1 (a) Using a calculator in SD mode; entering the 8 voltages,

Mean voltage = 1.0075 V, std dev = 0.00113

or in % = 0.113%

$1M\Omega \parallel 100\Omega = 99.99\Omega$

So Current = V/R = 10.076 mA indicated

Standard Uncertainty of value = $0.113/\sqrt{8}$ = 0.04%.

— " —

(b) DVM indicates HIGH now by $0.05 + 2 \times 0.01 = 0.07\%$

So correction $\delta_m = -0.07\%$

Resistor value high now by $0.02 + 3 \times 0.01 = +0.05\% = \delta_r$.

Actual Current $= \dfrac{V(1+\delta_m)}{R(1+\delta_r)} \simeq \frac{V}{R}(1+\delta_m)(1-\delta_r)$

$= \frac{V}{R}(1+\delta_m-\delta_r-\cancel{\delta_m\delta_r})$ small

$= 10.076(1-0.07\%-0.05\%)$

$= 10.076 \times 0.9988$

$= 10.064$ mA.

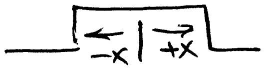
— " —

(c)

Source of Uncertainty	Value %	Probability	Divisor	Standard Uncertainty (%)
Cal of DVM	.02 × 2	Normal	2	0.02
Cal of R	.02 × 3	Normal	2	0.03
Temp Coeff, DVM	.02 × 5	Rectangular*	$\sqrt{3}$	0.058
" " , R	.01 × 5	"	$\sqrt{3}$	0.029
Observations on DVM	.04			→ 0.04

$\sqrt{\sum Unc^2}$ = 0.084

× 2 & rounded up 0.17%

* When data is given with a ±x limits, then it is assumed that  it has a rectangular shape probability over the range to the limits. (contd)

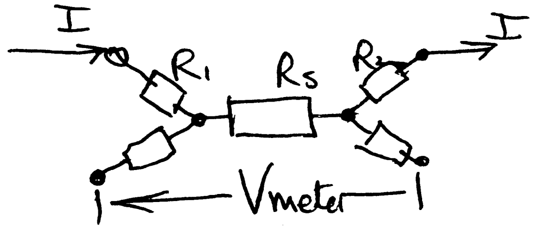
1 (c) (Contd)

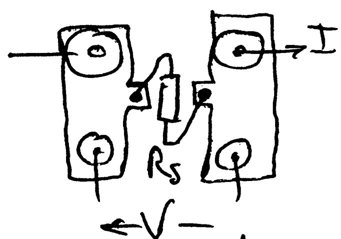
So sensor current is <u>10.064 mA</u> with a <u>0.17%</u> uncertainty, being a standard uncertainty multiplied by 2 to give a level of confidence of approximately 95%.

Clearly the DVM Temperature Coefficient is the biggest single contributor to a large uncertainty by a big factor. So <u>either</u> look for a DVM which has a lower temp' coeff' <u>or</u> control the temperature, say to ±2°C rather than the ±5°C given.

(d) A 4 terminal resistor, R_S, is shown with the lead + terminal resistances $R_1 - R_4$

<u>Circuit</u>





The terminals <u>are doubled up</u> allowing separation of where <u>the current flows</u> and where <u>the voltage is sensed</u>.

Now $R_1 + R_2$ are variable $\propto$ terminal tightness, etc, but <u>ONLY</u> <u>the voltage IR_S is passed</u> to DVM.

And R_3 & R_4 again are variable, so $IR_S - I_m(R_3+R_4)$ is actually what the DVM indicates.

But I_m typically a few μA in a good DVM

$R_3 + R_4$ " " mΩ.

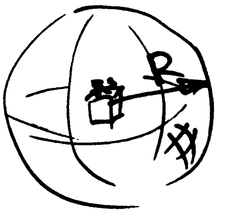
Their product is the error & is a <u>few nV</u> & is negligible.

2(a) Thermal power radiated, $W = \underbrace{\epsilon}_{\text{emissivity}} \underbrace{\sigma_{SB}}_{\text{S-B}} \underbrace{T^4}_{\text{temp}} \underbrace{A}_{\text{area}}$

$\therefore W = 0.73 \,.\, 5.6 \times 10^{-8} \,.\, \underbrace{(85+273)}_{358}{}^4 \,.\, 0.65$

$= \underline{436 \text{ W}}$

(b) At a range of 10m, thermal intensity



$= \dfrac{436}{4\pi \underbrace{R^2}_{10}} = 0.347 \text{ W/m}^2$

$\therefore$ Thermal radiation collected by 5cm dia. lens

$= \dfrac{\pi}{4} 0.05^2 \,.\, 0.347 = \underline{0.68 \text{ mW}}$

Array $\Rightarrow 256 \times 256 = 65536$ elements

$\times 12\% = 7864$ elements exposed to 0.68mW

$\therefore$ 86 nW per element exposed. Hence, for a thermal rating of 1.3×10^4 °C/W, the temperature rise $= 86 \times 10^{-9} . 1.3 \times 10^4 = 1.1 \times 10^{-3}$ °C

This gives a signal voltage of $1.1 \times 10^{-3} \times 67$ µV

$\therefore V_s = \underline{74 \text{ nV}}$

2 (c). • Noise voltage for a resistor (thermal) $V_n = (4kTRB)^{1/2}$ V_{rms} (4)

assume camera @ 300K

$\therefore$ for a $50\,\Omega$ resistor and 200Hz bandwidth

$V_{n_t} = 1.28 \times 10^{-8}$ V

• for the pre-amp. $V_{n_0} = 1.2 \times 10^{-9} . \sqrt{200}$

$= 1.70 \times 10^{-8}$ V

$\therefore$ Total noise voltage $V_n = \sqrt{V_{n_t}^2 + V_{n_0}^2}$

$\therefore$ $V_n = 21$ nV rms

Hence, the signal-to-noise ratio for the thermal image of the transformer: $\frac{V_s}{V_n} = \frac{74}{21} \triangleq 3.5$. This would be quite 'fuzzy' to look at, but quite useful nonetheless.

Since the thermal signal is proportional to T^4, the temperature for $V_s/V_n = 1$ would be: $3.5^{-1/4} . 358$ K

$\therefore$ $T = 262$ K

or -11 °C

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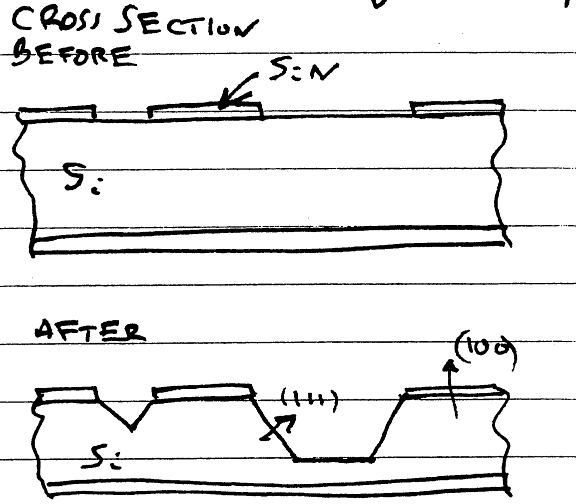
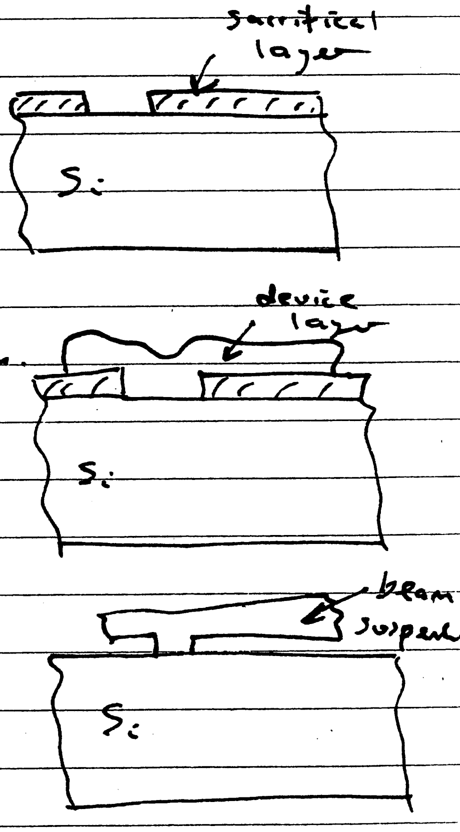
(i)



3 a) In bulk micromachining a masking layer such as a SiN thin film is used to define the regions of the silicon substrate to be etched.

After a sufficiently long etch time (~1μm/min in KOH + H_2O) the substrate etches through to make a SiN membrane on the back side provided the mask opening was large enough.

(ii)



In surface micromachining thinner structures are made on the surface of the wafer by successive deposition, patterning and etching of layers including a sacrificial layer. This process may be repeated several times.

To integrate MEMS with CMOS readout devices and circuitry for example it is necessary to restrict the range of MEMS processes to relatively low temperature processes using a small range of reagents and materials.

For example polysilicon micromechanical layers have been incorporated into mass produced BiCMOS acceleration sensors.
In Fig 3 the black area is the suspended proof mass. The holes ensure undercutting.
Other MEMS products are successful in spite of having separate chips for readout electronics eg. pressure sensors.

b) The x direction in the diagram overleaf is the sensitive axis because the micromechanical spring is stiff in the y direction.

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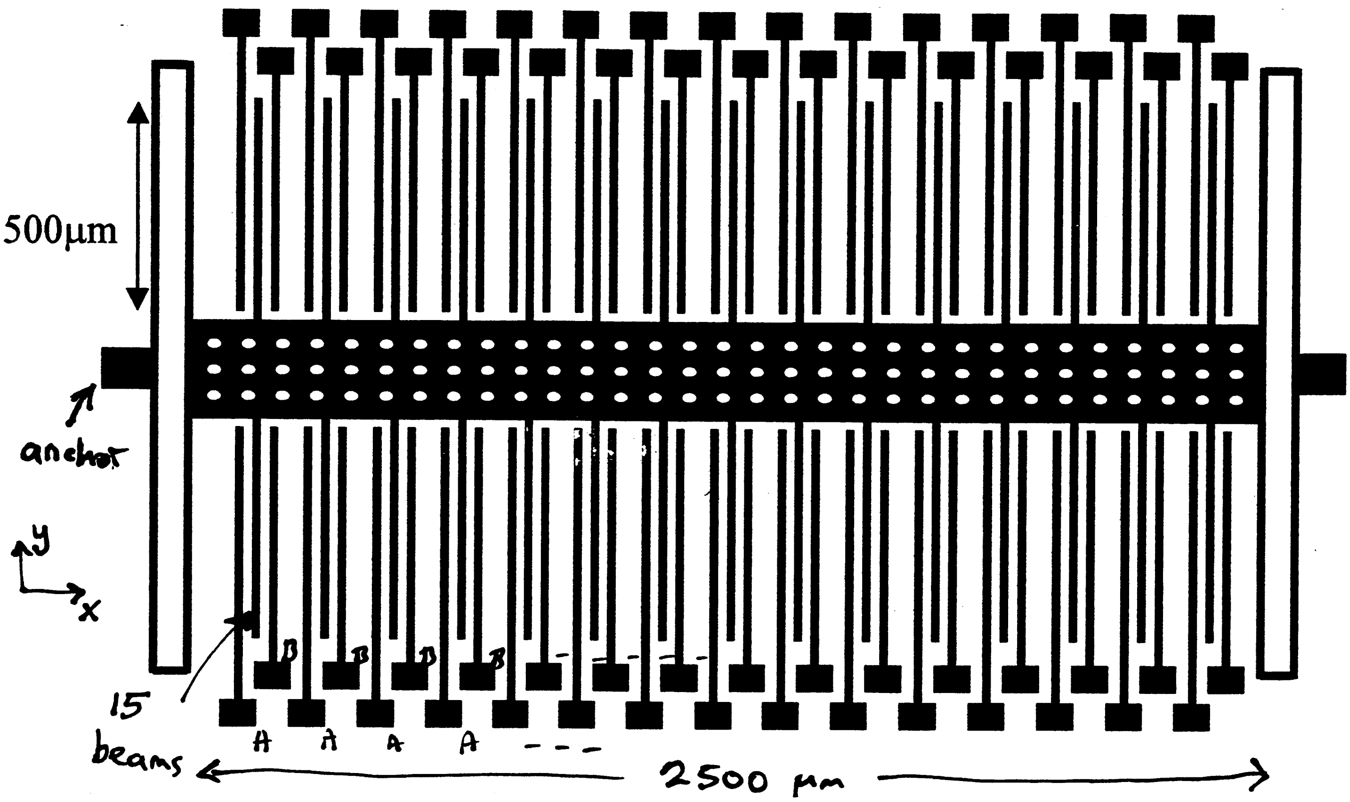
A differential capacitance measurement is made between the DFm 1·2·2004
set of beams A and the proof mass, and the set of beams B.



(i) open loop operation is when the change of differential capacitance is used to monitor the acceleration as a function of time (ii) in closed loop force-feedback some of the beam are driven with a voltage to counteract the acceleration.

c) Each beam capacitor A is 0.5 mm long and the depth is 3 μm. There are 30.

Total capacitance $= \frac{\text{Area} \times \varepsilon_0 \varepsilon_r}{d} = \frac{30 \times 0.5 \times 10^{-3} \times 8.9 \times 10^{-12} \times 1 \times 3 \times 10^{-6}}{0.8 \times 10^{-6}}$

Neglecting fringe effects the capacitance is $\boxed{5 \times 10^{-13}\ \text{F}}$

The volume of the proof mass is $\sim 250 \times 10^{-6} \times 2500 \times 10^{-6} \times 3 \times 10^{-6} = 1.9 \times 10^{-12}\ \text{m}^3$
(neglecting the beams and holes)

Taking the density to be $2000\ \text{kg m}^{-3}$ the mass is then 3.8×10^{-9} kg

The force when suffering $60\ \text{ms}^{-2}$ is mass $\times$ acceleration $= 3.8 \times 10^{-9} \times 60$
$= 2.25 \times 10^{-7}$ N

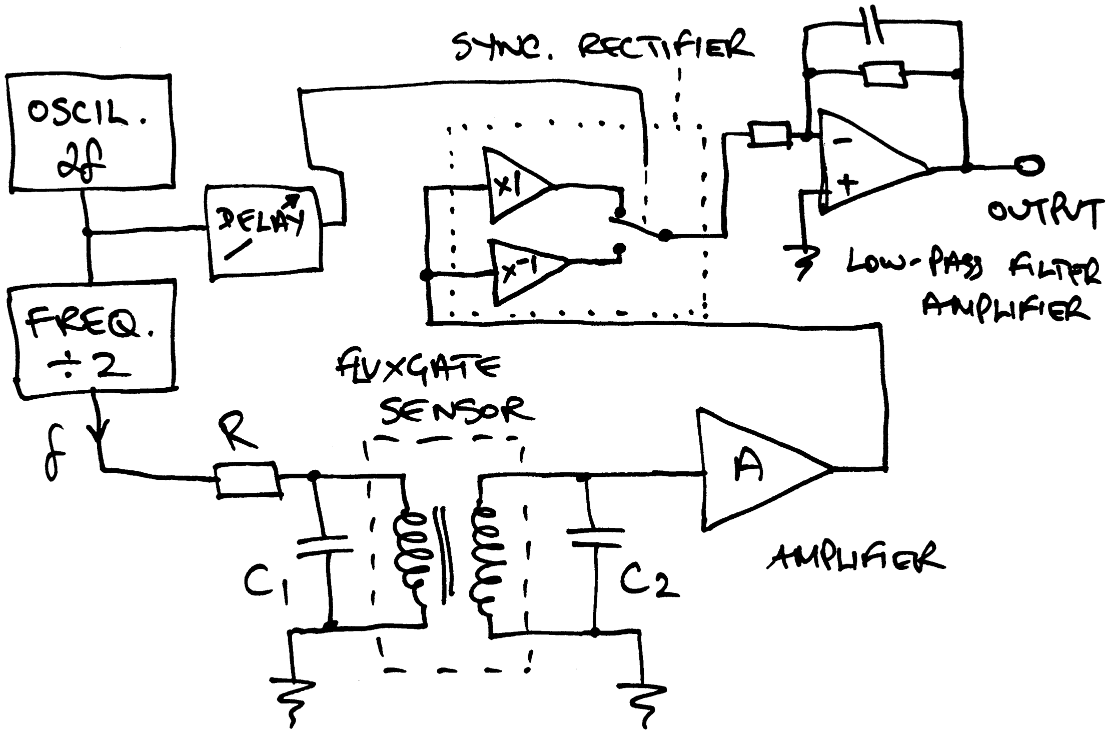
But the electrostatic force $= \frac{1}{2} QE = \frac{1}{2} CV \frac{V}{d} = \frac{1}{2} C \frac{V^2}{d}$

Only a small fraction of the total number of beams is used for feedback

Taking $C_{\text{feedback}} = 2 \times 10^{-13}$ F we have $V = \sqrt{\frac{2 \times \text{force} \times \text{separation}}{\text{capacitance}}}$

$V = \sqrt{\frac{2 \times 2.25 \times 10^{-7} \times 0.8 \times 10^{-6}}{200 \times 10^{-15}}} \simeq \boxed{1.3\ \text{Volt}}$

4 (a)



(b) The flux in the core is given by $B_{core} \times A_{core} = \phi$

with a demagnetising factor of D, $B_{core} = B/D$

(effective relative permeability $\approx 1/D$)

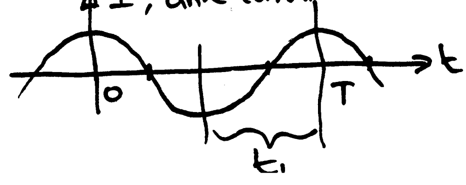
$$\therefore \quad \phi = \frac{\pi d^2}{4} \cdot \frac{B}{D} \quad \text{and} \quad D = \left(\frac{d}{l}\right)^2 \left[\ln \frac{2l}{d} - 1\right]$$

for core of diameter d and length l.

When driven by a gating current at frequency f, the core is saturated twice per cycle (once +ve and once -ve), hence the external field is switched in or out of the core 4 times.

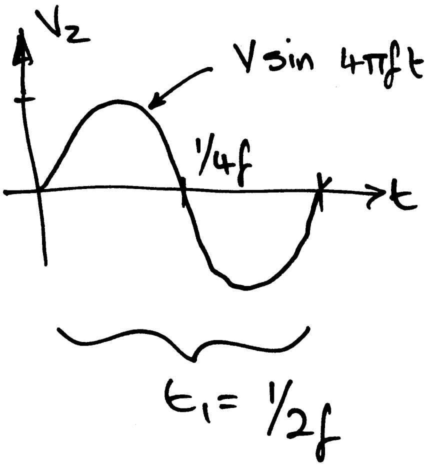
Now $V_2 = N \dfrac{d\phi}{dt}$ for N turns around the core, so

if we take the time to de-saturate the core from -ve to saturation +ve to be t_1:



4 (b) contd.

$t_1 = \frac{1}{2f}$ and during this time, the external field is switched into and back out of the core - causing a +ve and -ve voltage signal :-



If we assume the waveform is a sine wave and integrate a ½ cycle,

$$\int V_2 \, dt = N\phi$$

$$\therefore \quad N\phi = \int_0^{1/4f} V \sin 4\pi f t \, dt = \left[-\frac{V \cos 4\pi f t}{4\pi f} \right]_0^{1/4f} = \frac{V}{2\pi f}$$

$$\therefore \quad V_2 = 2\pi f N\phi = 2\pi f N \frac{\pi d^2}{4} \frac{B}{D}$$

$$V_2 = \frac{\pi^2 f N B l^2}{2\left(\ln\frac{2l}{d} - 1\right)}$$

The self-inductance is given by $L = \frac{N\phi}{I}$, for a current I in the coil. The H field in the coil is given by $H = \frac{NI}{l}$ and as $B = \mu H$ with $\mu = \mu_0/D$ for the core,

$$\phi = \frac{\pi d^2}{4} . B = \frac{\pi d^2}{4} \frac{NI}{l} \frac{\mu_0}{D} \quad \therefore \quad L = \frac{\pi d^2}{4} \frac{N^2}{l} \frac{\mu_0}{D}$$

$$\Rightarrow L = \frac{\pi l N^2 \mu_0}{4\left(\ln\frac{2l}{d} - 1\right)}$$

4(c) From (b)

$$V_2 = \frac{\pi^2 . 25\times10^3 . 400 . 25\times10^{-6} . 0.01^2}{2(\ln 100 - 1)}$$

$\therefore$ $V_2 = 0.034$ V (or $\approx$ 68 mV pp.)

Also

$$L = \frac{\pi . 0.01 . 400^2 . 4\pi\times10^{-7}}{4(\ln 100 - 1)}$$

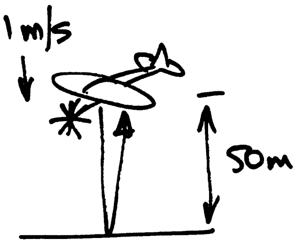
$\therefore$ $L = 0.438$ mH

So, to maximise $2f/f$ signal, we shall choose a capacitor to resonate with L at 50 kHz (2f).

$$50\times10^3 = \frac{1}{2\pi\sqrt{LC}} \qquad \therefore 1/C = (100\pi\times10^3)^2 . 0.438\times10^{-3}$$

$\therefore$ $C = 23$ nF

5(a) For 1 m/s descent rate: transit path length changes by 2 m/s.



$$\therefore\ f_{Doppler} = f_{u/sonic} \cdot \frac{2}{v_s}$$

with $f_{u/sonic} = 40\times10^{3}$ Hz and $v_s = 340$ m/s

$$\Rightarrow\ f_{Doppler} = 235 \text{ Hz}$$

$$\text{Pulse-echo transit time} = \frac{2\times 50 \text{ m}}{340 \text{ m/s}} = 0.294 \text{ s}$$

(b)

$$\text{Electrical power} = \left(\frac{240}{2\sqrt{2}}\right)^2 \Big/ 200 = \frac{V_{rms}^2}{R} = 36\text{W}$$

U/sonic power in transducer = 36 × 10% = 3.6W

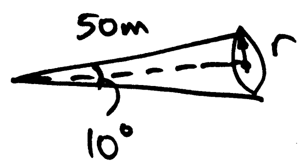
- U/sonic power coupled to air $= 3.6 \times \overbrace{\frac{4 Z_{air} Z_t}{(Z_{air}+Z_t)^2}}^{=0.522} = 1.88$W

  where $Z_{air} = 340 \times 1.29$ kg m^{-2} s^{-1}

  and $Z_t = 2400$ kg m^{-2} s^{-1}

- over 50m path, attenuation $= 10^{-50\times0.15/10} = \times 0.178$

- consider cone angle of transducer:



  r = 4.36m

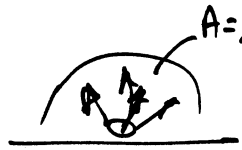
  ∴ u/sonic power spread over πr^2 area = 59.7 m^2

$$\therefore\ \text{Acoustic power density} = \frac{1.88 \times 0.178}{59.7} = 5.61 \text{ mW/m}^2$$

5(c) u/sonic energy scattered back $= 0{\cdot}25 \times 5{\cdot}61 \times 10^{-3} \times 59{\cdot}7$

$$= 0{\cdot}084 \text{ W}$$

This is sent back over a hemi-sphere



∴ intensity back up at the 'plane is:-

$$\underset{\text{Power}}{0{\cdot}084} \over \underset{\text{Area}}{2\pi\ 50^2}$$
$$\cdot \underbrace{0{\cdot}178}_{\text{Atten.}} = 9{\cdot}5 \times 10^{-7} \text{ W/m}^2$$

u/sonic power collected by transducer of dia. 7cm

$$P_{u/son.} = \underbrace{\frac{\pi\, 0{\cdot}07^2}{4}}_{\text{Area}} \cdot \underbrace{9{\cdot}5 \times 10^{-7}}_{\text{Intensity}} \cdot \underbrace{0{\cdot}522}_{\text{Acoustic coupling}} = 1{\cdot}9 \text{ nW}$$

Conversion to electrical signal $= 10\% \Rightarrow 0{\cdot}19$ nW

$\therefore 0{\cdot}19 \times 10^{-9} = \dfrac{V^2}{R}$ ($R \leftarrow 200$) into matched load $\therefore V = 0{\cdot}195$ mV

or 0·39 mV into open cct.

(d) In a quiet environment, the signals are quite reasonable; but with turbulence and vibration the signal recovery task would be rather challenging. For the ranging mode, signal averaging would be used but this would slow the response rate of the system. For the Doppler mode, vibration will coincide with the desired frequency range so it may not be practical.