Q1

(a) Ultimate Limit States

- strength of components & system to avoid collapse
- stability - overturning/bouyancy (include different loading states)
  - buckling of long compression members (e.g. legs)
- rupture & pressure on submerged tanks.
  (e.g. star-cell details which caused problems on Nth Sea structures)

Serviceability Limit States

- crack width - to avoid water penetration & ingress of sea water to cause corrosion
- dynamic response/vibrations - response from extreme wind & wave loading to ensure structure is habitable & doesn't respond in a way that might result in fatigue failure.
- durability - adequate detailing to concrete mix, & reinforcement protection to ensure long lasting performance
- deflection - no excess deflection under load for comfort & alignment.

(b) Structural failures (Name 3 only)

Name	Failure Mechanism	Prevention
Ronan Point 1968 Tower Block	Progressive collapse of prefabricated r.c. panels after gas explosion in cooker	Continuity + connectivity between elements (robustness)
Camden School 1973 Assembly Hall	(i) Insufficient bearing for beams (ii) R/f + Prestress detailing inadequate (iii) High-Alumina Cement Conversion	(i) Wider bearing seat (continuity at ends) (ii) R/f detailing at ends (iii) Not use HAC
Ynys-y-gwas 1984 Bridge	Tendon corrosion of post-tensioned segmental deck.	(i) Proper waterproofing of ducts + tendons (ii) Proper grouting (iii) Quality control on site

Q1 (b) cont.

Other examples from lecture notes

Parc des Princes 1979 – creep & shrinkage
Palau Bridge 1996 – " "
Murrah Building 1995 – robustness, r/f detailing
Stepney Pool 1974 – HAC
FDR Driveway 1989 – spalling/corrosion
Ferry Bridge Towers 1965 – design & construction errors
Oil rigs 1974/76/91 – design detailing/analysis errors.
Injaka Bridge 1998 – multiple design & construction errors.
Kufstein Bridge 1990 – scour
Tasman Bridge 1975 – impact
Earthquakes – detailing, ductility, robustness or extra loads exceeding economic design limit

(c) Characteristic strength $f_{ck} = 45$ MPa. (5% level)

$\sigma_c = 7$ MPa. $(= \sigma_R)$

(i) Mean strength $\mu_{CONCRETE} = \mu_R = 45 + 1.645 \times 7 = 56.4$ MPa

Design strength $f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{45}{1.5} = 30$ MPa

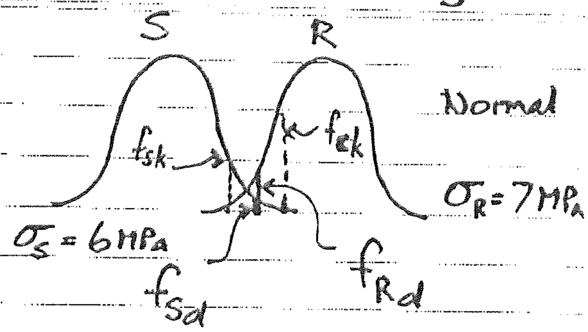
(ii) $P(x < f_{cd} = 30)$ $z = \frac{30 - 56.4}{7} = -3.771$

In tables: Area $-\infty$ to $z = +3.77 = 0.99991838$
Area below $z = -3.77 = 1.0 - 0.99991838 = 0.0000816$
$= 8.2 \times 10^{-5}$

$P(x < f_{ed} = 30) = 8.2 \times 10^{-5}$

Q1 (c) cont.

c(iii) Stress f_{Sd} = Resistance f_{Rd} $(= f_{cd})$



$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

$\mu_R = 56.4$ MPa $\quad \mu_S = 21.4 - 1.645 \times 6 = 11.5$ MPa

$\sigma_R = 7$ MPa $\quad \sigma_S = 6$ MPa.

$f_{Rd} = f_{cd} = 30$ MPa $= f_{Sd}$ $\quad f_{Sk} = \frac{30}{1.4} = 21.4$ MPa.

$$\therefore \beta = \frac{56.4 - 11.5}{\sqrt{7^2 + 6^2}} = \frac{44.9}{\sqrt{85}} = \frac{44.9}{9.22} = 4.87$$

$$\therefore P_f = 1 - 0.9^6 4420 = 5.6 \times 10^{-7} \simeq 6 \times 10^{-7}$$

(iv) Require $\beta = 3.5$

$$\beta = \frac{\mu_R - 11.5}{9.22} = 3.5$$

$\Rightarrow \mu_R = 3.5 \times 9.22 + 11.5 =$ 43.8 MPa (c.f. 56.4 MPa earlier)

$\therefore f_{ck} = \mu_R - 1.645 \times 7 = 43.8 - 1.645 \times 7 = 32.3$ MPa.

So specify 35 MPa

i.e. $f_{ck} = 35$ MPa c.f. 45 MPa earlier.
i.e. can reduce specified concrete strength

Q2

(a) 4C's plus w/c.

Cement Content – adequate cement in the mix ensures that the aggregate + sand is properly bound in a strong matrix + allows sufficient reaction to reduce Voids + hence capillaries in the matrix, thus improving the durability of the mix.

Cover – provides protection to r/f from ingress of deleterious substances such as chlorides, carbonation + sulphates. Also provides bond for restraining r/f bars.

Curing – adequate protection of concrete surface after casting prevents evaporation/loss of moisture in surface region which inhibits full hydration + can cause surface shrinkage cracks – both problems can lead to high permeability + lower durability. e.g. PVC sheeting or curing membranes/oils can be applied.

Compaction – adequate vibration/tamping removes air bubbles + avoids "honey", voided concrete which is porous/permeable.

Water/Cement Ratio – the correct/appropriate w/c ratio is critical in ensuring durable concrete. The strength is highly dependent on W/C as is the permeability + workability. Too little water + full hydration cannot occur + the workability is low, too much leads to large voids + capillaries + low strength + possible segregation.

(b) Mass concrete pour

First concern would be with high heat of hydration generating high temperatures. This can lead to differential temperatures + hence strains when the concrete cures, resulting in cracking + subsequent shrinkage + restraint cracking when the mix contracts on cooling. Can also cause problems of excessive pressure on formwork from expanded (hot) mix.

Several actions can mitigate the heat generated by the hydration process.

Q2 (b). cont.

These include

(i) Use of cement replacement materials e.g. gbfs + pfa – this directly leads to lower heat generation

(ii) Use of chilled water/ice in mix water to start with cold mix.

(iii) Inclusion of refrigeration system within casting e.g. refridgerated pipes through large dam pours – expensive but effective.

With large concrete pours one also needs to ensure continuity of supply to avoid "cold joints" between consecutive pours which may result from, for example, a failed concrete pump. Hence supply aspects of pour are critical.

Early set – must ensure set is not too rapid – add retardants to mix

Formwork pressures can be very high during large pours + great attention must be paid to allowance for subsequent shrinkage of cooling concrete.

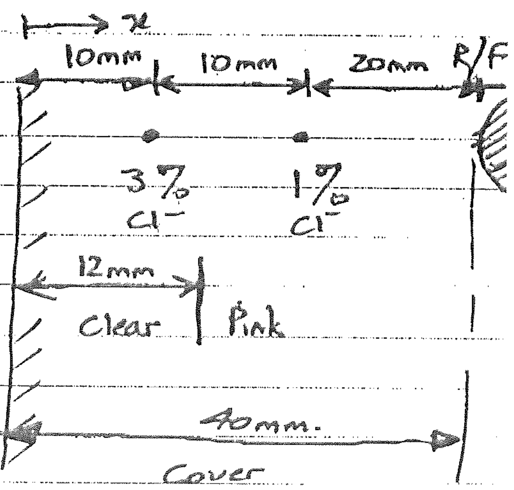
Shrinkage cracks – minimize scope for these – low w/c, superplasticizers

Compaction – it is critical to ensure all areas of formwork are readily accessible + no voids form in the mix, hence proper specification of vibration/tamping methods is important + also use of superplasticizers to improve workability.

Curing – this again is vital with large pours to ensure full hydration can occur.

(C) Time 6 years after construction.

Initial chloride content ~0.



Q2 (c) cont.

(i) Chloride ingress

From datasheet $C = C_0(1-\mathrm{erf}(z))$ ①; $z = \dfrac{x}{2\sqrt{D.t}}$ ②

At $t = 6$ years $x_1 = 10\,mm$; $C_1 = 0.03$
$x_2 = 20\,mm$; $C_2 = 0.01$

from ① $0.03 = C_0(1-\mathrm{erf}(z_1))$
$0.01 = C_0(1-\mathrm{erf}(z_2))$

$\therefore \dfrac{1-\mathrm{erf}(z_1)}{1-\mathrm{erf}(z_2)} = 3$

from ② $z_1 = \dfrac{10}{2\sqrt{D.6}}$; $z_2 = \dfrac{20}{2\sqrt{D.6}}$

$\therefore \dfrac{z_1}{z_2} = \dfrac{10}{20} = 0.5.$

Trial & error to find z_1 & z_2

Guess $z_1 = 0.5$; $z_2 = 1.0$

$\mathrm{erf}(z_1) = 0.52$; $\mathrm{erf}(z_2) = 0.84$

$\dfrac{1-\mathrm{erf}(z_1)}{1-\mathrm{erf}(z_2)} = \dfrac{0.48}{0.16} = 3.$ as required.

$\therefore z_1 = 0.5$; $z_2 = 1.0$

Q2(c) cont.

For given C, $x \propto \sqrt{t}$ assuming D constant.

i.e. $x = A\sqrt{t}$

At $t = 6$ years $x_1 = 10mm$ $C_1 = 0.03$

$x_2 = 20mm$ $C_2 = 0.01$

$x_3 = ?$ $C_3 = 0.004$

$$C_3 = C_0(1-\text{erf}(z_3)) = 0.004 \quad (3)$$

$$C_1 = C_0(1-\text{erf}(z_1)) = 0.03 \quad (4)$$

$\therefore$ (4) ÷ (3)

$$\frac{3}{0.4} = \frac{1-\text{erf}(z_1)}{1-\text{erf}(z_3)}$$ where $\text{erf}(z_1) = \text{erf}(0.5) = 0.52$

$\therefore 3(1-\text{erf}(z_3)) = 0.4 - 0.4\times0.52 = 0.192$

$\therefore \text{erf}(z_3) = 1 - \frac{0.192}{3} = 0.936 \simeq 0.94.$

from Table $z = 1.3$ $z = 1.4$

$\text{erf}\, z = 0.93$ $\text{erf}\, z = 0.95$

Linearly interpolate $z_3 = 1.35$ gives $\text{erf}(z_3) = 0.94$

$\therefore z_3 \simeq 1.35 = \dfrac{x_3}{2\sqrt{Dt_3}}$; $z_1 = \dfrac{x_1}{2\sqrt{Dt_1}} = 0.5.$

$\therefore \dfrac{z_3}{z_1} = \dfrac{1.35}{0.5} = \dfrac{x_3}{10} \Rightarrow x_3 = 27mm$

i.e. At $t = 6$ years, $C_3 = 0.004$ at depth $x_3 = 27mm.$

$x \propto \sqrt{t}$ $\therefore$ $\dfrac{40}{27} = \dfrac{\cancel{A}\sqrt{t_4}}{\cancel{A}\sqrt{6}}$ $\therefore t_4 = 6\times\left(\dfrac{40}{27}\right)^2 = 13.2$ years

i.e. 7.2 years after inspection

Q 2 (c) (i) cont.

Carbonation $x \propto \sqrt{t}$ OR $x = A\sqrt{t}$

$x_1 = 12\text{mm}$ at $t_1 = 6$ years

$x_2 = 40\text{mm}$ at t_2.

$$12 = A\sqrt{6}$$
$$40 = A\sqrt{t_2}$$

$$\therefore \frac{\sqrt{t_2}}{\sqrt{6}} = \frac{40}{12}$$

$$t_2 = \left[\sqrt{6} \times \frac{40}{12}\right]^2 = 66.7 \text{ years}$$

until carbonation front reaches steel bar & corrosion initiates.

⇒ Time after principal inspection is $67 - 6 = 61$ years until carbonation induced initiation of corrosion.

Thus Chloride corrosion governs & would initiate corrosion 7 years after the principal inspection.

(ii) $z_1 = \dfrac{x_1}{2\sqrt{Dt_1}}$ $\quad \therefore 2z_1\sqrt{Dt_1} = x_1$

$$\sqrt{Dt_1} = \frac{x_1}{2z_1}$$

$$D = \left(\frac{x_1}{2z_1}\right)^2 \cdot \frac{1}{t_1}$$

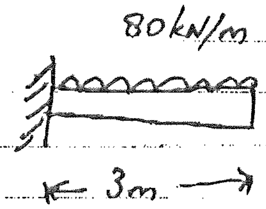
$$\therefore D = \left(\frac{0.010}{2 \times 0.5}\right)^2 \times \frac{1}{(6 \times 365 \times 24 \times 3600)} = 5.3 \times 10^{-13} \ m^2/s$$

t must be in seconds for D.

$$C_1 = C_0(1 - \text{erf}(z_1)) \quad \therefore C_0 = \frac{C_1}{1 - \text{erf}(z_1)} = \frac{0.03}{1 - 0.52} = 0.0625 \ (6.3\%$$ by weight of cement.)

Q3

(a) Moment at root $= 80 \times 3 \times 1.5 = 360\ kNm$



Assume steel yields in tension

$$T = \frac{f_y}{\gamma_{ms}} \cdot A_{st} = 400 \times 3 \times \frac{\pi (32)^2}{4} \times 10^{-3} = 965\ kN$$

$$C = \frac{\nu f_{cu}}{\gamma_{mc}} \times b \cdot x = 0.6 \times 40 \times 300 \times x \times 10^{-3} = 7.2x.\ kN.$$

$$\therefore x = \frac{965}{7.2} = 134\ mm$$ Neutral axis depth.

Lever arm $l = 450 - \frac{134}{2} = 383\ mm.$

$\therefore$ Moment capacity $M_u = T \times l = 965 \times 0.383 = 369.6 \simeq 370\ kNm$

$M_u > M_{applied} \Rightarrow$ OK.

(b) Shear force at root. $V = 80 \times 3 = 240\ kN$ $\tau_{Rd} = 0.5\ MPa$

From data sheet: $V_{Rd_1} = b_w d \left\{ \tau_{Rd} \cdot k \cdot (1.2 + 40\rho_1) + 0.15 \frac{N}{A_c} \right\}$

where $k = 1.6 - d = 1.6 - 0.45 = 1.15.$

$$\rho_1 = \frac{A_{st}}{bd} = \frac{3\pi\frac{32^2}{4}}{300 \times 450} = 0.0179$$

$$\therefore V_{Rd_1} = 300 \times 450 \left(0.5 \times 1.15 (1.2 + 40 \times 0.0179) + 0 \right) \times 10^{-3}$$

$$= 148.7\ kN.$$

$\therefore$ Required $V_s = 240 - 148.7 = 91.3\ kN$ $(= V_{Rd_3})$

$$V_{Rd_3} = \frac{A_{sw} f_{ywd} \cdot 0.9d}{S} \qquad \therefore S = \frac{A_{sw} f_{ywd}\ 0.9d}{V_{Rd_3}}$$

$$S = \frac{2 \times \frac{\pi}{4} 8^2 \times 220 \times 0.9 \times 450}{91.3 \times 10^3} = 98.1\ mm.$$ So choose $S = 100\ mm$

Q3 (c). 3m long cantilever divided into 4 sections each of 0.75m in length. $A_{stirrup} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5\,mm^2$

(i) Distance BE = 2.25 m.

Block A'B Total vertical force on BE is $2.25 \times 80 = 180$ kN.

$\therefore$ force/stirrup is $A_{stirrup} \times f_{wyd} = 100.5 \times 220 \times 10^{-3} = 22.1$ kN

$\therefore$ Require $\frac{180}{22.1} = 8.14$ stirrups in 750mm from A to B.

$\therefore$ Spacing $S_{AB} = \frac{750}{8.14} = 92$ mm

(If use whole no. of stirrups $\frac{750}{9} = 83$ mm

Block B'C Force to be carried $= 1.5 \times 80 = 120$ kN.

No. stirrups $= \frac{120}{22.1} = 5.4$

Spacing B'C $= \frac{750}{5.4} = 139$ mm (or $S = \frac{750}{6} = 125$ mm) (for whole no. of stirrups)

Block C'D Force to be carried $= 0.75 \times 80 = 60$ kN

No. stirrups $= \frac{60}{22.1} = 2.7$

Spacing C'D $= \frac{750}{2.7} = 278$ mm (twice that of B'C)

(OR $S = \frac{750}{3} = 250$ mm for whole no. of stirrups.)

(ii) From Mohr's Circle $\tau = \sigma_c \sin\theta \cos\theta$.

$\tan\theta = \frac{405}{750} = 0.54$ $\therefore \theta = 28.4°$.

$\tau = \frac{V}{b_w(0.9d)} = \frac{240 \times 10^3}{300 \times 405} = 1.975$ MPa.

$\sigma_c = \frac{\tau}{\sin\theta\cos\theta} = \frac{1.975}{\sin 28.4 \cos 28.4} = 4.72$ MPa This is quite small

(The applied shear stress is actually quite small).

Q 3 (c) cont.

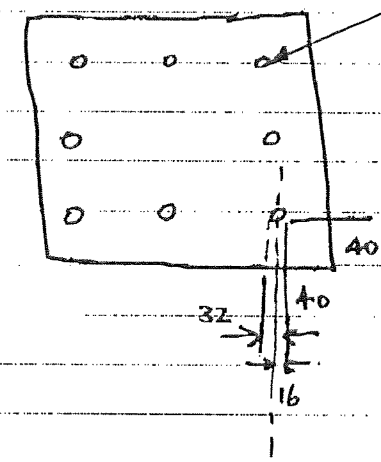
(iii) $\sigma_c \cos^2\theta = 4.72 \times \cos^2 28.4 = 3.66 \text{ MPa} \times 300 \times \frac{405}{2}$

$= 222.1 \text{ kN}$.

Required T is $\frac{360}{0.405} + 222.1 = 1110 \text{ kN}$

This cannot be taken by the main bars provided - so we must either increase longitudinal bars or increase θ - but main bars are just OK for moment, so not much in hand.

Q4.



$8 \times 32\phi$ $\quad A_{bar} = \frac{\pi d^2}{4} = 804\,mm^2$

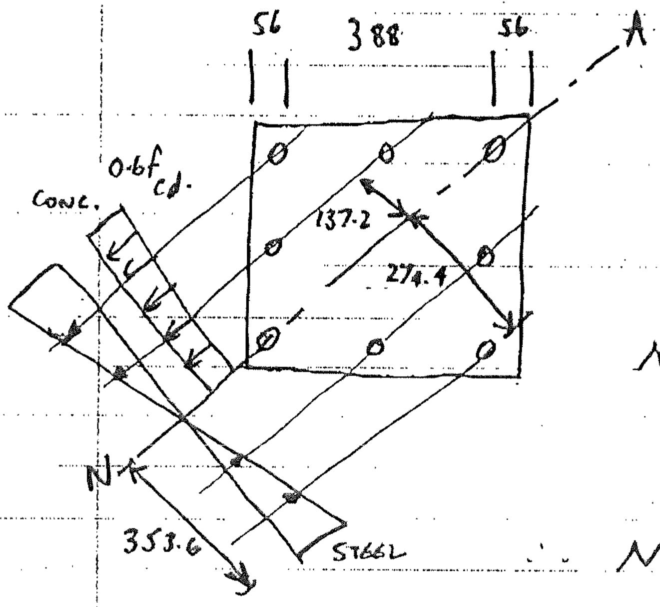
$f_{yd} = 400\,MPa$.

$f_{cd} = 30\,MPa$

$(\nu f_{cd} = 0.6 f_{cd} = 0.6 \times 30 = 18\,MPa)$

$T_{bar\ yield} = f_{yd} . A_{bar} = 804 \times 400 = 321.6 \times 10^3\,N$
$= 322\,kN$

(a)(i) N.A. at column centre (diagonal)



Axial load (N)

$N_{CONCRETE} = 0.6 f_{cd} \times \frac{A_{column}}{2}$

$= 18 \times \frac{500^2}{2} = 2.25 \times 10^6\,N = 2.25\,MN$

N_{STEEL} = force in bars on NA only since others cancel.
$= \pm 2 \times T_{bar} = \pm 2 \times 322 = \pm 644\,kN$

$\therefore$ $N = 2.25 + 0.644$ OR $2.25 - 0.644$

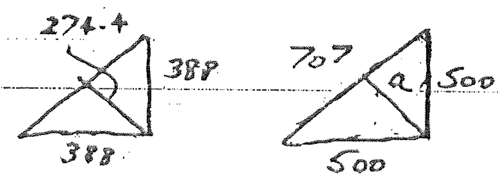
$\therefore$ $1.606\,MN \leq N \leq 2.894\,MN$

Moment (M)

centroid of area in compression

$M = 2.25 \times 10^3 \times \frac{0.354}{3}$ CONCRETE

$+ 322 \times 0.137 \times 2 \times 2$ – steel in 2 inner rows of bars
$+ 322 \times 0.274 \times 2$ – 2 outer corner bars

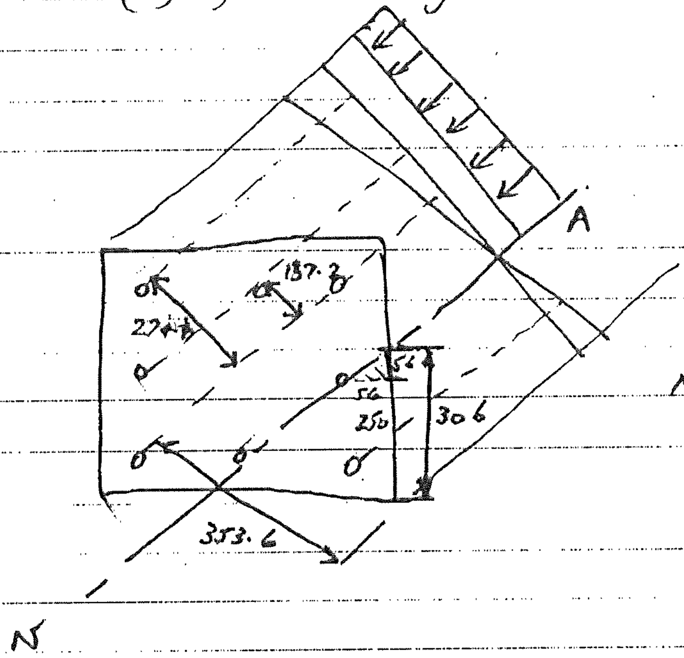


$a = \frac{500}{\sqrt{2}} = 353.6\,mm$

$b = \frac{353.6}{2} = 176.8\,mm$

$= 265.5 + 352.9$
$= 618.4\ kNm$

Q 4 (a)(ii) NA through centre of 2 bars in middle of adjacent sides.



Axial load (N)

$N_{conc} = 18 \times (500^2 - \frac{306^2}{2}) = 3.657 \times 10^6$

$= 3.66\ MN$

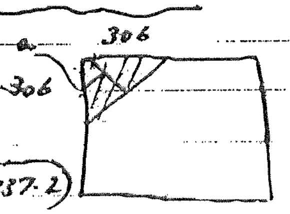
$N_{steel} = 4 \times 322 \pm 2 \times 322$ kN
(ON N.A.)
(2 outer corner bars cancel)

$= 1288 \pm 644$

$= 1932$ OR 644 kN

$\therefore\ 4.30\ MN \leq N \leq 5.59\ MN$

Moment (M) (about ℄)



$M_{conc} = \underbrace{18 \times \frac{1}{2} \times 216.4^2 \times 2}_{Force\ /\ Area} \times \underbrace{(\frac{216.4}{3} + 137.2)}_{offset}$

$= 176.4\ kNm$

$a = \frac{306}{\sqrt{2}} = 216.4\ mm.$

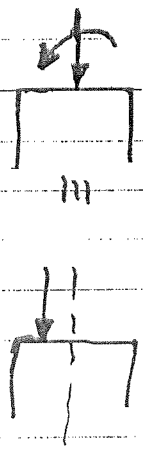
$M_{steel} = 2 \times 322 \times 0.274$ two outer bars

$+ 2 \times 322 \times 0.137$ second layer

$\pm 2 \times 322 \times 0.137$ ON N.A.

$+ 0$ (on diagonal)

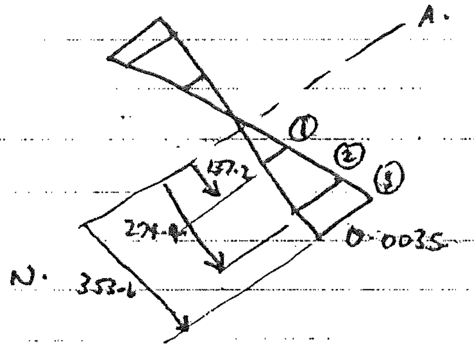
$= 176.5 + 88.2 \pm 88.2$

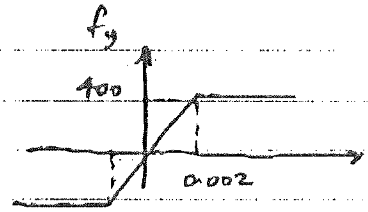


$\therefore\ 176.5\ kNm \leq M_{steel} \leq 353\ kNm$

$\therefore\ 353\ kNm \leq M \leq 530\ kNm$

Q4 (b) $\varepsilon_{cu} = 0.0035$





Steel layer ① $\varepsilon_{s_1} = \dfrac{137.2}{353.6} \times 0.0035 = 0.00136$.

$f_{s_1} = \dfrac{0.00136}{0.002} \times 400 = 271.6$ MPa.

$\therefore T_{s_1} = \underset{A_{bar}}{804} \times 271.6 \times 10^{-3} = 218.4\,kN \times \overset{bars}{2} = 437\,kN$

c.f. 644 kN earlier

i.e. 207.3 kN less

Steel layer No. ② $\varepsilon_{s_2} = \dfrac{274.4}{353.6} \times 0.0035 = 0.0027$

$> \varepsilon_{sy} \Rightarrow f_{sy} = 400$ MPa.

$\therefore T_2 = 322$ kN.

No stress (force) in bars on diagonal since $\varepsilon_s = 0$.

Forces in Bars in layers 1 & 2 cancel since forces in opposite directions.

$\Rightarrow N = N_{conc} = 2.25$ MN

Moment

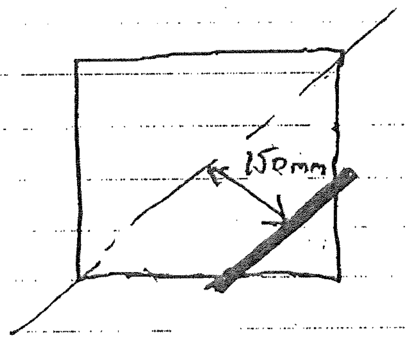
Moment is reduced due to reduction in force in 2 bars offset 137.2 mm from ℄ on either side.

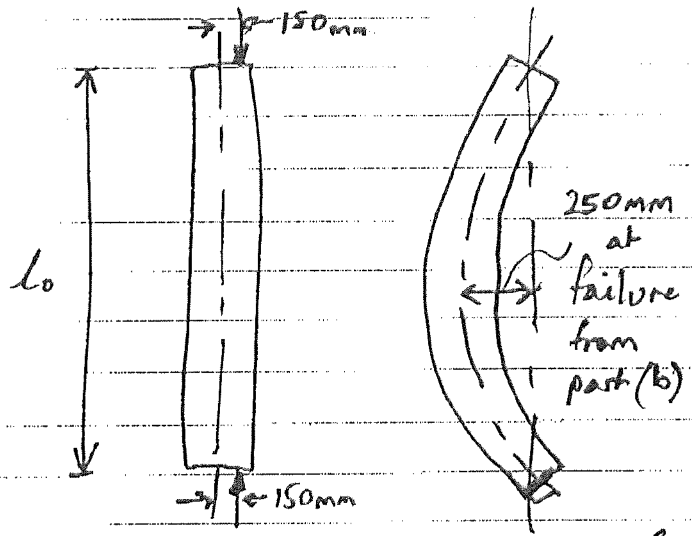
$$M = \underbrace{618.4}_{\substack{M\ from\\(a)(i)}} - \underbrace{207.3}_{\substack{reduction\\in\\bar\ force}} \times \underbrace{0.1372}_{offset} \times 2$$ layer either side of diagonal

$= 618.4 - 56.9 = 561.5$ kNm.

$\therefore\ e = \dfrac{M}{N} = \dfrac{561.5 \times 10^3}{2.25 \times 10^6} = 0.25\,m$

Q 4 (c) Note this is same as Example Sheet 2 Q7





$\varepsilon = \frac{y}{R} = \kappa y$

$N = 2.25\,MN$ (as per part (b))

From data sheet (Eccentricity due to secondary effects)

$$e_2 = \frac{l_o^2}{\pi^2} \cdot \kappa_m \qquad \text{Require } l_o? \qquad \text{i.e. } l_o \leq \sqrt{\frac{\pi^2 e_2}{\kappa_m}}$$

Max. curvature when concrete fails

$$\kappa = \frac{\varepsilon}{y} = \frac{0.0035}{0.3536} = 9.9 \times 10^{-3}$$

(dist. to extreme fibre of concrete)

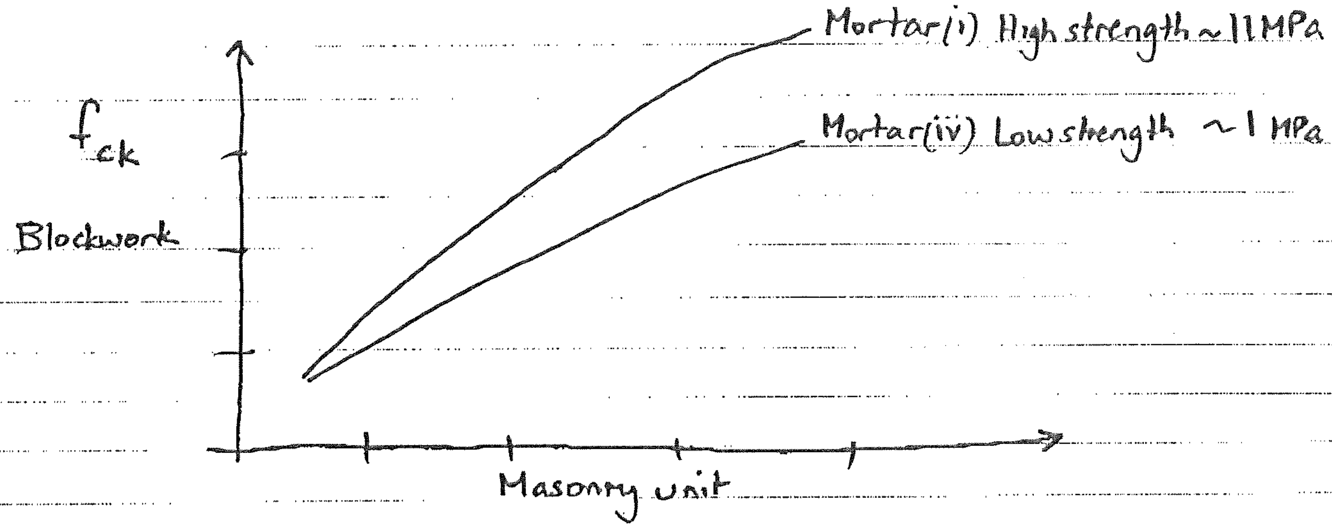
e_{total} at failure from part (b) is 250mm.
Load is at eccentricity of 150mm so maximum possible eccentricity due to secondary effects is $e_2 = 100$mm (i.e. remainder)

$$\therefore l_o \leq \sqrt{\frac{\pi^2 e_2}{\kappa_m}} = \sqrt{\frac{\pi^2 \times 0.1}{9.9 \times 10^{-3}}} = \underline{9.98\,m}$$

Check $r = \frac{500}{\sqrt{12}} = 144$ mm

$\frac{l_o}{r} = \frac{9.98}{0.144} = 69. > 35$ so Eurocode factor is 1.0.

Q5 (a) Refer to data sheet (Fig 5.6(a)).

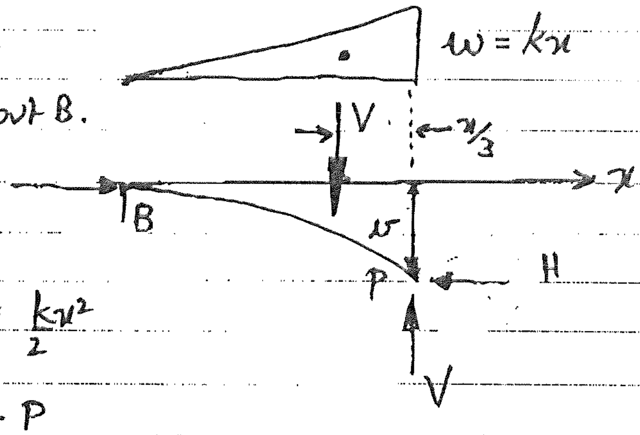


The overall strength of blockwork is intermediate between the strength of the units themselves & the mortar infilling between the units. This strength can exceed the mortar strength because of triaxial effects which result in confinement of the mortar by the stronger adjacent masonry units. This lateral compression in the mortar improves strength but corresponding lateral tension in the masonry units can reduce the units compressive strength. The strength of blockwork is also affected by bond, slenderness etc.

"Dressed" masonry, with thin joints, can approach the overall strength of the stone block units – this strength can be very high with good quality stone. However tensile strength is low so a good approach is to assume zero tensile & infinite compressive strength. So failure in arches etc. has to be by hinging about the outer edge – situation is ok if a line-of-thrust in equilibrium with the applied loads can be found within the masonry (lower bound plastic theory). OR can use mechanism approach (upper bound), with no internal work done, to discover whether a structure is stable or not.

Q 5(b) (i)

Symmetrical about B.



$V = \frac{1}{2} \cdot x \cdot kx = \frac{kx^2}{2}$

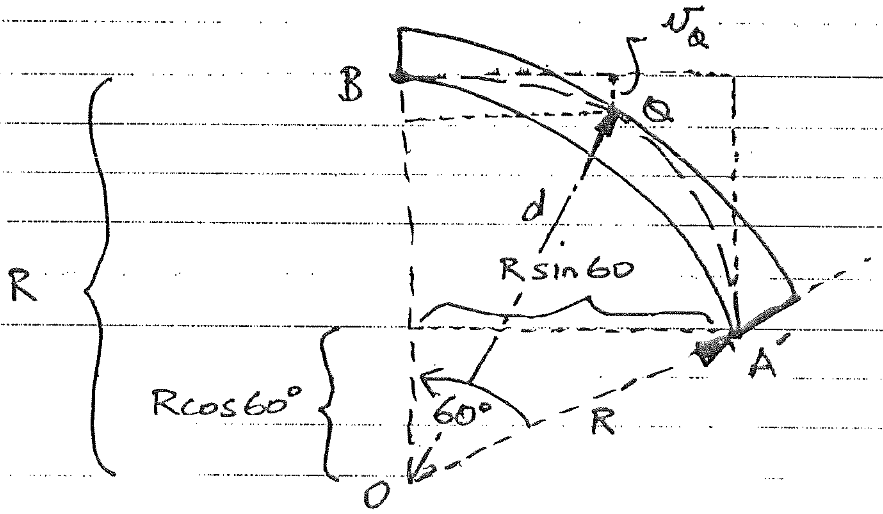
Moments about P

$V \cdot \frac{x}{3} = H \cdot v$

$\therefore v = \frac{V}{H}\frac{x}{3} = \frac{kx^3}{H\,6} = Ax^3$ where $A = \frac{k}{6H}$.

$\therefore$ Depth of line of thrust varies as a cubic either side of B.

(ii) Line of thrust through B & inner surface at springings, A′



$x_{A'} = R\sin 60^\circ$
$= 0.866R$
$v_{A'} = R - R\cos 60^\circ$
$= R(1-\cos 60^\circ)$
$= 0.5R$

Have $v = \frac{kx^3}{6H} = Ax^3$.

$v_{A'} = A x_{A'}^3$

$\therefore$ Line of thrust $\frac{v}{v_{A'}} = \left(\frac{x}{x_{A'}}\right)^3$

Q 5(b)(ii) cont.

$$\therefore \quad v = v_A'\left(\frac{x}{x_A'}\right)^3 = 0.5R \cdot \frac{x^3}{0.866^3 R^3} = 0.7698\frac{x^3}{R^2}$$

Distance from O to line of thrust at Q.

$$d^2 = x^2 + (R - v_Q)^2$$

$$= x^2 + \left(R - 0.7698\frac{x^3}{R^2}\right)^2 \quad (1)$$

Require maximum d^2 which will occur when $\frac{d(d^2)}{dx} = 0$.

$$\therefore 0 = \cancel{2}\cancel{x} + \cancel{2}\left(R - 0.7698\frac{x^3}{R^2}\right) \cdot \left(-3 \times 0.7698\frac{x^{\cancel{2}}}{R^2}\right)$$

$$= 1 - 3\frac{x}{R^2}(0.7698)\left(R - 0.7698\frac{x^3}{R^2}\right)$$

$$= 1 - 3(0.7698)\frac{x}{R}\left(1 - 0.7698\frac{x^3}{R^3}\right)$$

Let $z = x/R$.

$$\therefore 0 = 1 - 2.3094\, z\left(1 - 0.7698\, z^3\right)$$

$$\therefore z = \frac{1}{2.3094(1 - 0.7698 z^3)}$$

z	fn(z)
0.5	0.479
0.45	0.466
0.47	0.47

Solve numerically

$$\therefore z = 0.47 = x/R$$

$$\therefore \text{In } (1) \quad d^2 = x^2 + \left(R - 0.7698\frac{x^3}{R^2}\right)^2$$

$$\therefore d^2 = (0.47R)^2 + \left(R - 0.7698\frac{(0.47R)^3}{R^2}\right)^2$$

$$= 0.221R^2 + (R - 0.7698 \times 0.47^3 R)^2$$

$$= R^2(0.221 + (1 - 0.7698 \times 0.1038)^2)$$

$$= R^2\, 1.0676 \qquad \Rightarrow d = 1.033R$$