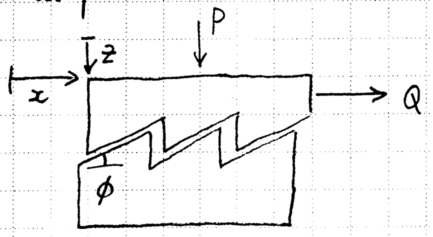
ENGINEERING TRIPOS 2003/2004.
MODULE 4D5 FOUNDATION ENGINEERING.

1) a, Quote : Dissipation, $D = f x \left(1 - \frac{\sin\psi}{\sin\phi}\right)\sec^2\psi$

Normality $\Rightarrow \sin\psi = \sin\phi \Rightarrow D = 0$

Or, graphically :



At failure $\Rightarrow \frac{Q}{P} = \tan\phi$

Compatibility $\Rightarrow \frac{\delta z}{\delta x} = -\tan\phi$

substitute in.

$$\text{Dissipation} = Q\,\delta x + P\,\delta z$$

$$= P\tan\phi\,\delta x - P\tan\phi\,\delta x$$

$$= 0$$

[30%]

b, Upper bound calculation – equate work done.

Purely frictional material $\Rightarrow$ no internal work dissipated.

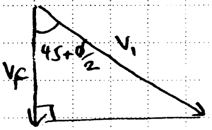
footing width $= B$

Length $BE = \frac{B}{2\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}$

BC is log spiral so $CE = BE\exp\left(\frac{\pi}{2}\tan\phi\right) = \frac{B\exp\left(\frac{\pi}{2}\tan\phi\right)}{2\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}$

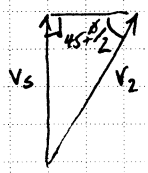
Length $DE = 2\,CE\sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = B\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left(\frac{\pi}{2}\tan\phi\right)$

If footing velocity $= V_f$, $V_1 = \frac{V_f}{\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}$



If surcharge upward velocity component $= V_s$,

$V_2 = \frac{V_s}{\sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}$



Work equation : (surcharge and footing load only).

$$q_f V_f B = \sigma_{v0} V_s B\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left(\frac{\pi}{2}\tan\phi\right)$$

$$q_f V_f B = \sigma_{v0} V_2 \sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right) B \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left(\frac{\pi}{2}\tan\phi\right)$$

$$q_f V_f B = \sigma_{v0} V_1 \sin\left(\frac{\pi}{4}+\frac{\phi}{2}\right) B \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left(\pi\tan\phi\right)$$

$$q_f V_f B = \sigma_{v0} V_f \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) B \exp\left(\pi\tan\phi\right)$$

$$\frac{q_f}{\sigma_{v0}} = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left(\pi\tan\phi\right).$$

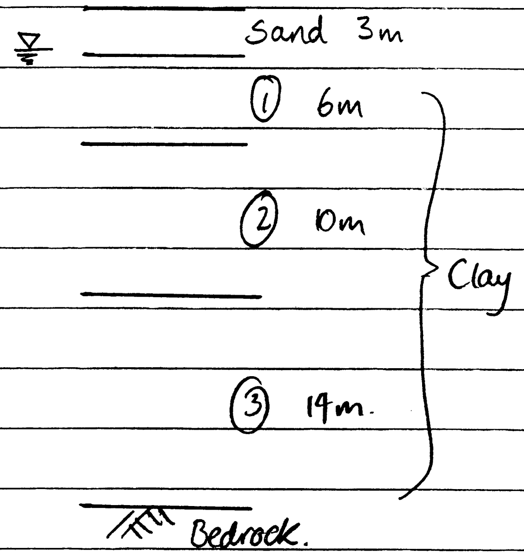
[70%]

2) a) Net bearing pressure is the pressure exerted by a structure and its foundation minus the previous in situ vertical total stress acting at the base of the foundation. It is therefore, the change in total vertical stress at the founding level due to the construction

$$q_{net} = \frac{30000}{30 \times 10} - 2 \times 20 = 60 \text{ kPa}$$ [10%]

b) Division into 3 layers.
(thinner at top where $\Delta\sigma_v/\Delta z$ is highest)

Consider two loaded rectangles.



for point A, L=30, B=10
for point B, L=15, B=5

x4 for superposition

Layer	Depth (below foundation base)	Entire n x m	Quarter n x m	I [Fadum] Entire	I [Fadum] Quarter	$\Delta\sigma_z$ (kPa) A (entire)	$\Delta\sigma_z$ (kPa) B (quarter)
①	4 m	7·5x2·5	3·75x1·25	0·24	0·215	14·4	51·6
②	12 m	2·5x0·83	1·25x0·42	0·185	0·11	11·1	26·4
③	24 m	1·25x0·42	0·63x0·21	0·11	0·05	6·6	12

For each area (entire and quarter) the foundation site is normalised by depth to give n x m. Fadum's chart is used to find the corresponding influence factor, I, and the resulting fraction of q_{net} acting at mid-depth in each layer.

Layer	Depth	σ'_{z0} (kPa)	σ'_{zf} (kPa) A	σ'_{zf} (kPa) B	$\bar{\sigma}_z$ (kPa) A	$\bar{\sigma}_z$ (kPa) B	Settlement (mm) A	Settlement (mm) B
①	4 m	- 84	98·8	135·6	91·4	109·8	7·3	23·1
②	12 m	- 164	175·1	190·1	169·6	177·1	6·1	14·1
③	24 m	- 284	290·6	296	287·3	290	3·5	6·3

3) continued

Settlement of a layer, $\Delta z = H \frac{\Delta \sigma_z'}{E} = H \frac{\Delta \sigma_v'}{500(\sigma_v')^{0.7}}$

layer thickness

Total settlement at A = 16.9 mm

—— " —— B = 43.5 mm

Differential settlement = 43.5 − 16.9 = 26.6 mm [70%]

[or $26.6/\sqrt{15000^2 + 5000^2} \sim 1/600$]

c) To reduce the differential settlement:

- construct a thicker/stiffer raft, use ground beams
- add piles beneath the centre of the raft.
-

[20%]

3) a) Design ultimate load = $20 \times 1{\cdot}35 + 10 \times 1{\cdot}5$ = 42 kPa / storey
= $42 \times 15 \times 15$ = 9450 kN/storey [10%]

b) Cast at ground level, square => $N_c = 6{\cdot}2$ (Databook, Skempton 1951)

$q_f = N_c s_u + \cancel{\gamma D} = 6{\cdot}2 \times 30 = 186\,\text{kPa}$

Design value, $q_{fD} = \dfrac{186}{1{\cdot}4} = 133\,\text{kPa}$

Number of storeys = 133/42 = 3 [30%]

c) Fleming et al (1992) calculated base resistance:

Pile diameters of 0·5 – 1m => Embed tip 5 diameters into sand, say 5 m.

$\sigma_{v} = 18 \times 35 + 20 \times 5 = 730\,\text{kPa}$

$u = 10 \times 35 = 350\,\text{kPa}$

$\sigma_v' = 380\,\text{kPa}.$

=> from Fleming et al (1992) chart: $q_b = 17$ MPa if $\phi = 30°$; $q_b = 26$ MPa if $\phi = 33°$

If $\alpha = 0{\cdot}8$, $\tau_s = 24$ kPa. => assume $q_b = 23$ MPa for $\phi = 32°$

Considering 1m diameter piles (@ 3 diameter spacings)

=> 5 × 5 grid = 25 piles

Total base area, = $25 \times \pi . 0{\cdot}5^2 = 19{\cdot}6\,\text{m}^2$

Total base capacity = 451 MN

Total shaft area = $25 \times \pi . 1 . 35 = 2749\,\text{m}^2$ [ignoring shaft capacity in sand]

Total shaft capacity = 66·0 MN

Design value of pile capacity = $\dfrac{451+66}{3} = 172$ MN

=> Number of storeys = $\dfrac{172000}{9450} = 18$ (rounded down).

Considering 0·5m diameter piles (@ 3 diameter spacings)

=> 10 × 10 grid = 100 piles [Base area and capacity unchanged]

Total shaft area = $100 \times \pi . 0{\cdot}5 . 35 = 5498\,\text{m}^2$

Total shaft capacity = 132 MN

Design value of pile capacity = $\dfrac{451+132}{3} = 194$ MN

=> Number of storeys = $\dfrac{194000}{9450} = 20$ (rounded down).

[40%]

CRIB

3) d) Yes. In clay, capacity is dominated by shaft friction.

Shaft area is proportional to $\frac{1}{\text{diameter}}$ for pile group filling a fixed area at a fixed spacing [10%]

e) few large diameter piles – quicker to install

Many small diameter piles – smaller rig, easier access to enclosed site.

– greater redundancy [10%]

4) a) API (2000) design parameters

$q_b = N_q \sigma'_{vo} \leq q_{b,lim}$

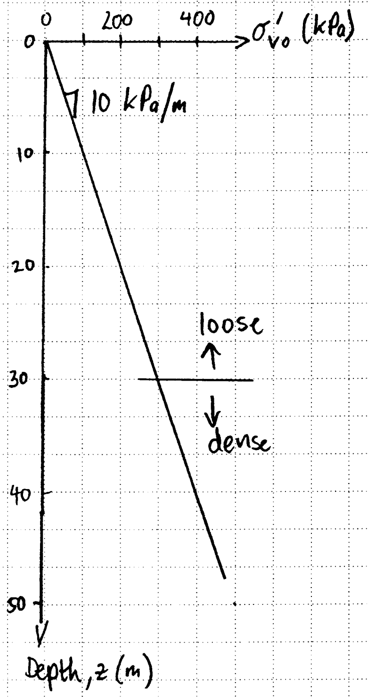
$\tau_s = K \sigma'_{vo} \tan\delta < \tau_{s,lim}$ $\sigma'_{vo} = 10z$ (z in m, σ'_{vo} in kPa)

Loose sand: ($z < 30$m) $N_q = 12$, $q_{b,lim} = 2.9$ MPa. $\Rightarrow q_b = 120z$, $z_{crit} = 24.1$m

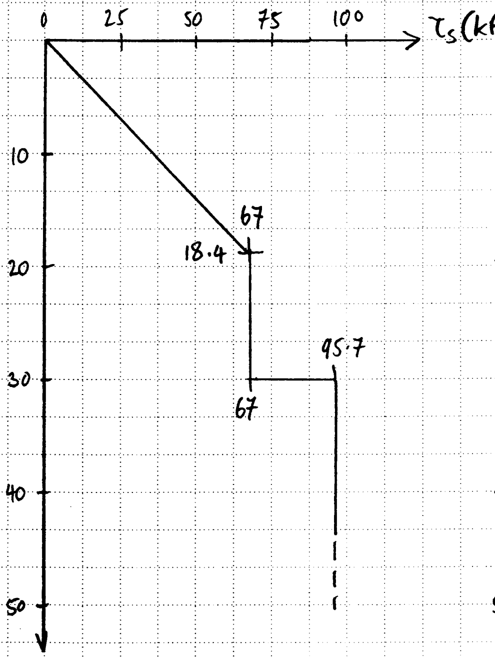
$K = 1.0$, $\delta = 20°$, $\tau_{s,lim} = 67$ kPa $\rightarrow \tau_s = 3.64z$, $z_{crit} = 18.4$m

Dense sand: ($z > 30$m) $N_q = 40$, $q_{b,lim} = 9.6$ MPa. $\Rightarrow$ At $q_{b,lim}$ @ 30m depth.

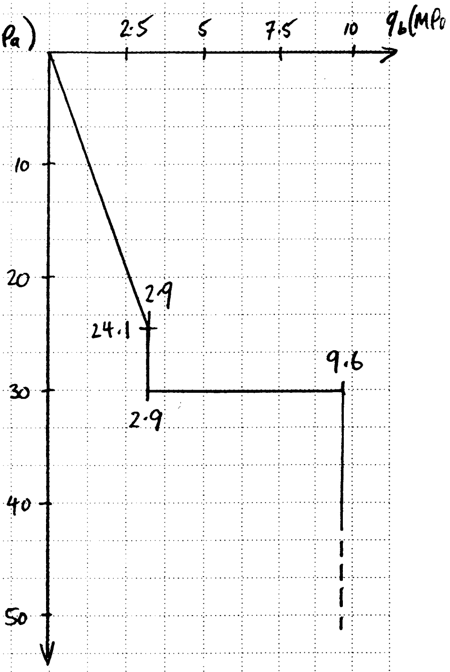
$K = 1.0$, $\delta = 30°$, $\tau_{s,lim} = 95.7$ kPa $\Rightarrow$ At $\tau_{s,lim}$ @ 30m depth.



Vertical effective stress



Unit shaft friction



Unit base resistance

[25%]

b) Shaft friction on region $z < 30$m:

$$Q_s = \bar{\tau}_s L \pi D = \left[\frac{1}{30} \left(\frac{67 \times 18.4}{2} + 67 \times 11.6 \right) \right] 30 . \pi . 1 = 4378 \text{ kN}$$

$\bar{\tau}_s$

Total shaft friction on pile $L > 30$, $Q_s = 4378 + (L-30)\, 95.7 . \pi . 1 = 301L - 4642$ kN

Base resistance on pile $L > 30$, $Q_b = q_b \pi (\frac{D}{2})^2 = 9.6 . \pi . (\frac{1}{2})^2 = 7540$ kN.

Ultimate compression capacity, $\underline{Q_c = 2898 + 301L}$ kN

Ultimate tension capacity, $\underline{Q_T = 301L - 4642}$ kN

[25%]

4) c) Design compression capacity (FOS=2), $Q_C = 1449 + 150{\cdot}5L$ kN

Design tension capacity , $Q_T = 150{\cdot}5L - 2321$ kN

for n piles, foundation compression capacity, $P_C = n(1449 + 150{\cdot}5L)$ kN

— " — — " — tension — " —, $P_T = n(150{\cdot}5L - 2321)$ kN

Optimal design, minimise nL =>

To satisfy compression requirement: $n > \dfrac{68000}{1449 + 150{\cdot}5L}$

— " — tension — " — : $n > \dfrac{34000}{150{\cdot}5L - 2321}$

Trial values

L=40 Comp $n > 9{\cdot}1$ } nL = 364m

Tens $n > 9{\cdot}1$

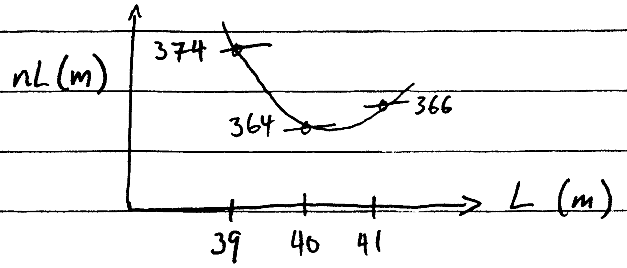
L=39 Comp $n > 9{\cdot}3$ → nL = 374m

Tens $n > 9{\cdot}6$

L=41 Comp $n > 8{\cdot}92$ → nL = 366m.

Tens $n > 8{\cdot}83$

Optimal value (rounding up to nearest m), L = 41m, n=9



[40%]

d) Long piles are more flexible - may take large settlement to mobilize full capacity due to compression of shaft, and any strain softening will lead to reduced capacity.

Long piles may suffer greater friction fatigue than short ones due to more cycles during installation. Upper part of shaft may have lower τ_s than shorter pile in same ground.

[10%]