MEMS Design Final Crib

4C15

1 (a)

$$k \text{ (spring constant)} = 8EH\left(\frac{W}{L}\right)^3$$

$$k = 8\times160\times10^{9}\times6\times10^{-6}\times\left(\frac{1.5}{150}\right)^3$$

$$k = 7.68 \text{ N/m}.$$

$$m = \rho V = 2330\times10^{-18}\times(6\times35\times60)$$

$$m = 2.936\times10^{-11} \text{ kg}.$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{7.68}{2.94\times10^{-11}}} = 81.4 \text{ kHz}.$$

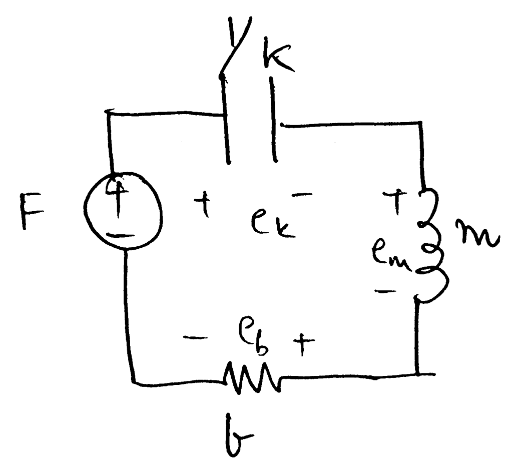
Damping modeled by Couette flow.

$$b = \frac{\eta A}{h} = \mu\left[\left(\frac{A}{y_0}\right)_{\text{fingers}} + \left(\frac{A}{y_0}\right)_{\text{substrate}}\right].$$

$$= 1.8\times10^{-5}\times1\times\left[\frac{35\times60}{3} + \frac{10\times10\times6}{1}\right]\times10^{-6}$$

$$= 2.34\times10^{-8} \text{ kg/s}.$$

(b)



$$F - e_k - e_m - e_b = 0$$

$$\frac{x(s)}{F(s)} = \frac{1}{s^2 m + bs + k}$$

$$\frac{\dot{x}(s)}{F(s)} = \frac{1}{sm + b + k/s}.$$

values of m, k, b from Question 1 (a).

(c). Static deflection $F/k = \frac{\frac{1}{2} N \epsilon_0 V^2 t/g}{k}$

$$= \frac{1}{2} \times 8.85 \times 10^{-12} \times 10 \times 1 \times 1 \times 6/1 \times \frac{1}{7.68}$$

$$= 3.46 \times 10^{-11} m.$$

Dynamic deflection $Q = \frac{\sqrt{km}}{b}$

$$Q = \frac{\sqrt{7.68 \times 2.94 \times 10^{-11}}}{2.34 \times 10^{-8}} \simeq 641.7$$

$$x_{dynamic} \simeq \frac{\epsilon_0 V_{AC} \cdot V_{DC} N t/g}{k} \cdot Q$$

$$\simeq 11.1 \text{ nm}$$

Q2 (a) Piezoresistors are located at positions of maximum induced stress (in response to measurand) in the membrane.

$w(x,y)$ — membrane deflection as a function of (x,y) coordinates.

$$w(x,y) = \frac{c_1}{4}\left[1+\cos\left(\frac{2\pi x}{L}\right)\right]\left[1+\cos\left(\frac{2\pi y}{L}\right)\right].$$

$$\frac{\partial^3 w}{\partial x^3} = 0 \quad \text{and} \quad \frac{\partial^3 w}{\partial y^3} = 0$$

$$\sin\left(\frac{2\pi x}{L}\right) = 0 \quad \text{and} \quad \sin\left(\frac{2\pi y}{L}\right) = 0.$$

$$x = \pm L/2 \quad \text{and} \quad y = \pm L/2.$$

Piezoresistors are located longitudinally along x-axis at $(\pm L/2, 0)$ or transversely along y-axis at $(0, \pm L/2)$.

argue by symmetry.

(b) Place resistors at $(\pm L/2, 0)$ longitudinally.

$$\sigma_x = \frac{EH}{2\rho_x} = \frac{EH}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)\Big|_{x=L/2}.$$

$$= \frac{EH}{2} \cdot \frac{c_1}{4}\left(\frac{2\pi}{L}\right)^2 \times 1 \times 2$$

$$c_1 = \frac{6P(1-\nu^2)L^4}{\pi^4 EH^3}$$

$$\Rightarrow \sigma_x = \left(\frac{L}{H}\right)^2 P \times 0.606$$

$$\simeq 0.606 \left(\frac{1000}{10}\right)^2 \times 10^3 \text{ Pa}$$

$$\simeq 6.06 \text{ MPa.}$$

$\frac{\Delta R}{R}$ for 1 piezoresistor $= \pi_{xl} \sigma_x$

$$\simeq -31.2 \times 10^{-11} \times 6.06 \times 10^6$$

$$\simeq -1.9 \times 10^{-3}.$$

(c) $\frac{\Delta R}{R}$ for 1 piezoresistor due to temperature drift

$$\simeq \alpha \Delta T$$

$$\simeq 2 \times 10^{-5} \times 100 \simeq 2 \times 10^{-3}.$$

$\therefore$ uncertainity in pressure reading

$$\simeq \frac{2}{1.9} \times 10^3 \text{ Pa} \simeq \pm 1.05 \text{ kPa.}$$

(d) Capacitive sensing – low noise, less prone to temperature drift, harder to implement in a bulk micromachining process as compared to piezoresistive sensing.

Q3. (a)

$$k = 4EH\left(\frac{W}{L}\right)^3$$

$$m = 4\times10^{-9}\ kg.$$

$$(2\pi\times5\times10^3)^2 = \frac{k}{m} = \frac{4\times1.6\times10^{11}\times10^{-5}}{4\times10^{-9}}\times\left(\frac{W}{L}\right)^3$$

$$\therefore \left(\frac{W}{L}\right)^3 = 6.17\times10^{-7} \Rightarrow L = 235\,\mu m.$$

(b)

$$x_{1g} = \frac{4\times10^{-9}\times9.81}{3.95} \simeq 9.94\,nm.$$

$$C = \frac{\epsilon_0 A}{g-x}$$

$$\Delta C = \frac{\epsilon_0 A}{g}\cdot\left(\frac{x}{g}\right) \quad \text{for } x \ll g.$$

$$\frac{\Delta C}{a} \simeq \frac{8.85\times10^{-12}\times100\times10\times10^{-6}\times8\times9.94\times10^{-3}}{1}\times\frac{1}{9.8}$$

$$\simeq 0.0717\ fF/m/s^2.$$

(c)

$$k_{el} = \partial F/\partial x$$

$$= \frac{-\epsilon_0 N t L V^2}{g^3}$$

$$\simeq \frac{-8.85\times10^{-12}\times8\times10\times100\times5^2}{10^{-6}}$$

$$\simeq -1.78\ N/m.$$

$$k_{new} = 3.95 - 1.78 \simeq 2.17\,N/m.$$

$$\therefore \left.\frac{\Delta C}{a}\right|_{new} = \frac{3.95}{2.17}\times0.0717 = 0.131\ fF/m/s^2$$

(d) (i) $$b = \frac{96\, N_{fingers}\, \eta\, L W^3}{\pi^4\, H^3}$$

$$= \frac{96 \times 1.8 \times 10^{-5} \times 8 \times 100 \times 10^{-6}}{\pi^4} \left(\frac{10}{1}\right)^3$$

$$= 1.419 \times 10^{-5} \ kg/s$$

(ii) $$\overline{F_n} = \sqrt{4 k_B T b}$$

$$= \sqrt{1.66 \times 10^{-20} \times 1.42 \times 10^{-5}}$$

$$\simeq 0.485 \ pN/\sqrt{Hz}$$

$$\overline{a_n} \simeq \frac{0.485 \times 10^{-12}}{4 \times 10^{-9}} \ m/s^2/\sqrt{Hz}$$

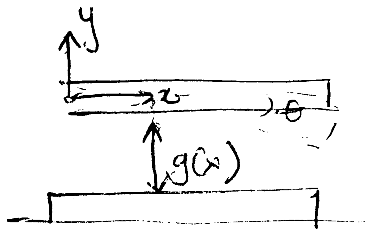
$$\simeq 12.36 \ \mu g/\sqrt{Hz}$$

4C15 MEMS Design.

4(a)

WIDTH OF PLATE
$b = \frac{A}{L}$

$$\tau = k_\theta \theta = \int_0^L x \, dF$$



$$= \int_0^L \frac{x \, \epsilon_0 b V^2}{2 g(x)^2} dx$$

$$= \frac{\epsilon_0 b V^2}{2} \int_0^L \frac{x \, dx}{(g - x\theta)^2}$$

$$= \frac{\epsilon_0 b V^2}{2\theta^2} \left[\ln(g - L\theta) - \ln(g) + \frac{g}{g - L\theta} - 1 \right]$$

$$\tau = k_\theta \theta = \frac{\epsilon_0 b V^2}{2\theta^2} \left[\ln\left(\frac{g - L\theta}{g}\right) + \frac{L\theta}{g - L\theta} \right] \quad \text{— (1)}.$$

$$V = \sqrt{\frac{2 k_\theta g^3}{b L^3 \epsilon_0} \frac{\left(\frac{L\theta}{g}\right)^3}{\ln\left[1 - \frac{L\theta}{g}\right] + \left(\frac{L\theta/g}{1 - L\theta/g}\right)}} \quad \text{— (2)}$$

(b) Pull-in instability $\frac{\partial \tau'}{\partial \theta} = 0$ or $\frac{\partial V}{\partial \theta} = 0$

$\tau' = \tau - k_\theta \theta$ — τ given by (1).

V is maximum

Substituting $x = L\theta/g$ in (2).

$$V^2 = \frac{2 k_\theta g^3}{b L^3 \epsilon_0} \frac{x^3}{\ln(1 - x) + \frac{x}{1 - x}} \quad \text{— (3)}$$

$$\frac{\partial V}{\partial \theta} = 0 \quad \Rightarrow \quad \frac{\partial V}{\partial x} = 0$$

Differentiating (3)

$$2V\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left[\frac{x^3}{\ln(1-x) + \frac{x}{1-x}}\right]\left(\frac{2K_\theta g^3}{bL^3 \epsilon_0}\right) = 0$$

$$\Rightarrow \quad \frac{x^2}{(1-x)^2} = 3\left[\ln(1-x) + \frac{x}{1-x}\right]$$

$$x^2 = 3\left[(1-x)^2 \ln(1-x) + x(1-x)\right]$$

substituting $x = 0.44$ we confirm this is a solution

and $\therefore$ $\theta_{PI} \simeq 0.44\, g/L$

(c) Substituting θ_{PI} in the relation for V we get :-

$$V_{PI} = 0.645\sqrt{\frac{2K_\theta g^3}{\epsilon_0 b L^3}}$$

The actual pull-in voltage as measured is likely to be a little lower as the beam translates as well as rotates.