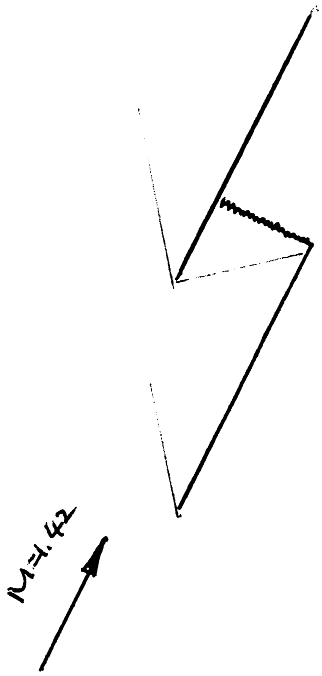
$T_{02} = T_{01} + U V_{\theta 2}/C_p = 288 + 71445/1004.5 = 71.12 + 288 = 359.12$ K

$P_{02} = P_2 \left(\frac{T_{02}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = 209111$ Pa, $\quad \pi = 2.064$

$T_{02,is} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 354.23$, $\quad \eta_{is} = \frac{354.23 - 288}{359.12 - 288} = \frac{66.23}{71.12} = 0.9312$

$\therefore \pi_s = 2.1858$, $\pi = 2.064$, $\eta_{is} = 0.9312$

c).



d).

tangential momentum equation:

$$(\rho_1 V_{x_1} \text{pitch})\cdot(V_{\theta 2}-V_{\theta 1}) = \int_0^1 \Delta p \cdot dx \cdot C_x \qquad (1)$$

$$\int_0^1 \Delta p \cdot dx = (1-K)(P_2 - P_1) = P_1(1-K)(\pi_s - 1)$$

$$\frac{C_x}{\text{Pitch}} = \frac{C\cdot\cos\beta}{\text{Pitch}} = \sigma\cdot\cos\beta \quad , \quad V_{\theta_1} = 0$$

(1) becomes.

$$V_{\theta 2}/V_{x_1} = \frac{P_1}{\rho V_{x_1}^2}\cdot(1-K)(\pi_s-1)\cdot\sigma\cdot\cos\beta = \frac{(1-K)(\pi_s-1)\,\sigma\cos\beta}{\gamma M_1^2}$$

$$\tan\alpha_2 = \cancel{tg\alpha_2} = \frac{V_{\theta 2}}{V_{x2}} = \frac{V_{\theta 2}}{V_{x1}}\cdot\frac{V_{x1}}{V_{x2}} = \frac{W_1}{W_2}\cdot\frac{(1-K)(\pi_s-1)\sigma\cdot\cos\beta}{\gamma M_1^2} = (1-K)(\pi_s-1)\frac{\sigma\cos\beta}{M_1^2}\cdot\frac{M_1^{rel}}{M_2^{rel}}\sqrt{\frac{T_1}{T_2}}$$

end. q1.

2.

a). For free vortex design. $\frac{\partial r V_\theta}{\partial r} = 0$. with uniform inlet and cyclindrical hub and casing line. $V_r << 1$ and $\frac{\partial V_r}{\partial r} << 1$ the only terms left in streamline curvature ~~of~~ equation are $\frac{\partial p}{\partial r}$ and $\rho \frac{V_\theta^2}{r}$, thus simple radial equilibrium can be used.

The ~~[illegible]~~ flow parameters at midspan:

$V_1 = 10\ m/s$. $V_2 = V_{max} = V_x/\cos\beta_2 = 30\ m/s$. ⇒ flow is ~~incompress~~ incompressible. $V_{\theta\,mid} = \cancel{V_x/tg\beta} \; V_x \cdot tg\beta_2 = 28.3\ m/s$

$rV_{\theta\,mid} = rV_\theta(r) = 12.73\ m^2/s$

from radial equilibrium. $\frac{dp}{dr} = \rho \frac{V_\theta^2}{r}$

$$\int dp = \int \rho \frac{(rV_\theta)^2}{r^3} dr = -\rho (rV_\theta)^2_{mid} \frac{1}{2r^2}\Big|_{r_h}^{r_t}$$

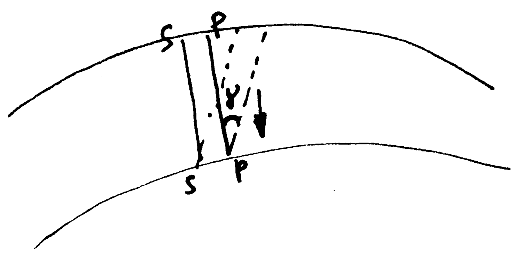
$$P(r) = P_{hub} + \frac{\rho (rV_\theta)^2_{mid}}{2}\left[\frac{1}{r_h^2} - \frac{1}{r^2}\right]$$

$$P_{tip} - P_{hub} = \Delta p = \frac{\rho (rV_\theta)^2_{mid}}{2} \cdot \frac{r_t^2 - r_h^2}{r_h^2 r_t^2} = \frac{\rho (rV_\theta)^2_{mid} \cdot r_{mid} \cdot h}{r_h^2 r_t^2}$$

$$\frac{\Delta P}{h} = \frac{\rho (rV_\theta)^2_{mid} \cdot r_{mid}}{r_h^2 \cdot r_t^2} \quad ; \quad r_{mid} = 0.45,\ r_h = 0.4\ \ r_t = 0.5$$

$$\frac{\Delta P}{h} = \frac{223.515}{0.1} = 2235.15\ kg/m^3$$

b). By leaning the blade, the radial component of the blade force is no longer zero and it acts to balance the centripedal accelaration term $\rho \frac{V_\theta^2}{r}$. thus the static pressure difference between the hub and the tip is off-set. However the blade should be leant circumferentially in such a way to generate downward force; that is, to lean the blade to its pressure surface: Reaction at hub is increased as the static pressure there increases.



c). A bodyforce represents the radial component of the blade force should be added. The mean blade force in circumference direction is $\overline{\Delta p}|_c$, its component in r direction is ~~$\frac{\overline{\Delta p}|_c}{s} \cdot h$~~ $\overline{\Delta p}|_c \cdot \frac{h}{s} \cdot tg\gamma$. h is the blade height and S the blade pitch. γ is the blade lean angle. thus the modified radial equilibrium equation takes form.

$$\rho \frac{V_\theta^2}{r} = \frac{dp}{dr} + \frac{\overline{\Delta p}|_c}{s} h \cdot tg\gamma$$

d). to reduce the $\Delta p|_{HT} = P_{tip} - P_{hub}$ by 50% ;

$$\overline{\Delta p}|_c \cdot \frac{h}{s} tg\gamma = \frac{1}{2} \Delta P|_{HT}$$

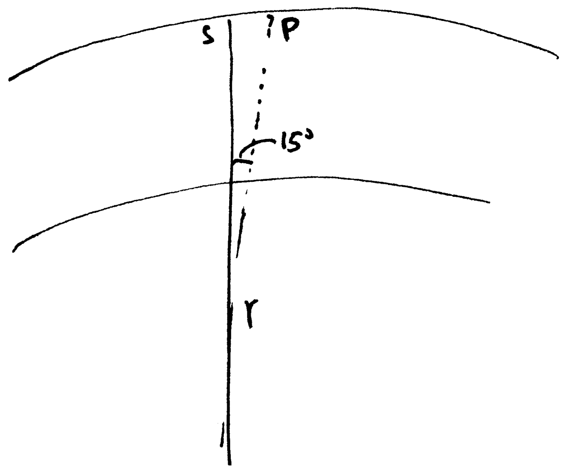
from tangential momentum equation at midspan:

$$S \cdot \rho V_x \cdot V_\theta = \overline{\Delta p}|_c \cdot C_x \quad \Rightarrow \quad \overline{\Delta P}|_c = 468.95 \text{ Pa}$$

$$\tan\gamma \; \cancel{tg\gamma} = \frac{1}{2} \cdot \Delta P|_{HT} \cdot \frac{S}{h} \cdot \frac{1}{\overline{\Delta p}|_c} = \frac{1}{2} \frac{\Delta p|_{HT}}{\overline{\Delta p}|_c} \cdot \frac{S}{C_x} \cdot \frac{C_x}{h} = \frac{1}{2} \frac{\Delta P|_{HT}}{\overline{\Delta p}|_c} \cdot S/C_x \left(\frac{h}{C_x}\right)^{-1}$$

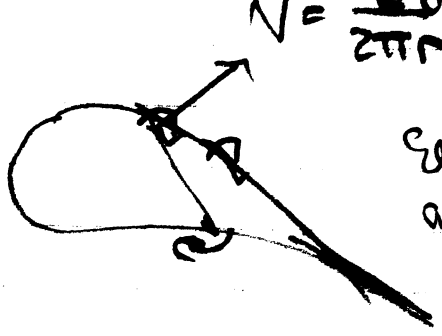
$$= \frac{1}{2} \frac{223.515}{468.95} \cdot \frac{1.35}{1.2} = 0.2681 \qquad \gamma = 15°$$

the blade should be leant by 15° to pressure surface (hub and pressure surface to form acute angle).



end q2.

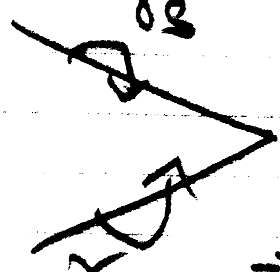
3. Panel method



$V = \frac{\gamma \delta l}{2\pi r}$

series of vortices around surface of blade

interaction of all vortices integrated to satisfy b.c. of zero tangential velocity inside blade



γ_s

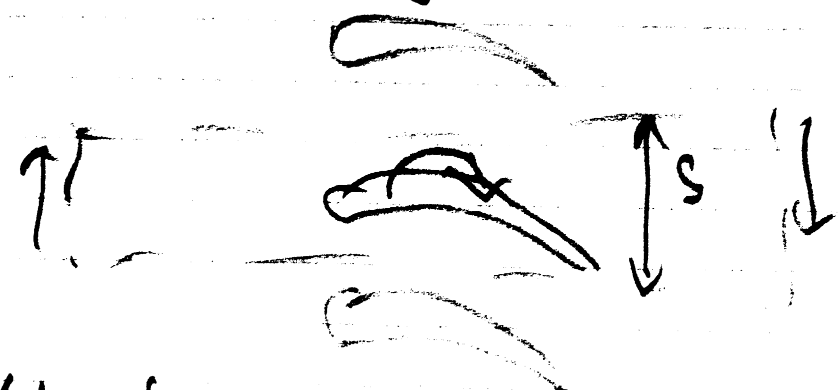
γ_p

Kutta condition is $V_s = V_p$

$\Rightarrow \gamma_s = -\gamma_p$

(jump in velocity is γ) where γ is per unit length

Turbomachinery b.c.



s

induced vel $= \frac{\Gamma}{2s}$

<u>Advantages</u>

Fast

Simple

good resolution

good for inverse method

<u>Disadvantages</u>

no shocks / only potential flo

Euler/Navier Stokes

evolve $\frac{\partial \rho}{\partial t} \delta Vol + \sum \rho \underline{V} \cdot \underline{A} = 0$

& $\frac{\partial \rho \underline{V}}{\partial t} \delta Vol + \sum (\rho \underline{V} \cdot \underline{A}) \underline{V} = \sum$

+ energy eqn

through small control volumes in the (

u/s & d/s

b.c. is periodic & zero normal veloc against solid walls

inflow/outflow – follow characteristics

Advantages

robust, reliable

N/S contains losses

fully compressible

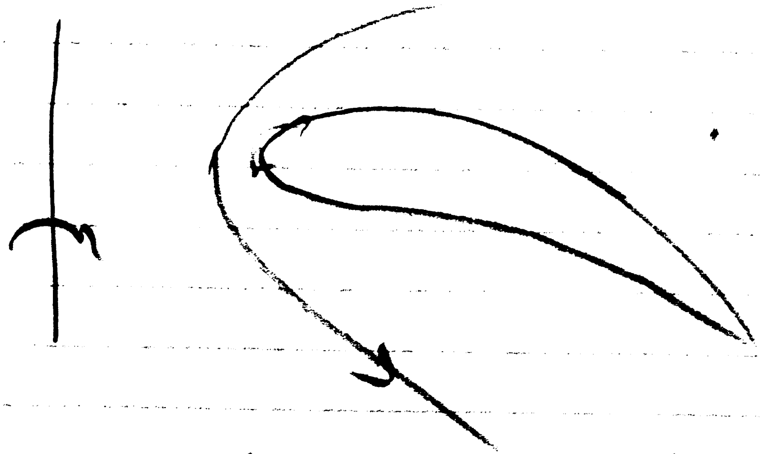
Disadvantages

relatively slow

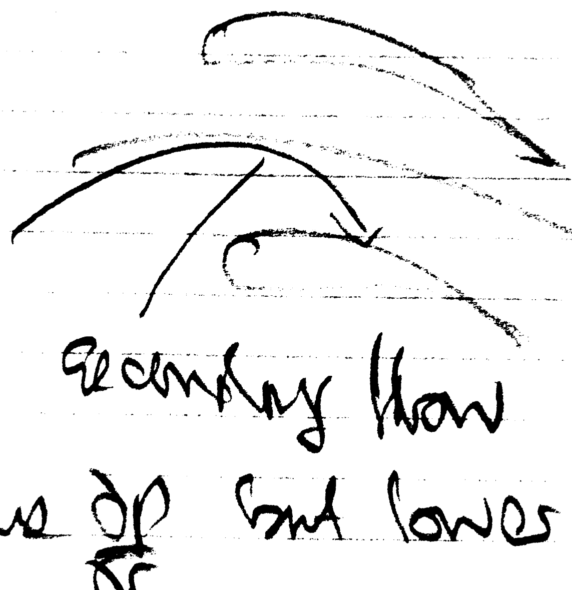
mesh dependent in some c

5) Secondary flow is any component of velocity that is not in the main flow direction

vorticity arguments are also suitable.



wrapping of inlet vorticity around LE to form horseshoe vortex

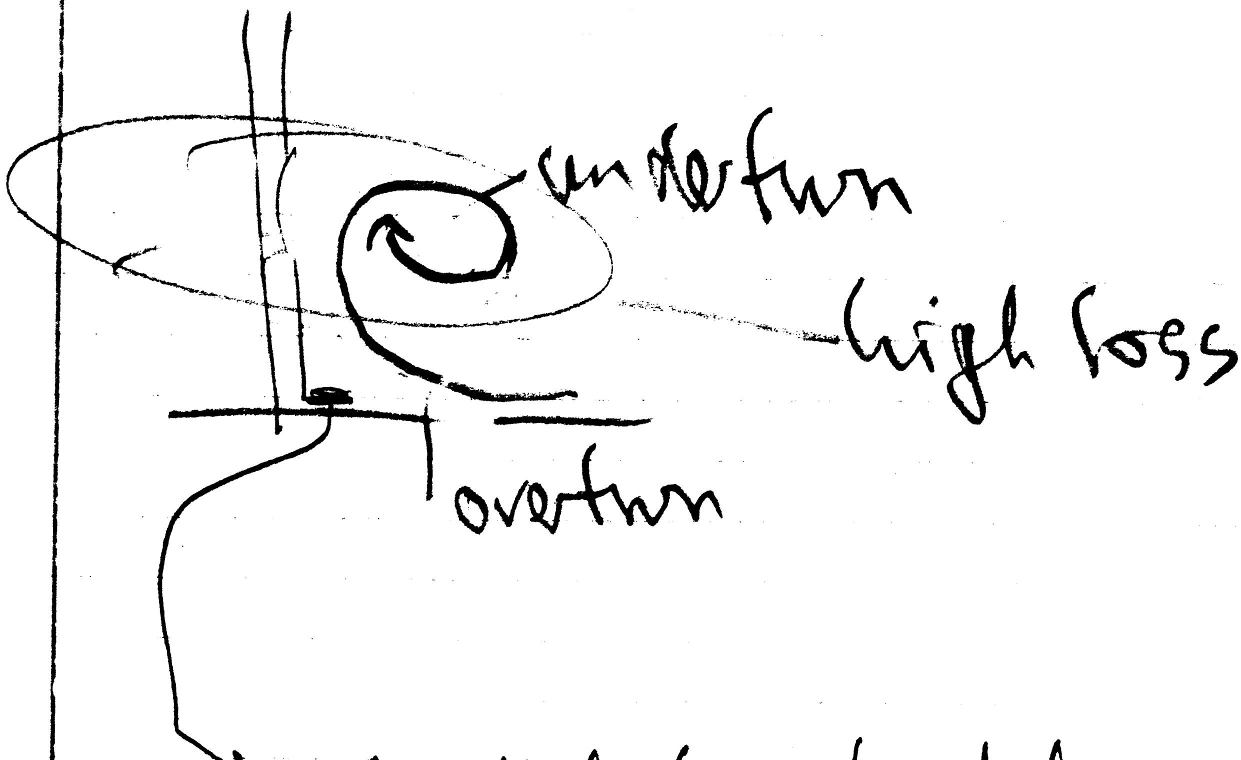


$$\frac{\partial p}{\partial r} = \frac{\rho V^2}{r}$$

Same $\frac{\partial p}{\partial r}$ but lower V $\boxed{\Rightarrow \text{lower } r}$

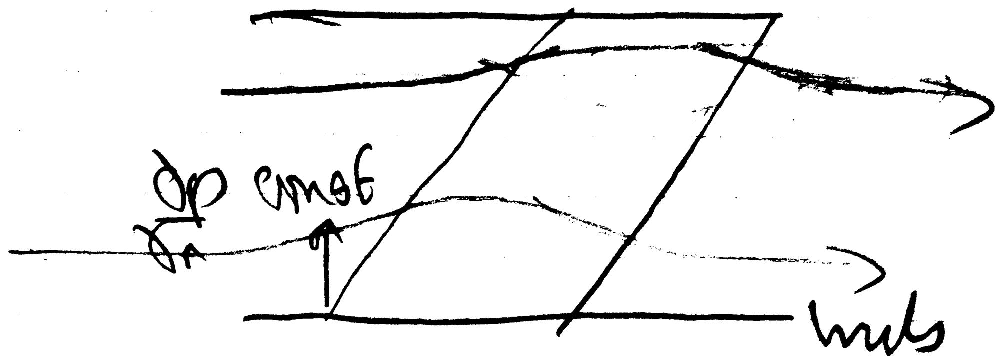
Most b.l. fluid ends up in the passage vortex near the suction surface

c) endwalls: overturning



endwall b.l => high loss

d) sweep: l.e. or t.e. not ⊥ to flow direction



=> low V at hub high V at casing & movement of streamlines as shown

& low hub loading high casing loading

and q3