ENGINEERING TRIPOS PART II B CRIB 2003/2004

MODULE 4C2: DESIGNING WITH COMPOSITES

1. (a) $\nu_{21} = \nu_{12} E_2/E_1 = 0.01$

with $E = E_1/0.999$

$$Q = \begin{pmatrix} E_1/0.999 & \frac{0.01 E_1}{0.999} & 0 \\ \frac{0.01 E_1}{0.999} & E_2/0.999 & 0 \\ 0 & 0 & E_{12} \end{pmatrix} = E \begin{pmatrix} 1 & 0.01 & 0 \\ 0.01 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}$$

[B] is zero (balanced symmetric laminate)

Balanced $\Rightarrow$ $A_{16} = A_{26} = 0$ (terms cancel out)

For +45° ply, $c^2 = s^2 = 0.5$, $c^4 = s^4 = 0.25$

$\frac{\bar{Q}_{11}}{E} = 0.25 + 0.1 \times 0.25 + 2(0.01 + 2 \times 0.1)0.25 = 0.38$ (same for −45° ply)

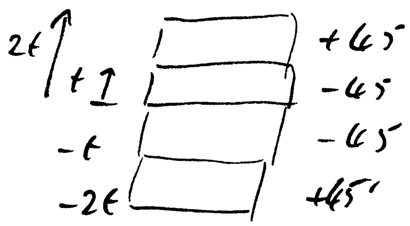
$\frac{\bar{Q}_{12}}{E} = (1 + 0.1 - 4 \times 0.1)\frac{1}{4} + 0.01(0.5) = 0.13$ "

$\frac{\bar{Q}_{22}}{E} = \frac{1}{4} + 0.1 \times \frac{1}{4} + 2(0.01 + 2 \times 0.1)\frac{1}{4} = 0.38$ "

$\frac{\bar{Q}_{16}}{E} = \frac{\bar{Q}_{26}}{E} = \frac{1}{4}\left((1 - 0.01 - 0.2) - (0.1 - 0.01 - 0.2)\right) = 0.225$ (−ve for −45° ply)

$\bar{Q}_{66} = (1 + 0.1 - 2 \times 0.01 - 2 \times 0.1)\frac{1}{4} + 0.1(0.5) = 0.27$ (same for −45° ply)

$$A = \sum_{k=1}^{N} \bar{Q}_k (z_k - z_{k-1})$$



t is ply thickness

4 plies

$\frac{A_{11}}{E} = \bar{Q}_{11}(2t - t) \times 4$

$\frac{A_{11}}{Et} = 4 \times 0.38 = 1.52$

$$[A] = tE \begin{pmatrix} 1.52 & 0.72 & 0 \\ 0.72 & 1.52 & 0 \\ 0 & 0 & 1.08 \end{pmatrix}$$

Similarly other A's

$$D = \frac{1}{3} \sum_{1}^{N} \bar{Q}_k (z_k^3 - z_{k-1}^3) = \frac{1}{3} t^2 (2^3 - 1)\bar{Q}_{45} + (1^3 - 0)\bar{Q}_{-45}$$

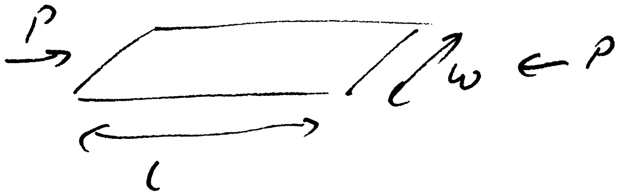
$$+ (-0^3 + 1^3)\bar{Q}_{-45} + (-1^3 + 2^3)\bar{Q}_{45} = \frac{1}{3} t^2 (14 \bar{Q}_{45} + 2 \bar{Q}_{-45})$$

$$= t^3 E \begin{bmatrix} 2.03 & 0.96 & 0.9 \\ 0.96 & 2.03 & 0.9 \\ 0.9 & 0.9 & 1.44 \end{bmatrix}$$

[60%]

1. (cont)

(b)



Data Bk. $P_E = \dfrac{\pi^2 EI}{l^2}$: EI is bending stiffness = $D_{11} w$

$$\Rightarrow P_E = \frac{\pi^2 t^3 E \,.\, 2{\cdot}03}{l^2}$$ [10%]

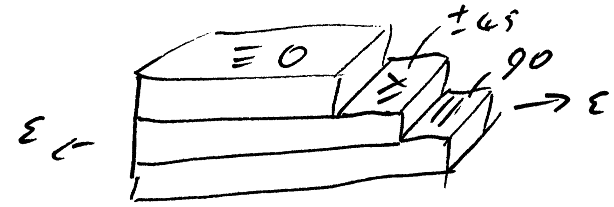
(c) The most important problem will be to get the load into the column. Glued fittings and reinforced ends may be useful. Otherwise there will be a tendency to have premature failure at the ends, perhaps due to splitting or delamination.

In any case a $(\pm 45)_s$ plate is not a very efficient strut. 0° fibres would increase the bending stiffness, but better would be to increase the second moment of area, perhaps by using a tube or square section, or a sandwich panel construction. Again end fittings will be needed.

Strength may be sensitive to imperfections in lay-up, perhaps from ply angle inaccuracy.

[30%]

2(a)



Strain allowables are used by considering that a laminate is made up of 0, ±45 and 90° plies. Since all plies are subject to the same strain, the ply orientation with the smallest ε to failure will be critical (to give the ε allowable). Hence it is possible to predict laminate failure from a laminate calculation, without going into the stresses at the ply level. The laminate stiffness is conveniently found using carpet plots.

[20%]

2(b) To get a reasonable first guess of a laminate which carries the load, model plies independently, neglect ν

0°: assumed to carry $N_x \Rightarrow t_0 = \frac{N_x}{E_1 e^+} = \frac{3\times10^5}{160\times10^9 \cdot 0.004} = 0.56$ mm

±45°: carries $N_{xy} \Rightarrow t_{45} = \frac{N_{xy}}{G e_{LT}} = \frac{2\times10^5}{38\times10^9 \cdot 0.005} = 1.05$ mm

Adding 0's will increase mass but reduce δ - lets check what the trade-off is - compare laminates A and B

Using: $m = \rho t A = 6000\, t$ kg/m

$\delta = \frac{N_x}{E_x t} \cdot 2 = \frac{6\times10^5}{E_x t}$ (2 = side length)

Compare:	0°	±45	E_x/GPa (from carpet plot)	m/kg	δ/mm	2m + δ
A	0.56 mm (36%)	1.05 mm (66%)	62	9.54	6.09	25.17
B	1.05 mm (50%)	1.05 mm (50%)	81	12.6	3.52	28.72

$\Rightarrow$ it looks better to keep the weight down at the expense of reduced stiffness

2 (b) cont.

Now use carpet plots to optimise the design.

With a good mix of 0° and 45°, probably OK to do without 90° plies.

groups of 4 ⇒ balanced

Then conceptual design → 0° : 45° = 4 : 8 = 33 : 67%

Total thickness = 1.5 mm

$$E_x = 62\ GPa,\ G = 26.5\ GPa \Rightarrow t_x = \frac{N_x}{E_x e_L^+} = \frac{3\times10^5}{62\times10^9 \cdot 0.004} = 1.20\ mm$$

$$t_{xy} = \frac{N_{xy}}{G e_{LT}} = \frac{2\times10^5}{26.5\times0.005} = 1.50\ mm$$

$$m + 2S = 21.9$$

So the design is just OK for N_{xy}, slightly over designed for N_x. Not much scope for rearranging laminate as taking any 0's out will cause failure under the N_{xy} load.

Suitable layup $[(\pm45)_2, 0_2]_s$ [80%]

3 (a)

- Q matrix from E_1, E_2, ν_{12}
- Rotate to $\bar{Q}$ for each ply using transformation eqns
- Calculate laminate stiffness matrix A by adding Q's
- Calculate laminate strains ε_x etc from $[\varepsilon] = [A^{-1}][N]$
- Transform to local ply strains along fibre directions
- Find local stresses using Q matrices.
- ~~Finna~~ Finally apply failure criterion [30%]

(b) $\varepsilon_{xy} = 0$ from symmetry.

$(A) = 2tQ + 2t\,\bar{Q}_{30} + 2t\,\bar{Q}_{-30}$ t – ply thickness

$= 2t\begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix} + 4t\begin{pmatrix} 85 & 24 \\ 24 & 20 \end{pmatrix}$ Note $\bar{Q}_{16}$ $\bar{Q}_{26}$ cancel, $\bar{Q}_{66}$ not needed

$= t\begin{pmatrix} 618 & 101.4 \\ 101.4 & 98 \end{pmatrix}$ GPa

Put $[N] = 6t\,(\lambda\ 0\ 0\ 0\ 0\ 0)^T$ (λ is stress)

$$[\varepsilon] = [A^{-1}][N] = \frac{6t}{t(618.98 - 101.4^2)}\begin{pmatrix} 98 & -101.4 \\ -101.4 & 618 \end{pmatrix}\begin{pmatrix} \lambda \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0117 \\ -0.0121 \end{pmatrix} \times 10^{-9}\,\lambda\ \mathrm{m^2/N}$$

$\underset{\sim}{0°}$:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = (Q)(\varepsilon) = \begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix}\begin{pmatrix} 0.0117 \\ -0.0121 \end{pmatrix}\lambda = \begin{pmatrix} 1.59 \\ -0.077 \end{pmatrix}\lambda$$

Tsai Hill:
$$\left(\frac{1.59}{1448}\right)^2 - \frac{1.57 \times (-0.077)}{1448^2} + \left(\frac{0.077}{248}\right)^2 = \frac{1}{\lambda^2} \Rightarrow \lambda = 857\ \mathrm{MPa}$$

$\frac{1}{S_C}$ (MPa) $\frac{1}{S_T}$

$\underset{\sim}{+30°}$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = T^{-1}\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} 0.75 & 0.25 & 0.433 \\ 0.25 & 0.75 & -0.433 \\ -0.866 & 0.866 & 0.5 \end{pmatrix}\begin{pmatrix} 0.0117 \\ -0.0121 \\ 0 \end{pmatrix} \times 10^{-9}\,\lambda\ \mathrm{m^2/N}$$

$$= \begin{pmatrix} 5.75 \\ -6.15 \\ -20.6 \end{pmatrix} \times 10^{-6}\,\lambda\ \mathrm{m^2/N}$$

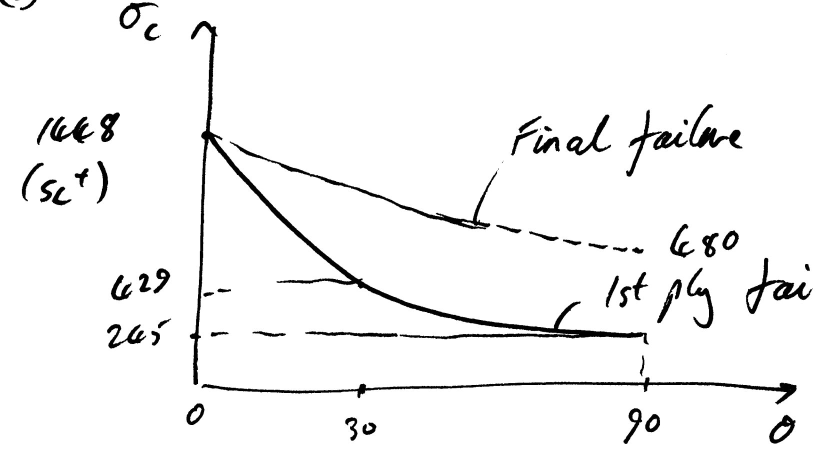
$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = Q\,\varepsilon = \begin{pmatrix} 139 & 2.7 & 0 \\ 2.7 & 9 & 0 \\ 0 & 0 & 6.9 \end{pmatrix}\begin{pmatrix} 5.75 \\ -6.15 \\ -20.6 \end{pmatrix} \times 10^{+9} \times 10^{-6}\,\lambda = \begin{pmatrix} 0.78 \\ -0.040 \\ -0.14 \end{pmatrix}\lambda$$

[45%]

Tsai Hill:
$$\left(\frac{0.78}{1448}\right)^2 - \frac{0.78(-0.04)}{1448^2} + \left(\frac{0.040}{248}\right)^2 + \left(\frac{0.14}{62}\right)^2 = \frac{1}{\lambda^2} \Rightarrow \lambda = 429\ \mathrm{MPa}$$

This is the critical ply (or −30°)

3(c)



For $(0\,90_2)_s$ estimate 1st ply from ε allowable

$$E_x \approx \frac{E_1}{3}, \quad \varepsilon_{90} = \frac{\sigma_T^+}{E_2} = \frac{48\times10^6}{9\times10^9}$$

$$\sigma_c = E_x\,\varepsilon_{90} = \frac{138\times10^9}{3}\times\frac{48\times10^6}{9\times10^9} = 245 \text{ MPa}$$

Expect final failure = 1st ply failure for 0_6 since there is no intact material.

[30%]

For $(0,90_2)_s$, the 0's could carry $\frac{S_c^+}{3} = 480$ MPa.

[25%]

4.

(a) In general CFRP laminates are popular for lightweight structures due to their good specific strength and stiffness. Laminates are needed to cope with loads in more than one direction, and to avoid splitting. It is difficult to design against and prevent splitting and delamination in composites, hence the increasing use of tough epoxy matrix, which limits these modes of failure. The compressive strength of the composite is governed normally by the shear strength of the matrix, rather than toughness, so that tough epoxies may have reduced compressive strength.

[25%]

(b) The primary purpose of torque shafts - to transmit torque - is best served having ±45° plies which carry the loads along the fibres. However it is often the case that axial stiffness is required, perhaps for bending stiffness or to prevent whirl in high-speed applications. Here some axial fibres are needed. The motivation for using composite tubes in the first place is often weight, but may also be for whirl (e.g. in the Renault Espace where the E/ρ ratio of other materials is insufficient to prevent whirl without a central bearing), or perhaps on account of its good thermal expansion properties (e.g. cooling towers) or corrosion resistance.

[25%]

(c) Yachts need high stiffness and strength with low weight. This is best achieved using CFRP composite material with well aligned and consolidated fibres. This is most easily achieved using hand lay-up and autoclaving, to give good consolidation and strength. However this method of production is expensive (particularly the autoclave), but the market for top-rate (e.g. racing) yachts can bear this. By contrast leisure boats need to be much more price-conscious, and here a cheaper alternative is chopped strands mat, laid dry and impregnated by hand. Although the strength and stiffness is not so good, particularly with the random fibre orientations, it is a simple but relatively tough construction with good mechanical properties, good corrosion resistance and easy to make into relatively complex shapes.

[25%]

(d) The bending strength of the beam is predicted assuming that the face sheets yield. In practice other failure mechanisms come into play, for example shear failure of the core, indentation of the skin, skin wrinkling and delamination. Indentation is a particular problem for thin face sheets or at the load points, and can often be the limiting factor in design. Care in load introduction is needed to ensure that these failure modes don't dominate, to exploit the full potential of a sandwich beam.

[25%]