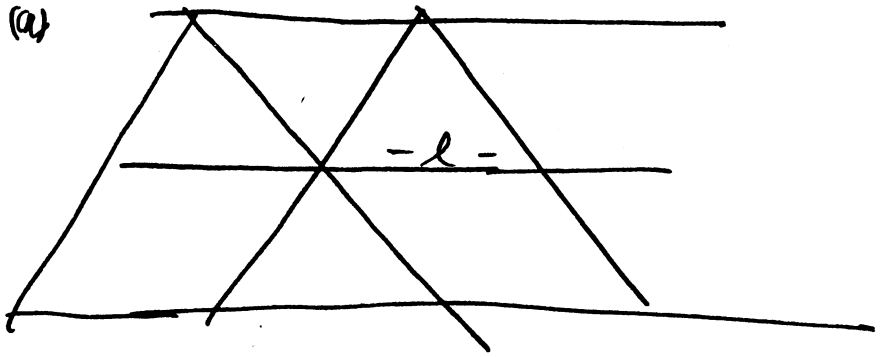
4C14 : CRIB 2004

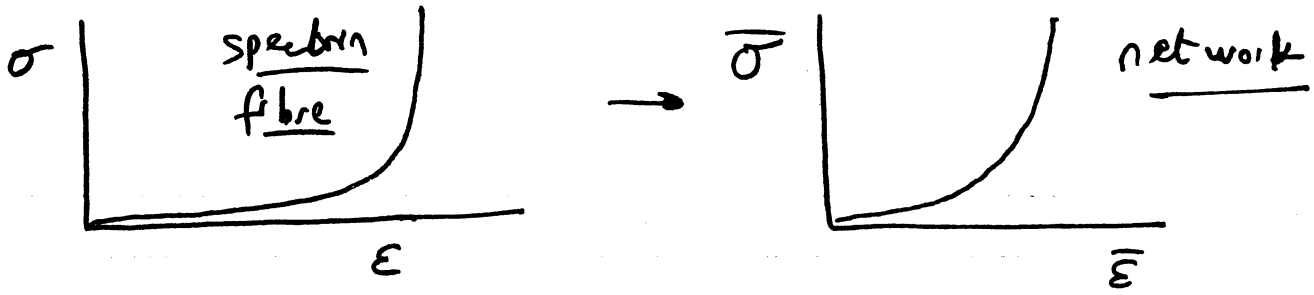
1. (a)



Spectrin elastomeric fibres are arranged in a fully triangulated manner on the outer membrane of the red blood cell.

Persistence length ξ_p of spectrin is much less than the length l, so spectrin behaves as a rubber, with a high lock-up strain.

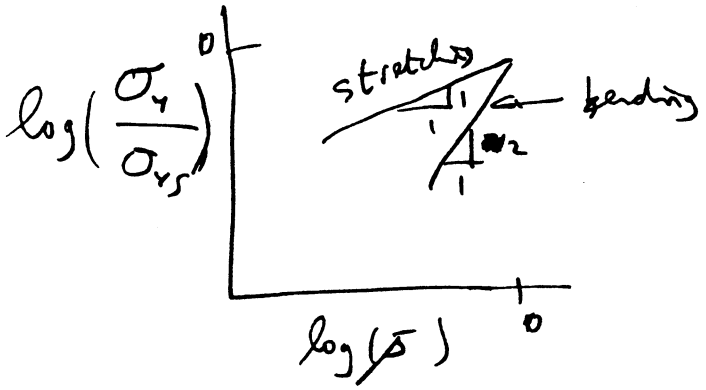
The structure (fully triangulated) is a redundant stretching structure.

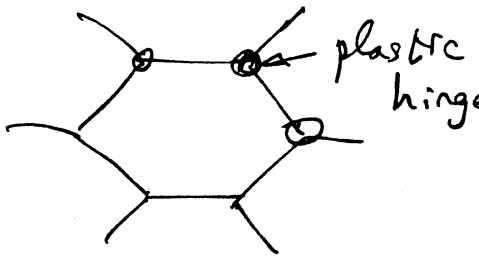


The elastin fibres when pulled straight has high modulus and strength due to the covalently bonded backbone.

(25%)

(b) Wood resembles a honeycomb structure — along the grain it is a stretching structure, while across the grain it is a bending structure





across the grain

(25%)

(c) ~~(iii)~~ Proteins are transported from the centrosome (adjacent to the cell nucleus) by the motors kinesins and dynein along the microtubules, and also along the actin networks of the cell's cytoskeleton.

These motors are powered by ATP and consume energy, analogous to muscle activation.

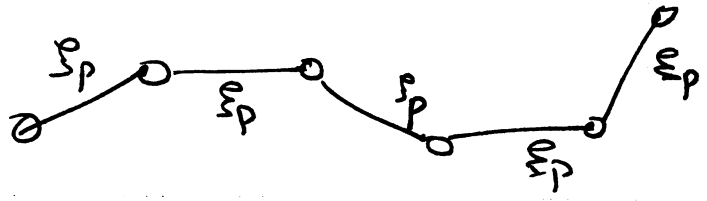
Diffusion, in contrast, as a passive concentration-gradient driven process and is very slow for large molecules.

(25%)

2. (a) (i) Persistence length, ξ_p is a material property dependent upon the bending stiffness D of the filament and thermal activation energy kT:

$$\xi_p \equiv \frac{D}{kT}$$

Over a length of ξ_p angular correlation is lost, and the filament can be thought of as a rubber-like link with freely-jointed ends at a spacing of ξ_p.



If $\xi_p <<$ distance l between nodes of cross-links of the filament, then the fibre is rubber-like, with a low modulus dictated by entropy.

If $\xi_p >> l$ then the fibre behaves like a stiff beam, eg. microtubules.

(25%)

(d)

Fungi secrete digestive enzymes outside of their bodies and absorb digested nutrients.

Plants have chloroplasts for capturing light energy and for storing this energy as glucose (and starches).
They consume the glucose in their mitochondria and generate ATP.

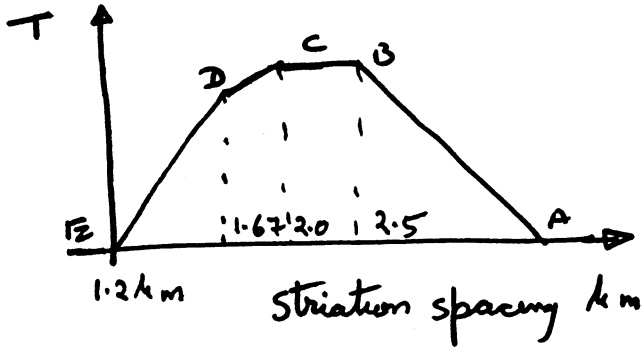
Animal cells do not have chloroplasts but do have mitochondria. These have proton-driven pumps to use the energy liberated by aerobic and anaerobic hydrolysis of glucose to produce ATP from ADP.

(25%)

2 (b) With a high Ca^{2+} concentration in the sarcoplasm, the myosin head is tightly bound to an adjacent actin filament. Before long ATP binds to the myosin & a conformational change reduces the affinity of myosin to actin & the two seperate, simultaneously causing the myosin to shift a distance of 5nm towards the +ve end of the actin filament, where it rebinds at a new location. Hydrolysis then causes the release of P_i from the ATP to produce ADP & the associated conformational change triggers the power stroke that drives the actin filament in the direction of the -ve end. During the power stroke the ADP is released returning the myosin to its original state, ready for the next ATP to come along & bond.

(25%)

(c)



The tension changes with change in overlap between the thick & thin filaments. Between B & C the overlap is a constant & so is the tension.

2. (a) (ii) nodal connectivity z dictates whether a cellular structure is bending or stretching dominated.

2D : need $z \geq 4$ for stretching

3D : need $z \geq 6$ for stretching.

Thus, in 2D,

compliant bar ($e.g.\ \xi_p << l$) + 'mechanism' $z < 4$ $\Rightarrow$ very compliant

+ stretching, redundant structure $z > 4$ $\Rightarrow$ moderate stiffness

stiff bar ($\xi_p >> l$) + mechanism $\Rightarrow$ moderate stiffness

+ stretching structure $\Rightarrow$ high stiffness

(25%)

If the force-length relation between A & B were nonlinear the force could still be produced by crossbridges working independently. It would be necessary to propose that crossbridges are stronger at one end or the other of the thick filament.

(25%)

Q3

(a) $-v\frac{dn}{dx} = (1-n)f(x) - ng(x)$

Under isometric conditions $v = 0$

ie $(1-n)f = ng$

$$n = \frac{f(x)}{f(x)+g(x)}$$

(10%)

(b) For shortening

$n = 0$ $x > h$ as no crossbridges are dragged there & the probability of attachment $= 0$ for $x > h$.

$\underline{0 < x < h}$

$$-v\frac{dn}{dx} = (1-n)f_0 - ng_0$$

$$-v\frac{dn}{dx} = -n(f_0+g_0) + f_0$$

Homogeneous soln. is

$$n = A\exp\left[\frac{(f_0+g_0)x}{v}\right]$$

Particular soln. is $n = B$

$$\Rightarrow B = \frac{f_0}{f_0+g_0}$$

ie $$n(x) = A\exp\left[\frac{(f_0+g_0)x}{v}\right] + \frac{f_0}{f_0+g_0}$$

$$n(h) = A \exp\left[\frac{(f_0+g_0)\,h}{v}\right] + \frac{f_0}{f_0+g_0} = 0$$

$$A = \frac{-f_0}{(f_0+g_0)\exp\left[\frac{(f_0+g_0)h}{v}\right]}$$

$$n(x) = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{\frac{(f_0+g_0)}{v}(x-h)\right\}\right]$$

$$\text{ie } n(0) = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{-\frac{(f_0+g_0)h}{v}\right\}\right]$$

$\underline{x<0}$

$$-v\frac{dn}{dx} = -ng_1$$

$$n = C\exp\left[\frac{g_1 x}{v}\right]$$

$$n(0) = C = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{-\frac{(f_0+g_0)h}{v}\right\}\right]$$

$$n(x) = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{-\frac{(f_0+g_0)h}{v}\right\}\right]\exp\left(\frac{g_1 x}{v}\right)$$

(50%)

(c) Suppose the isometric tension T_0 decreases to $T_1 < T_0$. The extension of each crossbridge decreases by an unknown amount ΔL, so that the crossbridge distribution changes from

$$n_s(x) = \frac{f(x)}{f(x)+g(x)} \quad \text{to} \quad n_s(x+\Delta L).$$

ΔL may be obtained by solving

$$T_1 = \frac{mSA}{2} \int_{-\infty}^{\infty} k x \, n(x+\Delta L)\, dx$$

To obtain $v(t)$ recall that $\frac{\partial T_1}{\partial t} = 0$ ie.

$$0 = \int_{-\infty}^{\infty} k x \frac{\partial n}{\partial t}(x,t)\, dx$$

with initial condition $n(x,0) = n_s(x+\Delta L)$

Also $$\frac{\partial n}{\partial t} - v\frac{\partial n}{\partial x} = (1-n)f - ng$$

Thus

$$0 = \int_{-\infty}^{\infty} k x \left[v\frac{\partial n}{\partial x} + (1-n)f(x) - ng(x) \right] dx$$

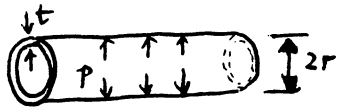
which can be solved with the above initial conditions to obtain $v(t)$ (40%)

4. (a) Arteries are elastic, muscular tubes that carry blood from the left ventricle to the capillaries. The walls of arteries tend to be thicker than other blood vessels due to the high pressure.

The veins carry blood from capillaries to the heart, following a path parallel with the arteries. Located within the veins are one-way valves that allow blood to flow toward the heart but does not blood to flow backwards. (15%)

(b) The capillaries are the smallest working unit in the blood vessels that connect the arterioles to the venules. The walls of the capillaries are only 1 cell thick, allowing for the exchange of nutrients and other substances like O_2 and CO_2. The cells in the walls of the capillaries have openings between them so that the exchange can take place. (15%)

(c)



$A = A_0 + CP$ ← compliance per unit length

$A = \pi r^2, \quad A_0 = \pi r_0^2$

$\sigma_H = \frac{Pr}{t} = E\varepsilon_H, \quad \varepsilon_H = \frac{u}{r} = \frac{r - r_0}{r}$

$\Rightarrow \pi(r^2 - r_0^2) = CEt(r - r_0)/r^2$

$\Rightarrow C = \frac{\pi r^2 (r^2 - r_0^2)}{Et(r - r_0)} = \frac{\pi r^2 (r + r_0)}{Et}$

$r \approx r_0 \Rightarrow \boxed{C = \frac{2\pi r^3}{Et}}$

(25%)

(d) The radius of pulmonary and systemic capillaries is much smaller than large arteries. The compliance of a blood vessel scales as r^3, and hence the compliance of large arteries is much larger than capillaries. In general, capillaries can be modelled as resistance vessels with zero change in volume, whereas arteries cannot. On the other hand the walls of veins are weaker (smaller E) than arteries, → veins are more compliant than arteries (for the same radius). (15%)

4. (e)

$$Q = \frac{\pi r^4}{24\mu L} \cdot \frac{(1+3\gamma P_0 + 3\gamma^2 P_0^2 + \gamma^3 P_0^3) - (1+3\gamma P_1 + 3\gamma^2 P_1^2 + \gamma^3 P_1^3)}{\gamma}$$

$$= \frac{P_0 - P_1}{8\mu L/\pi r^4} \cdot \left[1 + \gamma(P_0+P_1) + \frac{\gamma^2}{3}(P_0^2 + P_0P_1 + P_1^2)\right]$$

In the limit $\gamma \to 0$,

$$Q = \frac{P_0 - P_1}{8\mu L/\pi r^4} = \frac{P_0 - P_1}{R}$$

Here, $R = 8\mu L/\pi r^4$ is the resistance of the vessel of length L.

But $R = \rho L \Rightarrow \boxed{\rho = 8\mu/\pi r^4}$

Since Q is an increasing function of γ, it follows that a given fluid flux can be driven by a smaller pressure drop in a compliance vessel than in a noncompliance vessel. As veins are more compliant than arteries, the pressure drop in veins can be much less than in the arteries.

(30%)