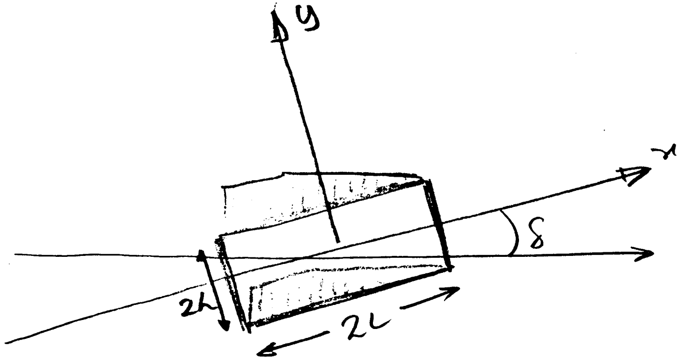
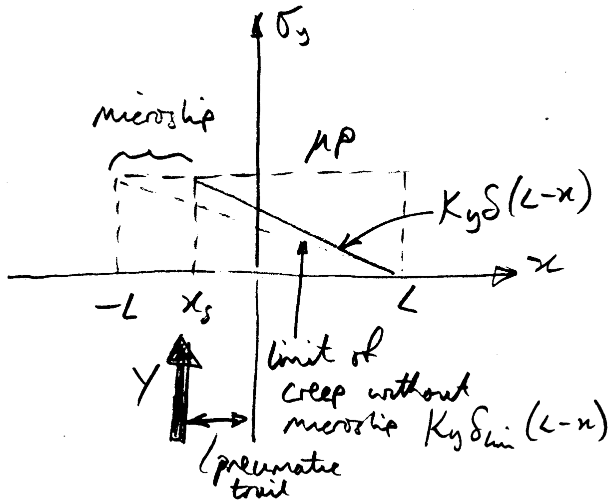
PART II 4C8 EXAM 2004 — SOLUTIONS

1.





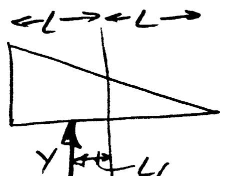
(a) Small δ — no microslip:

Displacement of bristles is $q_y = \delta(L-x)$ — (1)

No longitudinal slip ∴ $q_x = 0 \Rightarrow \sigma_x = 0$

Brush model (data sheet) $\sigma_y = k_y q_y = K_y \delta (L-x)$ — (2)

Realigning moment (data sheet):



$$N = \iint_A (x\sigma_y - y\sigma_x)\, dA = \int_{-h}^{h}\int_{-L}^{L} x K_y \delta (L-x)\, dx = -\frac{4}{3} L^3 h K_y \delta \quad (3)$$

Pneumatic trail $\ell = N/Y$ = distance of centre of pressure from centre of contact patch $\Rightarrow \ell = L/3$ [30%]

(b) The region of microslip starts at the location in the contact patch where

$$\sigma_y = \mu p = K_y \delta (L-x) \Rightarrow x_s = L - \frac{\mu p}{K_y \delta} \quad (4)$$

Once microslip begins, (3) has to be evaluated in 2 parts

$$N = 2h\int_{-L}^{x_s} x \mu p\, dx + 2h \int_{x_s}^{L} x K_y \delta (L-x)\, dx \quad (5)$$

Integrate (5) and use $p = \frac{Z}{4hL}$ & $\lambda = \frac{4L^2 h K_y \delta}{\mu Z}$ — (6)

To be consistent with part (a), the two solutions should have the same values of $N(\delta_{lim})$ and $\left.\frac{dN}{d\delta}\right|_{\delta_{lim}}$ where δ_{lim} is the steer angle at which microslip first starts at the rear of the contact area.

1 cont

δ_{lim} is found from (2) as:

$$\left|K_y \delta_{lim}(L - - L)\right| = \mu p \quad \text{ie} \quad \delta_{lim} = \frac{\mu (Z/4Lh)}{2K_y L}$$

$$\Rightarrow \delta_{lim} = \frac{\mu Z}{8L^2 h K_y} \Rightarrow \lambda_{lim} = 1/2 \qquad (7)$$

from (3) $N(\delta_{lim}) = -\frac{4}{3} L^3 h K_y \left(\frac{\mu Z}{8L^2 h K_y}\right) = -\frac{\mu Z L}{6}$

From (6) $\lambda_{lim} = \frac{4L^2 h K_y}{\mu Z}\left(\frac{\mu Z}{8L^2 h K_y}\right) = 1/2$

& from Question sheet $\frac{N}{\mu Z L} = \frac{1}{4\lambda}\left(\frac{1}{3\lambda} - 1\right) = \frac{1-3\lambda}{12\lambda^2}$ (8)

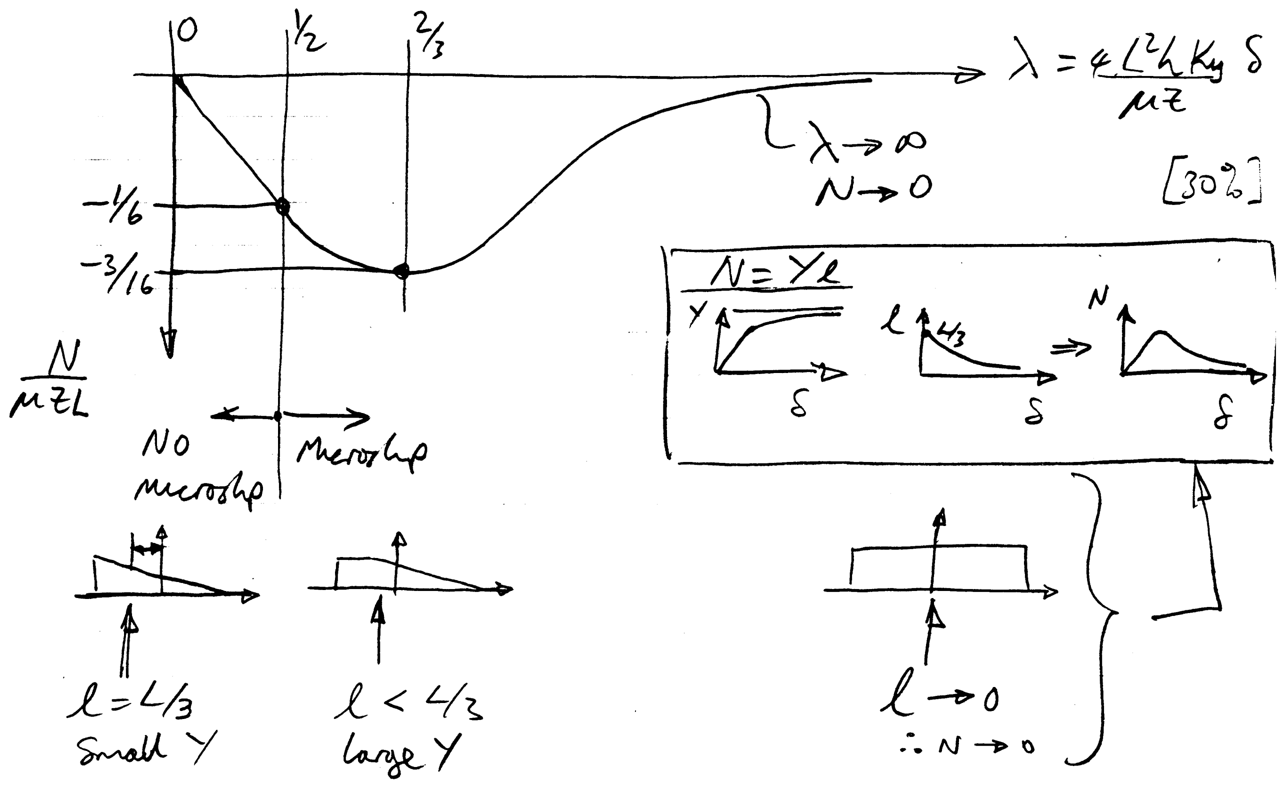
$$\frac{N(\delta_{lim})}{\mu Z L} = \frac{1}{2}\left(2/3 - 1\right) = -1/6 \checkmark$$

[20%]

(c) N_{max} found from

$$\frac{1}{\mu Z L}\left(dN/d\lambda\right) = \frac{12\lambda^2(-3) - (1-3\lambda)24\lambda}{144\lambda^4} = 0 \Rightarrow \lambda = 2/3$$

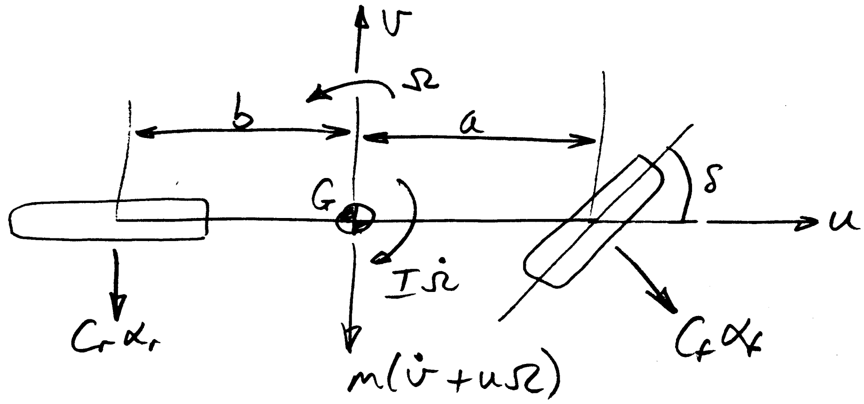
$$\Rightarrow \frac{N_{max}}{\mu Z L} = -3/16$$



[30%]

1 cont (d) Realigning moment is usually neglected in simple models of vehicle dynamics because the pneumatic trail is small relative to length of the vehicle. ie N is small relative to yawing moments on the vehicle body due to Y. [20%]

2.



(a) $\Sigma F: \quad m(\dot{v}+u\Omega) + C_f\alpha_f + C_r\alpha_r = 0$

$\Sigma M_G: \quad I\dot{\Omega} + aC_f\alpha_f - bC_r\alpha_r = 0$ (1)

Slip angles $\alpha_f = \frac{v+a\Omega}{u} - \delta$, $\alpha_r = \frac{v-b\Omega}{u}$ (2)

Combining (1) & (2)

$$m(\dot{v}+u\Omega) + (C_f+C_r)v/u + (aC_f-bC_r)\frac{\Omega}{u} = C_f\delta$$
$$I\dot{\Omega} + (aC_f-bC_r)\frac{v}{u} + (a^2C_f+b^2C_r)\frac{\Omega}{u} = aC_f\delta \quad (3)$$

Assumptions : All angles small
Neglect tyre realigning moment
Tyres behave linearly
δ is average of 2 steered front wheels
Neglect motion of sprung mass on suspension
[40%]

(b) Steady turning: $\dot{\Omega} = \dot{v} = 0$ & $\Omega = u/R$ (4)

(4) into (3) gives

$$\begin{bmatrix} C & Cs+mu^2 \\ Cs & Cq^2 \end{bmatrix} \begin{Bmatrix} \beta \\ 1/R \end{Bmatrix} = \begin{Bmatrix} C_f\delta \\ aC_f\delta \end{Bmatrix} \quad (5)$$

where $C = C_f + C_r$, $l = a+b$, $s = \frac{aC_f - bC_r}{C_f+C_r}$, $q^2 = \frac{a^2C_f + b^2C_r}{C_f+C_r}$, $\beta = v/u$

Solving (5) for $1/R$ gives

$$\frac{1/R}{\delta} = \frac{C\,C_f\,(a-s)}{C_fC_rl^2 - Csmu^2} = \frac{lC_fC_r}{C_fC_rl^2 - Csmu^2} \quad (6)$$

or $\delta = \frac{l}{R}\left(1 - \frac{Csmu^2}{l^2C_fC_r}\right)$ (7)

2 cont

Differentiate (7) $\Rightarrow$ $\dfrac{d\delta}{du} = -\dfrac{2l}{R}\left(\dfrac{csm}{l^2 C_f C_r}\right) u$ (8)

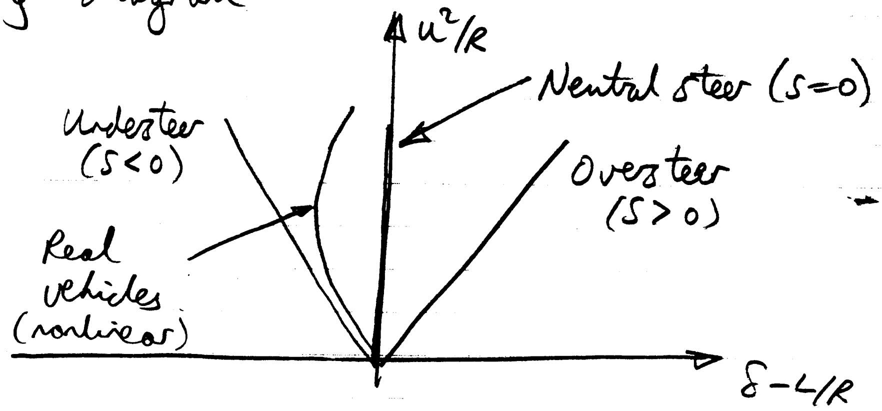
Neutral steer $(s=0)$ $\longrightarrow$ $\delta = l/R$ & $d\delta/du = 0$

Understeer $(s<0)$ $\longrightarrow$ $d\delta/du > 0$ all speeds

Oversteer $(s>0)$ $\longrightarrow$ $d\delta/du < 0$ all speeds

Vehicle becomes unstable when $u \geq \sqrt{\dfrac{C_f C_r l^2}{csm}}$

Handling diagram



[40%]

(c) Real vehicles understeer at low speeds & oversteer at high speeds. This is caused by two main effects

(i) Saturation of the tyres ie Nonlinear Y vs δ at high lateral accelerations

(ii) Effects of suspension compliance at high lat acc, particularly roll-steer (steering of axles due to body roll)

[20%]

3a) $Z_R = Z_v + Z_\phi$

Z_v and Z_ϕ uncorrelated, so

$$S_{Z_R}(n) = S_{Z_v}(n) + S_{Z_\phi}(n)$$

$$= S_{Z_\phi}(n)\left(\frac{1}{|G(n)|^2} + 1\right)$$

$$\therefore S_{Z_\phi}(n) = S_{Z_R}(n)\frac{|G(n)|^2}{1+|G(n)|^2}$$

$$= S_{Z_R}(n)\frac{\frac{n^2}{n_c^2+n^2}}{1+\frac{n^2}{n_c^2+n^2}} = \frac{\frac{n^2}{n_c^2+n^2}}{\frac{n_c^2+2n^2}{n_c^2+n^2}}$$

$$\underline{\underline{S_{Z_\phi}(n) = S_{Z_R}(n)\frac{n^2}{n_c^2+2n^2}}}$$

[15%]

b) $S_Z(n) = K n^{-2}$

but $\omega = 2\pi U n$

so $S_Z(n) = K n^{-2} = K\left(\frac{\omega}{2\pi U}\right)^{-2}$

must ensure that

$$\int_{n=0}^{n=\infty} S_Z(n)\,dn = \int_{\omega=0}^{\omega=\infty} S_Z(\omega)\,d\omega$$

but $d\omega = 2\pi U\,dn$

so $$\int_{\omega=0}^{\omega=\infty} K\left(\frac{\omega}{2\pi U}\right)^{-2}\frac{d\omega}{2\pi U} = \int_{\omega=0}^{\omega=\infty} S_Z(\omega)\,d\omega$$

$$\therefore\ S_z(\omega) = \frac{\kappa}{2\pi U}\left(\frac{\omega}{2\pi U}\right)^{-2}$$

$$\underline{\underline{S_z(\omega) = 2\pi U \kappa \omega^{-2}}}$$

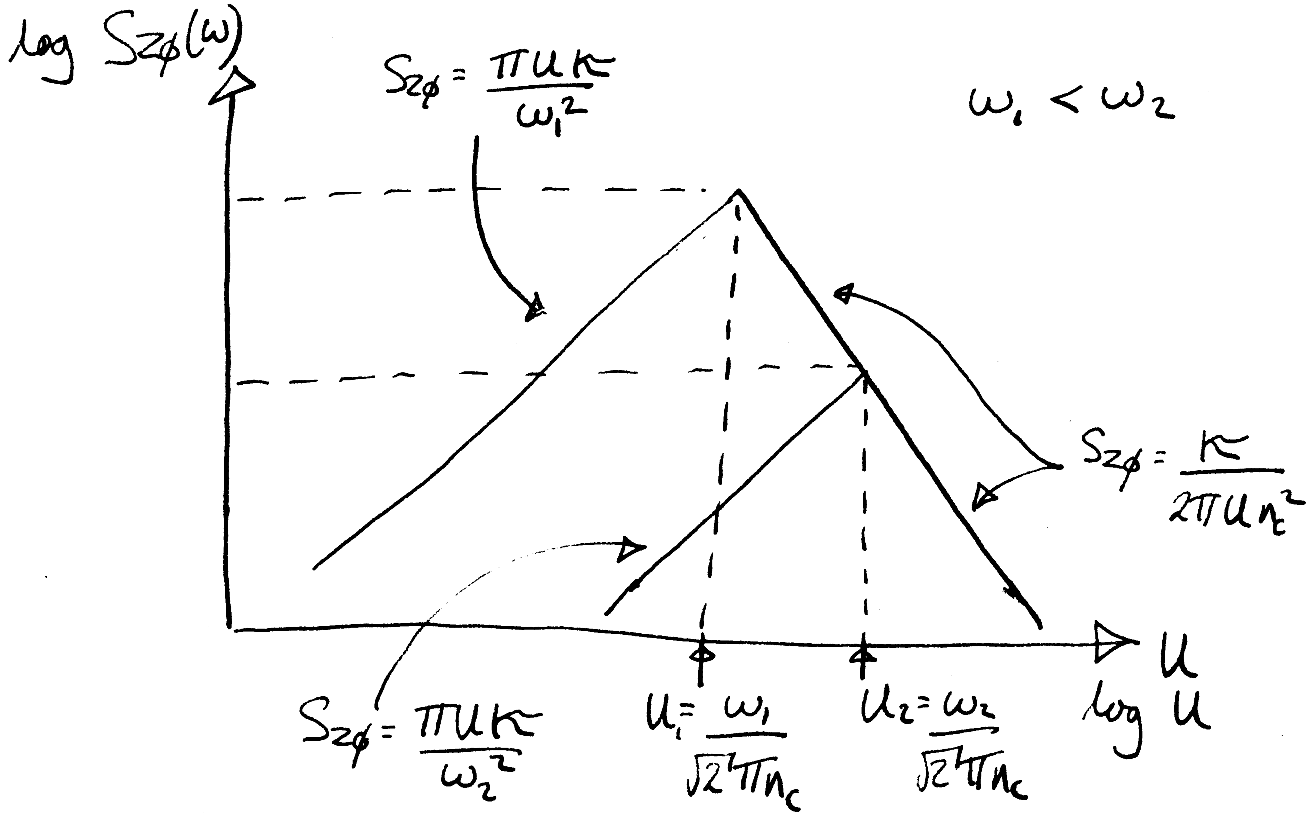
[15%]

c)
$$S_{z\phi}(n) = S_z(n)\frac{n^2}{n_c^2+2n^2}$$

$$S_{z\phi}(\omega) = S_z(\omega)\frac{\omega^2}{(2\pi U)^2 n_c^2 + 2\omega^2}$$

$$= \frac{2\pi U\kappa}{\omega^2}\ \frac{\omega^2}{(2\pi U)^2 n_c^2 + 2\omega^2}$$

$$S_{z\phi}(\omega) = \frac{2\pi U\kappa}{(2\pi U)^2 n_c^2 + 2\omega^2}$$



[40%]

d) At low speed, both modes are excited in roll, with the excitation increasing with speed. Eventually, the excitation of the low frequency mode begins to decrease as the speed increases, whilst the high frequency mode's excitation continues to increase.

For $n_c = 0.1$ cycle/m, $\omega_1 = 5$ rad/s, $\omega_2 = 50$ rad/s
$U_1 = 11.3$ m/s (40.7 km/hr) and $U_2 = 113$ m/s (407 km/hr), so in practice, the speed at which both modes see a reduction in excitation with speed is not reached.

[30%]

4a)

Newton's 2nd Law on each mass

$$m_s \ddot{z}_s = k(z_u - z_s) + c(\dot{z}_u - \dot{z}_s)$$

$$m_u \ddot{z}_u = k(z_s - z_u) + c(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u)$$

adding

$$\underline{\underline{m_s \ddot{z}_s + m_u \ddot{z}_u = k_t(z_r - z_u)}}$$

[15%]

b) from definition of transfer functions

$$(z_r - z_u) = H_{TF} \frac{\dot{z}_r}{k_t}$$

$$\therefore \quad z_u = z_r - \frac{H_{TF}\, j\omega z_r}{k_t}$$

$$z_u = z_r\left(1 - j\omega \frac{H_{TF}}{k_t}\right)$$

and

$$z_s - z_u = H_{ws}\, \dot{z}_r$$

$$z_s = H_{ws}\, j\omega z_r + z_u$$

$$= H_{ws}\, j\omega z_r + z_r\left(1 - j\omega \frac{H_{TF}}{k_t}\right)$$

$$z_s = \left(j\omega H_{ws} + 1 - j\omega \frac{H_{TF}}{k_t}\right) z_r$$

substituting for z_u and z_s into eqn in part (a):

$$-m_s\omega^2\left(j\omega H_{ws} + 1 - j\omega \frac{H_{TF}}{k_t}\right) z_r - m_u \omega^2 z_r \left(1 - j\omega \frac{H_{TF}}{k_t}\right)$$

$$= k_t(z_r - z_u)$$

$$= k_t\left(z_r - z_r\left(1 - j\omega \frac{H_{TF}}{k_t}\right)\right)$$

$$-m_s\omega^2 j\omega H_{WS} - m_s\omega^2 + jm_s\omega^3 \frac{H_{TF}}{k_t} - m_u\omega^2 + jm_u\omega^3 \frac{H_{TF}}{k_t}$$
$$= k_t - k_t + j\omega H_{TF}$$

$$-m_s j\omega^3 H_{WS} + H_{TF}\left(j\frac{m_s\omega^3}{k_t} + j\frac{m_u\omega^3}{k_t} - j\omega\right) = m_s\omega^2 + m_u\omega^2$$

$$m_s\omega^2 H_{WS} + H_{TF}\left(1 - \frac{(m_s+m_u)\omega^2}{k_t}\right) = j\omega(m_s+m_u) \qquad [50\%]$$

c) Once H_{TF} is specified, then H_{WS} is also specified i.e. cannot tune suspension parameters k and c to set H_{TF} and H_{WS} independently.

Also, there is an 'invariant point' in the H_{WS} transfer function at $\omega_1 = \sqrt{\frac{k_t}{m_s+m_u}}$, where

$$H_{WS}(\omega_1) = j\,\frac{m_s+m_u}{m_s}\sqrt{\cancel{\frac{m_s+m_u}{k_t}}\frac{k_t}{m_s+m_u}}$$

The value of $H_{WS}(\omega_1)$ is independent of the suspension parameters k and c. [35%]