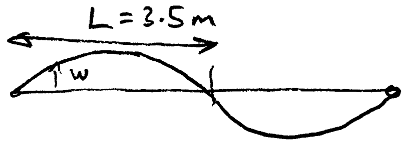
ENGINEERING TRIPOS PART IIB 2003-2004

Module 4D6 DYNAMICS IN CIVIL ENGINEERING.

CRIBS



Q1.a) Assume $w \approx A \sin \frac{\pi x}{L}$

or $\bar{u} = \sin \frac{\pi x}{L}$

$$M_{eq} = \int_0^{2L} m \bar{u}^2 \, dx = 2m \int_0^L \underbrace{\sin^2 \frac{\pi x}{L} \, dx}_{= L/2} = 2m \frac{L}{2} = mL$$

$$K_{eq} = \int_0^{2L} EI \left(\frac{d^2 \bar{u}}{dx^2} \right)^2 dx = 2 EI \int_0^L \left(\frac{\pi}{L} \right)^4 \sin^2 \frac{\pi x}{L} \, dx = 2 EI \frac{\pi^4}{L^4} \frac{L}{2}$$

$$= \frac{EI \pi^4}{L^3}$$

$$\omega = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{EI \pi^4}{L^3 (mL)}} = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{m}}$$

457 × 152 × 60 UB $m = 59.8$ kg/m

$I = 25500 \text{ cm}^4 = 25500 \times 10^{-8} \text{ m}^4$

$E = 210 \times 10^9 \text{ N/m}^2$

$$\omega = \frac{\pi^2}{L^2} \sqrt{\frac{210 \times 10^9 \times 25.5 \times 10^{+3} \times 10^{-8}}{59.8}} = 762.4 \text{ rads/s.}$$

3.5

$$f = \frac{\omega}{2\pi} = 121.3 \text{ Hz} \approx \underline{120 \text{ Hz}}.$$

Also, $M_{eq} = m.L = (59.8)(3.5) = 209.3$ kg

$$K_{eq} = \frac{\pi^4 EI}{L^3} = \frac{\pi^4 . 210 \times 10^9 . 25.5 \times 10^{-5}}{3.5^3} = 121.7 \times 10^6 \text{ N/m}$$

1(b) $F_{eq} = \text{"}\sum_i F_i \bar{u}_i\text{"} = F\,\bar{u}\big|_{x=0.8}$

$$= (10 \times 10^3\ \text{N}) \times \underbrace{\sin\left(\frac{0.8\pi}{3.5}\right)}_{0.658}$$

$$= \underline{\underline{6\,580\ \text{N}}}$$

$$u_{static} = \frac{F_{eq}}{K_{eq}} = \frac{6580\ \text{N}}{121.7 \times 10^6\ \text{N/m}} = \underline{\underline{54 \times 10^{-6}\,\text{m}}} \quad (\text{small !})$$

$$t_d = 20 \times 10^{-3}\ \text{s}$$

$$T = \frac{1}{f} = \frac{1}{121.3} = 0.0082 = 8.2 \times 10^{-3}\ \text{s}$$

$$\frac{t_d}{T} = \frac{20 \times 10^{-3}}{8.2 \times 10^{-3}} = 2.43 \qquad \underset{(\text{from data sheet})}{\text{DAF}} \approx 1.03$$

$$u_{dynamic} = u_{static} \times \text{DAF} = (54 \times 10^{-6}\,\text{m})(1.03) = \underline{\underline{56 \times 10^{-6}\,\text{m}}}$$

This occurs at midspans of AB and of BC.

1(c). $F_{eq} = \text{"}\int f\,\bar{u}\,dx\text{"} = \int_0^L (130)\sin\frac{\pi x}{L}\,dx = 130\,L\,\frac{2}{\pi} \quad (L = 3.5\,\text{m})$

$$= (130)(3.5)\frac{2}{\pi} = \underline{\underline{290\,\text{N}}}$$

Worst case is resonance, when frequency of forcing ~ natural frequency

$\text{DAF} \simeq \frac{1}{2\zeta}$ for ζ small (Mechanics Data Book, (IA material))

ζ = fraction of critical damping $= 1.2 \times 10^{-2}$

$$u_{static} = \frac{F_{eq}}{K_{eq}} = \frac{290\,\text{N}}{121.7 \times 10^6\,\text{N/m}} = \underline{\underline{2.38 \times 10^{-6}\,\text{m}}}$$

$$u_{dynamic} = u_{static} \times \text{DAF} = 2.38 \times 10^{-6} \times \overbrace{\frac{1}{2(1.2 \times 10^{-2})}}^{41.67}$$

$$= 99 \times 10^{-6}\,\text{m} = \underline{\underline{0.1\ \text{mm}}}$$

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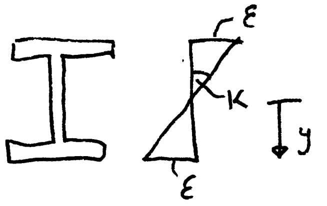
1(c) cont'd.



$$\varepsilon_{max} = (Ky)_{max}$$

$$\sigma_{max} = E\varepsilon_{max} = EKy_{max}$$

$$\therefore K = \frac{d^2u}{dx^2} = (1\times10^{-4}\,m)\times \frac{d^2}{dx^2}\sin\frac{\pi x}{L}\Big|_{\text{at midspan}}$$

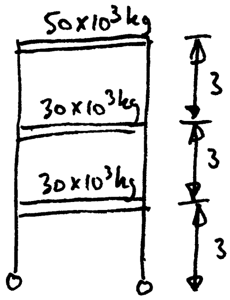
$$= 1\times10^{-4}\,m\ \frac{\pi^2}{L^2}$$

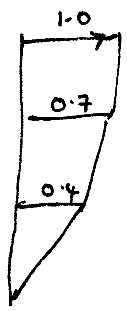
$$= \frac{1\times10^{-4}\,\pi^2}{3.5^2} = \underline{\underline{0.806\times10^{-4}\ m^{-1}}}$$

$$\therefore \sigma_{max} = EKy_{max} = (210\times10^9\,N/m)(0.806\times10^{-4}\,m^{-1})\left(\frac{4546}{2}\times10^{-3}\,m\right)$$

$$= 3.8\times10^6\ N/m^2 = \underline{\underline{3.8\ MPa}}$$

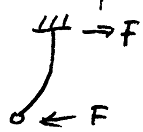
Q2





Assumed mode.

First floor

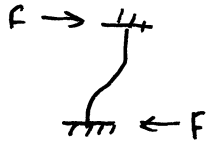


$$K_1 = \frac{3EI}{L^3}\times 8$$

$$= \frac{3(63\times10^6\,Nm^2)}{3^3\ m^3}\,8$$

$$= \underline{56\times10^6\ N/m}$$

Top two floors



$$K = \frac{12EI}{L^3}\times 8$$

$$= 4\times K_1$$

$$= \underline{224\times10^6\ N/m}$$

$$M_{eq} = [30(0.4)^2 + 30(0.7)^2 + 50.\ 1^2]\times10^3\ kg$$

$$= \underline{69.5\times10^3\ kg}$$

$$K_{eq} = 56\times10^6(0.4)^2 + 224\times10^6(0.7-0.4)^2 + 224\times10^6(1-0.7)^2$$

$$= \underline{49.28\times10^6\ N/m}$$

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Q2 cont'd.

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{49.28 \times 10^6}{69.5 \times 10^3}} = \underline{\underline{4.24\,Hz}} \approx 4\,Hz$$

Modal participation factor $\Gamma = \frac{\sum M\bar{u}}{\sum M\bar{u}^2} = \frac{30(0.4) + 30(0.7) + (50)(1)}{69.5}$

$$= 1.194 \approx 1.2$$

Definition: The modal participation factor Γ compares the mass participating in the forcing function with the mass participating in the inertia effects M_{eq}.

2(b). From Figure 2c, with $T \approx \frac{1}{4} = 0.25\,s$

$\frac{S_{da}}{a_g} = 2.5$ so $S_{da} = 2.5\,a_g = (2.5)(0.15g) = \underline{\underline{3.68\,m/s^2}}$

$\ddot{y}|_{max} = \Gamma S_{da} = 1.194 \times 3.68 = 4.39\,ms^{-2}$

$\ddot{u}_i|_{max} = \ddot{y}|_{max}\,u_i \rightarrow$
$\ddot{u}_3 = 4.39 \times 1 = 4.39\,m/s^2$
$\ddot{u}_2 = 4.39 \times 0.7 = 3.08\,m/s^2$
$\ddot{u}_1 = 4.39 \times 0.4 = 1.76\,m/s^2$

Inertial forces		Column Shears.	Per Column
$F_3 = 4.39 \times 50 \times 10^3 =$	219.5 kN	219.5 kN	27.4 kN.
$F_2 = 3.08 \times 30 \times 10^3 =$	92.4 kN	311.9 kN	39.0 kN
$F_1 = 1.76 \times 30 \times 10^3 =$	52.8 kN	364.7 kN	45.5 kN

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Q4.

(a) Quasi-static assumption is that time of travel for fluid across structure is very short compared with representative time-scale of structural oscillations. This is rarely the case for bridges. Theodorsen's theory does not make the quasi-static assumption - it assumes the structure is undergoing small amplitude harmonic oscillations in the two coupled structural modes (vertical and torsional) and that there is a harmonically varying wake of shed vorticity behind the structure whose influence can still be felt by the structure. There are limitations:

- only applies to flat plates, and bridge decks are bluff bodies with much more complex flow patterns around them
- it is two-dimensional
- it assumes the incident wind speed is steady in magnitude and direction (- no turbulence in the approach flow).

b). Rayleigh's Principle states that for a structure undergoing small, free vibrations, in a particular mode, ~~the~~ with natural frequency ω, then equating the kinetic energy at the mean position with the potential energy (strain energy) at the position of maximum displacement leads to an expression that may be solved for ω. The principle states that if the mode shape is unknown, but a reasonable mode shape is assumed, then the method will lead to a good approximation to the exact frequency ω. If the assumed mode shape is applied to the <u>fundamental</u> mode the estimated frequency should be slightly <u>higher</u> than the exact value. The expression for ω^2 involves a ratio of strain energy terms to kinetic energy terms - the former depending on stiffnesses and the latter depending on densities and the mass distribution. None of these will be known exactly, so although the approximate ω will be greater than the exact ω (if parameters, geometry etc were known exactly) - in general the approximate ω will only be an approximation, possibly higher, possibly lower than the <u>real</u> ~~value~~ frequency of the <u>real</u> structure.

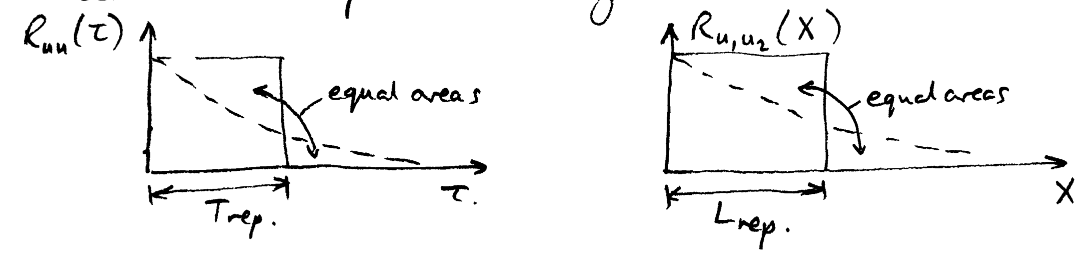
In long span suspension bridges gravity causes tension-stiffening and pendulum effects. These can be difficult to analyse (and are often omitted completely by engineers who do not realise their significance). In general then, one should expect the answer to an approximation.

Q4 c)

Marks would be awarded for explaining how the equations of motion of free vibration may be written in the form $\underline{\underline{M}}\ddot{\underline{X}} + \underline{\underline{K}}\underline{X} = 0$, and how these decouple by eigenvalue/eigenvector analysis, with any eigenvector $\underline{\Phi}_i$ having the property $\phi_i^T M \phi_j = 0 \ (i \neq j)$ and $\phi_i^T K \phi_j = 0 \ (i \neq j)$ (provided $\omega_i \neq \omega_j$).

Q4 d). Thunderstorms, hurricanes, tornadoes. Marks would be awarded for describing the basic flow fields and mechanisms of each phenomenon as per the lecture notes.

Q4 e). Marks would be awarded for definitions and sketch graphs as per page 9 of the lecture handouts, together with the comment that there is one representative time-scale and 9 representative length scales.



Q 3. a) When an earthquake arrives at a bed rock, vertically propagating horizontal shear waves (S_h) waves are set up in the soil strata overlying the bed rock. These S_h waves will reach the soil surface and subject any structure to horizontal shaking via its foundations. The structure will start to oscillate subjecting the foundation soil to horizontal, vertical and rocking modes of vibration. These vibrations can lead to changes in soil stiffness and will set up outwardly radiating stress waves away from the structure. This type of interaction between the soil and the structure under earthquake or wind loading is called dynamic soil-structure interaction.

[10%]

3 b) A saturated sand layer can suffer a degradation in its stiffness under following conditions.

i) when subjected to large cyclic shear strains induced by the earthquake loading the soil will suffer a reduction in its stiffness. This is because the stiffness of the soil reduces with increased shear strains.

ii) loose saturated sands can loose their stiffness or suffer partial reduction of their stiffness due to generation of excess pore water pressures. Loose soils have tendency of volumetric densification. This translates into excess pore water pressure as the rate of earthquake loading does not allow for pore water to escape. This causes an increase pore pressure and a corresponding drop in effective stress. leading to a degradation in soil stiffness.

[10%]

3 c) Loose saturated sands have a tendency to suffere volumetric densification on shearing as shown below.

The pore space reduces as particles reorganise themselves into a tighter packing. However in case of saturated sands pore water needs to escape out (drain out) for this volume reduction to happen. In case of earthquake loading, the rate of shearing applied by cyclic stresses does not allow for drainage to occur. Under these circumstances the pore water pressure increases as the soil tries to reduce its volume.

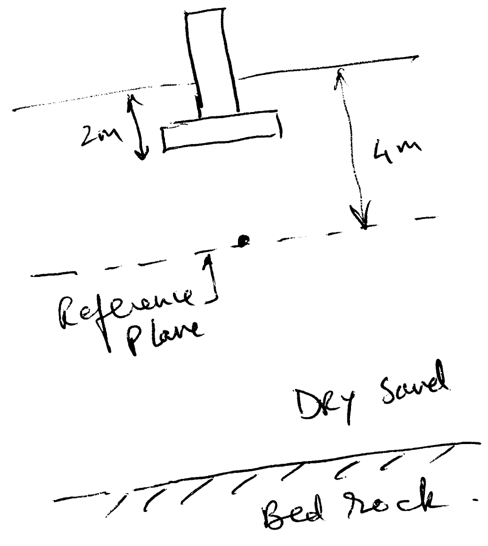
If the increase in pore water pressure matches the intergranular stress completely then the particles will be in suspension and full liquefaction is said to have reached.

$$\sigma' = \sigma - u = 0$$

On the other hand if the rise in pore pressure is such that it matches only a portion of the inter granular stress σ, then the soil is suffers 'partial liquefaction'. The partial liquefaction can also result in significant damage as the natural frequency of a initially stiff system can reduce causing it resonate.

[20%]

3 d)



Soil properties :

Unit weight of Sand $= 15\ kN/m^3$

$e = 0.6$

$K_0 = 0.45$

$\phi'_{crit} = 30°$.

First we need to find effective vertical stress in reference plane.

Height of soil from ref plane to foundation = 2 m.

$\therefore\ \sigma_v' = \sigma_v = \gamma z = 15 \times 2 = 30\ kPa$.

vertical stress due to structure $= 50\ kPa$.

Total vertical stress $= \sigma_v' = 80\ kPa$.

Mean confining pressure $p' = p = \left(\frac{1+2K_0}{3}\right)\sigma_v'$

$= \left(\frac{1+2\times0.45}{3}\right) 80 = 50.67\ kPa$.

Small strain Shear Modulus G_{max} (from Data sheet)

$$G_{max} = 100\ \frac{[3-e]^2}{1+e}\ (p')^{0.5}$$

G_{max} in MPa, p' in MPa

$$= 100\ \frac{[3-0.6]^2}{1+0.6} \left(\frac{50.67}{1000}\right)^{0.5}$$

$$= 81.033\ MPa.$$

Using this G_{max} we can calculate the shear wave velocity

$$v_s = \sqrt{\frac{G}{\rho}}$$

$\rho = 1530\ kg/m^3$ $G_{max} = 81.033\ MPa$.

$\therefore\ v_s = 230.14\ m/s$.

Distance from bed rock to foundation of structure

$= 10 - 2 = 8m$.

$\therefore$ Travel time for S_h waves $= \dfrac{8\ m}{230.14} = 0.0347\ s$

or $\underline{34.76\ ms}$

[30%]

3 e) Peak shear strain $\gamma_{max} = 0.25 \times 10^{-3}$ during BAM earthquake

We need to estimate the reduction in shear modulus due to this shear strain. From data sheets

$$G = \frac{G_{max}}{1 + \gamma_h}.$$

Hyperbolic shear strain

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a\, e^{-b(\gamma/\gamma_r)}\right]$$

Reference shear strain γ_r depends on the Mohr circle.

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

$$\tau_{max} = \sqrt{\left(\frac{1+K_0}{2}\sigma'_v \sin\phi'\right)^2 - \left(\frac{1-K_0}{2}\sigma'_v \sin\phi\right)^2}$$

$$= \sqrt{\left(\frac{1+0.45}{2}\right)^2 - \left(\frac{1-0.45}{2}\right)^2}\ \sigma'_v \sin\phi'$$

$$= 0.6708\ \sigma'_v \sin\phi'$$

But $\sigma'_v = 80$ kPa $\phi'_{crit} = 30°$.

$$\therefore \tau_{max} = 0.6708 \times 80 \times \sin 30$$

$$= 26.83 \text{ kPa}.$$

$$\Rightarrow \gamma_r = \frac{\tau_{max}}{G_{max}} = \frac{26.83 \times 10^3}{81.033 \times 10^6} = 0.3311 \times 10^{-3}$$

$$\gamma/\gamma_r = \frac{0.25 \times 10^{-3}}{0.3311 \times 10^{-3}} = 0.75498$$

$$\therefore \gamma_h = 0.75498\left[1 + a\, e^{-b \times 0.75498}\right]$$

No of cycles in the quake

For sands $b = 0.16$ $a = -0.2 \ln 9 = -0.4394449$

$$\therefore \gamma_h = 0.4609$$

$$\therefore G = \frac{G_{max}}{1+\gamma_h} = \frac{G_{max}}{1.4609} = \underline{55.466 \text{ MPa}}.$$

The shear modulus has decreased by about 68% due to the shear strain amplitude.

The travel time will consequently increase to:

$$v_s = \sqrt{\frac{G}{\rho}} = 190.39 \text{ m/s} \qquad = \frac{8\text{ m}}{190.4 \text{ m/s}} = 0.042 \text{ sec}$$

or $\underline{42 \text{ ms}}$.