Solutions

1

a) $A = A_0 [1/(1 + f / f_{c1})][1/(1 + f / f_{c2})]$

b) First pole is at $f_{c1}$ - from 100Hz to 200kHz : -20dB/dec

Second pole is at $f_{c2}$ - from 200kHz to 2 MHz: -40db/dec

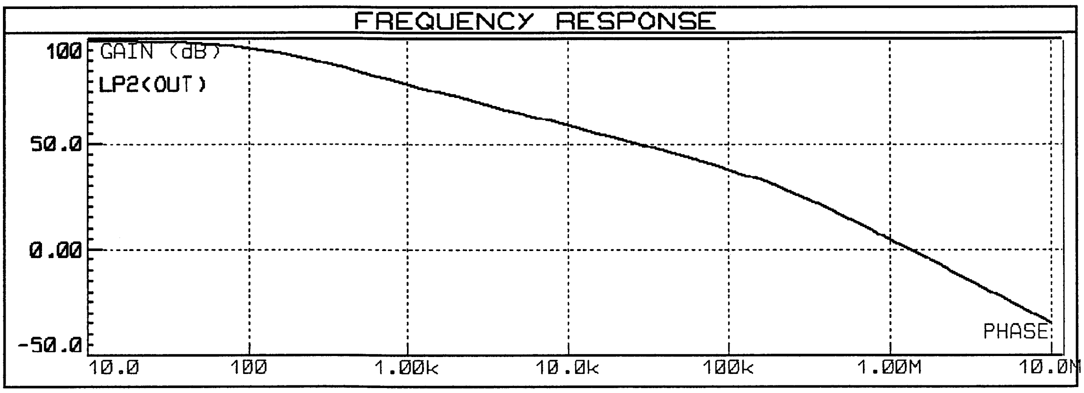



Fig 1

c) Frequency response for unity gain will be as shown in Fig 2:

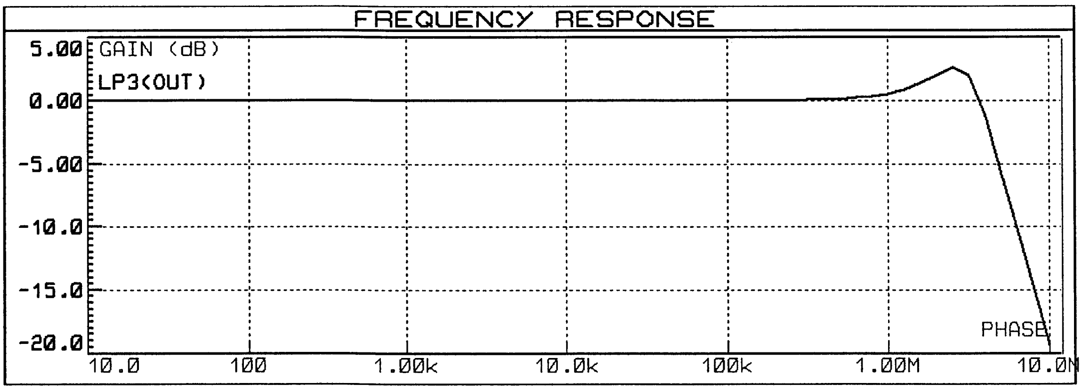



Fig 2

Step response will have tendency of ringing as it is shown in Fig 3:

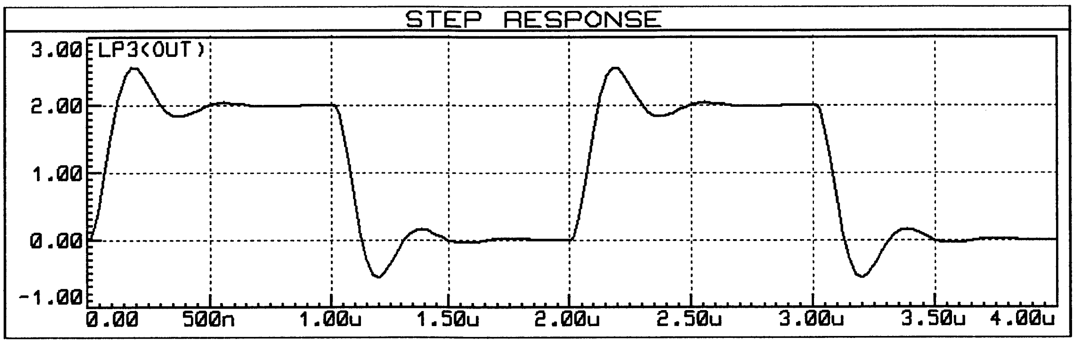
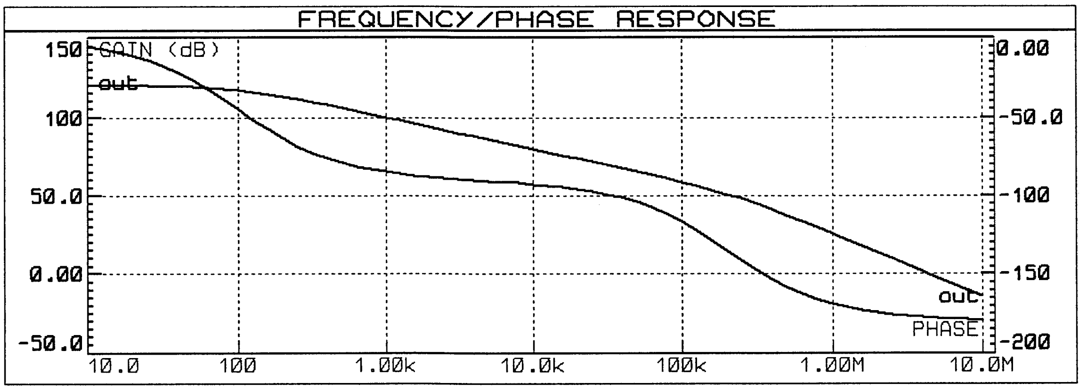
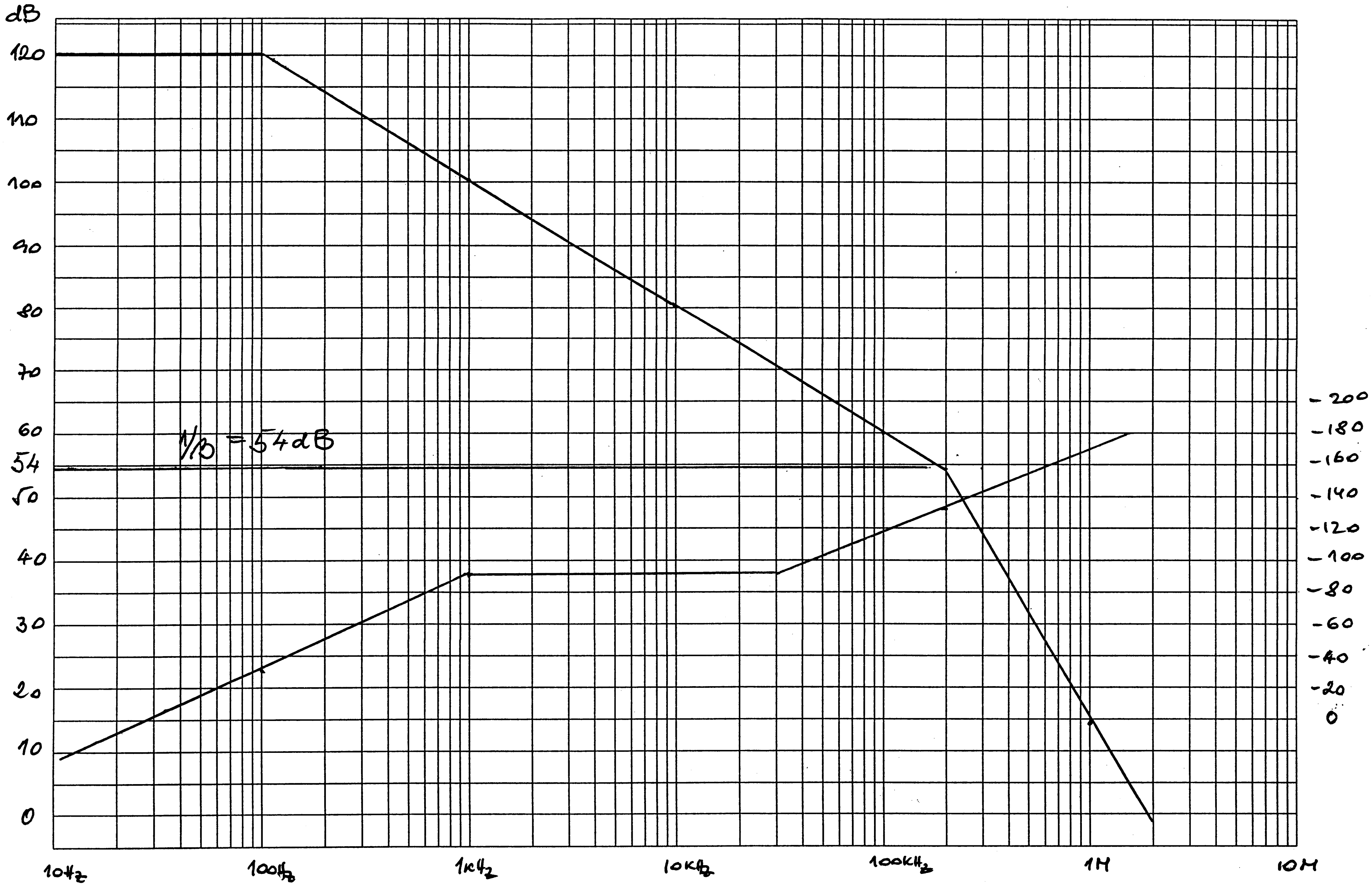



Fig 3

d) Graph Frequency/Phase Response shows Bode plot of open loop amplifier, with the idealised phase shift.




The closed loop gain is defined as $A_f = A(j\omega) / [1 + \beta^* A(j\omega)]$ and stability criterion states that phase shift of $\beta^* A(j\omega)$ must be less than $180^0$ when magnitude of $\beta^* A(j\omega)$ falls to zero. As gain is defined as:
Gain = $1 / \beta$ => 54 dB phase shift for the Gain of 500 is already $135^0$ it is shown in Fig 4.


dB
120
110
100
90
80
70
60
54
50
40
30
20
10
0
1/β = 54 dB
-200
-180
-160
-140
-120
-100
-80
-60
-40
-20
0
10Hz
100Hz
1kHz
10kHz
100kHz
1M
10M

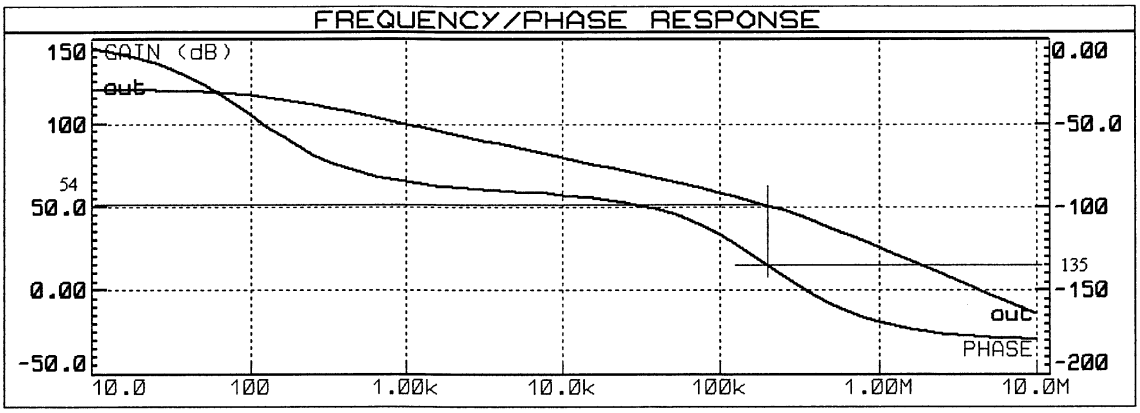



Fig 4

To maintain this $45^0$ phase margin, the break introduced by capacitive load should come at point where Gain is 1 i.e at 2 MHz. Then

$1 / RC = 2\pi * 2 * 10^6$

where:

R = amplifier output resistance
C = introduced load capacitance

For R = 150 Ω it follows:

$C = 1 / (150 * 2 * \pi * 2 * 10^6) = 1 / (6 * \pi * 10^8) = 10^{-8} / (6 * \pi) = 530 * 10^{-12}$ F

2

a) Hold step: This is an error that occurs each time a sample-and-hold goes from sample mode to hold mode causing that small voltage being held that makes it different from the input voltage at the time of sampling.
Droop rate: This is a slow change in output voltage, when in hold mode, caused by effects such as leakage currents.

b) Assumed that capacitor is initially discharged and neglecting effect of $i_b$ it follows (acquisition time):

$V_c = V_{in} (1 - e^{-t/RC})$

Since it is required $V_c / V_{in} >= 0.05\%$ it follows:

$99.95/100 >= (1 - e^{-t/RC})$

$e^{-t/RC} >= 0.05 * 10^{-2}$

$-t/RC >= \ln(0.05 * 10^{-2})$

$-t / RC >= -7.601$

$C <= t / ( R * 7.601)$

As $t = 10 * 10^{-6}$ and R during "on" time is 40 Ω it follows:

$C <= 10 * 10^{-6} / ( 40 * 7.601)$

$C <= 10 * 10^{-6} / 304.0361$

$C <= 0.0329 * 10^{-6}$

$C <= 32.9$ nF

From the definition for Hold step:

$\Delta V = Q / C$

where: Q is charge injection and $\Delta V$ is Hold step

$Q / C < 1$ mV

$C > Q * 10^{3}$

$C > 20 * 10^{-12} * 10^{3}$

$C > 20$ nF

From the request for Droop rate it follows:

$i_{worst\_case} = C * dV/dt$

As

$dV/dt < 5$ mV/ms

$i_{worst\_case} / C < 5$ mV/ms

$C > i_{worst\_case} / 5$

$i_{worst\_case} = i_{leakage\_current} + i_{input\_bias\_current}$

$i_{worst\_case} = 30$ nA + 50 nA

$i_{worst\_case} = 80$ nA

$C > 16$ nF

The required capacitor should be 20 nF < C <= 32.9 nF.

c) From

$V_c = V_{in} (1 - e^{-t/RC})$

It follows:

$V_c = V_{in} - V_{in} e^{-t/RC}$

$dV_c / dt = V_{in}/ RC * e^{-t/RC}$

Slew rate at t = 0 is as follows:

$dV_c / dt = V_{in}/ RC$

As required $dV_c / dt <= 8$ V/µs it follows:

$V_{in} / RC <= 8 * 10^6$

$V_{in} <= 8 * 10^6 * 40 * C$

For C = 20 nF

$V_{in} <= 8 * 10^6 * 40 * 20 * 10^{-9}$

$V_{in} <= 6.4$ V

For C = 32.9 nF

$V_{in} <= 8 * 10^{6} * 40 * 32.9 * 10^{-9}$

$V_{in} <= 10.5$ V

3

a) Rewriting the formula with $s = j\,\omega$ it follows:

Gain = $V_2 / V_1 = A / [\,\alpha + j\,(\,\omega / \omega_0 - \omega_0 / \omega\,)\,]$

When $\omega = \omega_0$

Gain = $A / \alpha$

Bandwidth = $\omega_0\,\alpha$ ( small $\alpha$ == high Q )

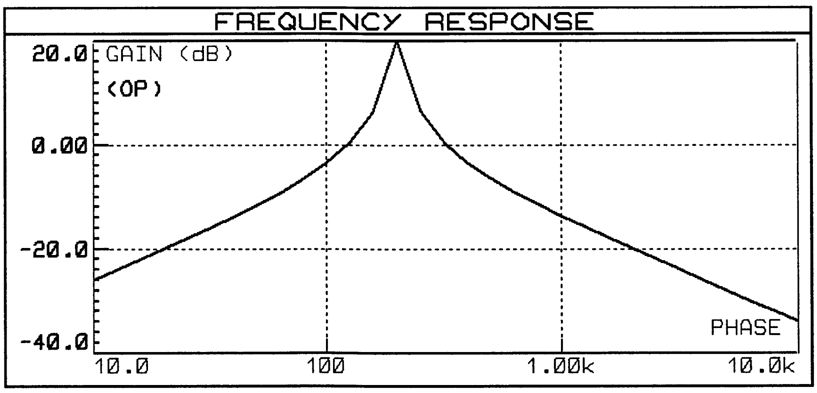



Fig 5

b) Comparing equations in part (a) and (b) of the question:

If $Y_1 = 1 / R_1$ , resistor and $Y_3 = s\, C_3$ to give single *s* in the numerator. Further $Y_4 = s\, C_4$, $Y_5 = 1 / R_5$ and $Y_2 = 1 / R_2$ (other substitutions do not give the required real, s, $s^2$ terms).

$$\text{Gain} = V_2 / V_1$$
$$\text{Gain} = -1 / R_1 * s\, C_3 / [\, 1 / R_5 (1 / R_1 + 1 / R_2 + s\, C_3 + s\, C_4 ) + s^2\, C_3 C_4]$$

Dividing all terms by $C_3C_4$

$$\text{Gain} = -s / (R_1C_4) / [\, 1/(R_5\, C_3\, C_4)(1/R_1 + 1/R_2) + s\, ( 1/C_3 + 1 /C_4 )\, 1/R_5 + s^2\, ]$$

Now matching with the equation in part (a) it follows:

$A\omega_0 = -1 / (R_1\, C_4)$ (1)
$\alpha\, \omega_0 = ( 1/C_3 + 1 /C_4 )\, 1/R_5$ (2)
$\omega_0^2 = 1/( R_5\, C_3\, C_4)(1/R_1 + 1/R_2 )$ (3)

Dividing equation (1) by (2) gain of the circuit is as follows:

$\text{Gain} = A / \alpha = - R_5 / (2\, R_1)$ if $C_3 = C_4$ (4)

c) Equation (1) from part b) gives:

$R_1 = -1/ (A * \omega_0 * C_4) = -1 / ( -1 * 2 * \pi * 200 * 100 * 10^{-9} ) = 7958\ \Omega$

where:

$\text{Gain} = -10 = A/ \alpha \rightarrow \alpha = 0.1$ and $A = -1$; $C_3 = C_4 = 100 * 10^{-9}$ F

Equation (4) from part b) gives:

$R_5 = - 2 * R_1 * A / \alpha = -2 * 7958 * (-10) = 159.15\ k\Omega$

Equation (3) from part b) gives:

$R_2 = 1 / (R_5 * C_3 * C_4 * \omega_0^2 - 1/R_1)$
$R_2 = 1 / (159.15 * 10^3 * 10^4 * 10^{-18} * 4 * \pi^2 * 4 * 10^4 - 1 / 7958 )$
$R_2 = 418.4\ \Omega$

d)

i) The Gain x Bandwidth product of the Op-Amp must be >> 200 Hz x 10 from peak gain needed in say 100 * 200 * 10 = 200 kHz so that the performance meets the theoretical case with reasonable precision.

Slewing rate limitation to examine: For amplitude of 10 V needed at 200 Hz slewing rate is as follows:

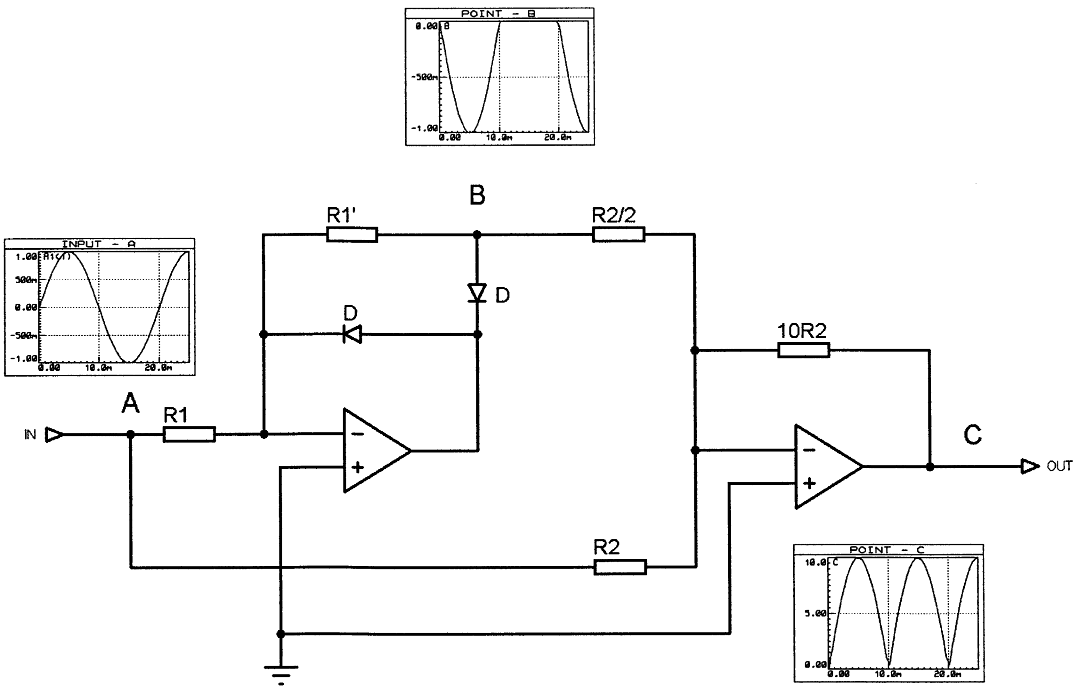
$S = 2 * \pi * 200 * 10 \sim 10^4$ V/s = 0.5 V/μs

Which is within capability of LM741.

ii) Important use – To selectively amplify just one frequency – for example used in an AC Bridge circuit or AC Servomechanism. *Induced noise* at other frequencies is attenuated and this helps the performance.

4

a) It is important to remember both op-amp stages INVERT, so to get a positive output at (C), the input at point (B) must be negative formed by the first op-amp. So 1V peak signal at input (A) is shown at the circuit:



Of importance:

i) First stage has UNITY gain, usually, for economy in component selection so $R_1 = R_1'$.

ii) Direction of the diodes D give negative going half waves is as shown.

iii) Ratio of two resistors leading to second stage adds twice signal at (B) to original signal from (A) so that final stage subtracts positive period of (A) and negative period of (B).

iv) Diode leakage current, $I'$, allows part of the expected current in $R_2'$ to be lost; so the output is $I' * R_1'$ less than the expected 1 V peak defined by equal resistance $R_1$ & $R_1'$.

b)

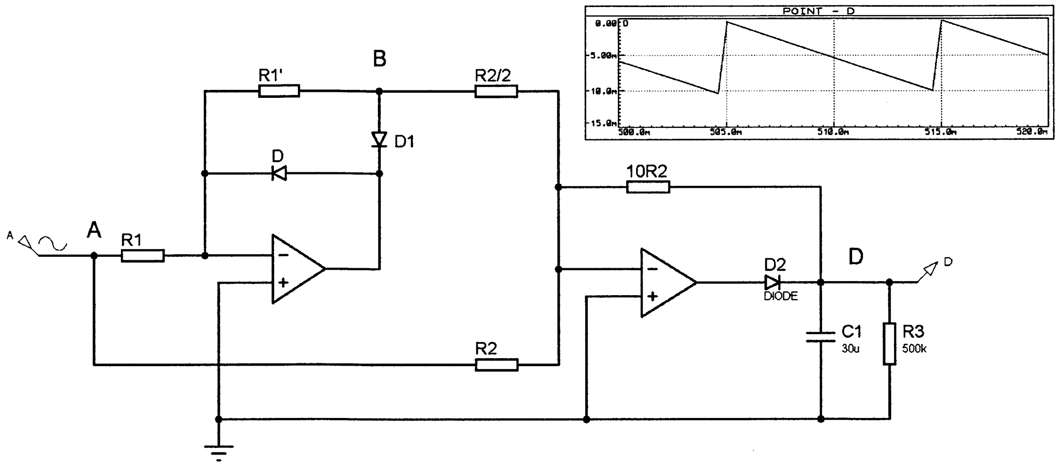
i) The input resistance to the circuit is parallel value of $R_1$ and $R_2$. If a value of 50 kΩ needed, $R_1 = R_2 = 100$ kΩ (similar resistors have usually the same temp. coefficients).

ii) To get unity gain in the first stage, $R_1' = 100$ kΩ.

iii) Diode leakage current $I'$ will give
$I' * 100$ kΩ = (0.1% of 1 V) = 1 mV
$I' = 10$ nA

iv) As stable 100 kΩ resistor already chosen, make $R_2/2 = 50$ kΩ from two 100 kΩ resistors joined in parallel (again temp coefficient tracking helps performance as some errors will be nulled). Then for the final stage for Gain = 10 - select resistor $10R_2 = 1$ MΩ.
Following components are required:
Resistors: 1 MΩ, 5 x 100 kΩ
Diodes: 2 – leakage current < 10 nA.

c) New output waveform (D) shows 10V decaying to 9.99V (0.1%less):




Capacitor discharge is defined as follows:
$9.99 = 10 * e^{(-t/RC)}$
where R = 1 MΩ, 500 kΩ and t = (½ * wave period) = 10 ms (approx.)
$e^{(-t/RC)} = 0.999$
$-t/RC = \ln(0.999)$
$C = - t / [ R * \ln(0.999)]$
C = 30 μF

5

a) The coefficient of interaction between two loops is called mutual inductance. A mutual inductance $L_M$ injects a noise voltage $U_M$ proportional to the current in the driving (primary) loop.

b) The injected voltage in the loop B is defined as follows:

$U_M = L_M * dI_A / dt$ (5b1)

where:
$U_M$ = noise voltage proportional to the mutual inductance
$L_M$ = mutual inductance
$dI_A / dt$ = current change in the driving circuit (loop).

Change of the signal amplitude in the driving circuit is defined as follow:

$dV_A / dt = \Delta V_A / T_R$

where:
$T_R$ = signal rise time

As
$V_A = R_A * I_A$

$dV_A / dt = d(R_A * I_A) / dt = R_A * (dI_A / dt)$

From equation (5b1) it follows:

$U_M = L_M / R_A * (dV_A / dt) = (L_M / R_A) * (\Delta V_A / T_R)$

The mutual inductance is as follows:

$L_M = R_A * T_R * U_M / \Delta V_A$
$L_M = 50 * 800 * 10^{-12} * 80 * 10^{-3} / 2.7$
$L_M = 3.2 * 10^{-9} / 2.7$
$L_M = 1.185$ nH

Measured value of the calculated mutual inductance must be doubled because the real measured induced voltage on resistor $R_B$ is divided by two (because of the coaxial cable and oscilloscope terminating resistor). The real mutual inductance of the two parallel resistors, terminated by 50 Ω is as follows:

$L_M = 1.185$ nH * 2 = 2.37 nH

c) An ideal transmission line consist of two perfect conductors. These two conductors have zero resistance, are uniform in cross section and extend forever. An ideal transmission line has three following properties:
   1) It is infinite
   2) Signals propagating on the line are not distorted as they progress
   3) Signals propagating on the line are not attenuated as they progress.

d) At two points along transmission line time delay is defined as follows:

$t_2 - t_1 = (Y - X)(L*C)^{1/2}$

$$T = (Y - X)(L_{length} * C_{length})^{1/2} \qquad (5d1)$$

The capacitance between these two points is defined as follows:

$C_{YX} = (Y - X) * C_{length}$ (5d2)

Charge between points X and Y along transmission line is defined as follows:

$Q = C_{YX} * V$ (5d3)

The average current along transmission line is defined as follows:

$I = Q / T$ (5d4)

Rewriting equation (5d4) using equations (5d1) and (5d3) it results in:

$I = ( C_{YX} * V ) / [(Y - X)(L_{length} * C_{length})^{1/2}]$ (5d5)

Rearranging equation (5d5) and including equation (5d2) it results in:

$V / I = (Y - X)(L_{length} * C_{length})^{1/2} / [(Y - X) * C_{length}]$

$Z_0 = V / I$ (characteristic impedance)

$Z_0 = (L_{length} / C_{length})^{½}$

For the coaxial cable with above measured inductance and capacitance per unit of length, characteristic impedance is as follows:

$Z_0 = (6.8 * 10^{-9} / 2.6 * 10^{-12})^{1/2}$

$Z_0 = 51.14\ \Omega$