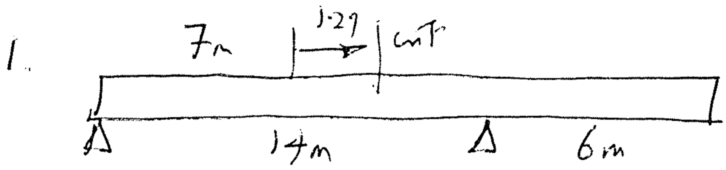
4D8 2004

4D8 2004 Solutions

1.



load 60 kN/m

(a) Given that critical position is as shown.

Max. hogging moment there, load on cantilever

$$M_h = 60 \times 6 \times 3 \times \left(\frac{8.29}{14}\right) = 0.639 \text{ MNm}$$

For max sagging moment, load all of 14 m span, parabolic diagram

$$M_s = \frac{60 \times 14^2}{8}\left(1 - \left(\frac{1.29}{7}\right)^2\right) = 1.420 \text{ MNm}$$

$\therefore$ max moment range is $\underline{2.06 \text{ MNm}}$

$\therefore$ required $Z_{min} = \dfrac{2.06 \times}{22} = \underline{\underline{0.0936 \text{ m}^3}}$

[20%] suggest overall depth about 1 m, flanges 0.5×0.2 m
refine to maybe 1.1 m deep, 0.6×0.2 flange

(b) section adopted has $A = 0.40 \text{ m}^2$ $Z_t = -0.17 \text{ m}^3$

$Z_b = +0.14 \text{ m}^3$. dead load $0.4 \times 24 = 9.6$ kN/m

moment due to dead load, applied everywhere

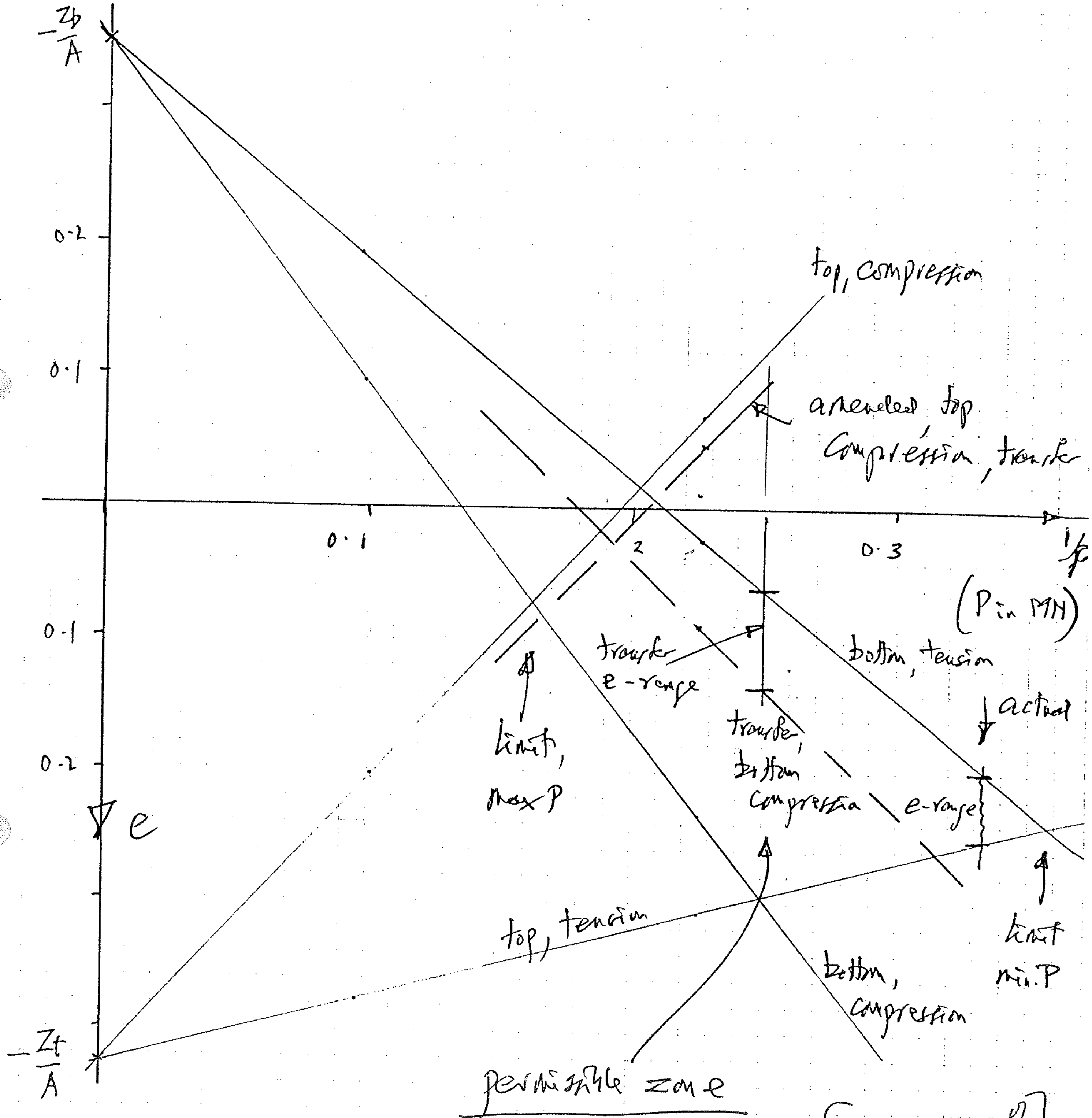
$$M_d = +\frac{9.6 \times 14^2}{8}\left\{1 - \frac{1.29^2}{49}\right\} - 9.6 \times 6 \times 3\left(\frac{8.29}{14}\right)$$

$= 124.9$ say $\underline{\underline{125 \text{ kNm}}}$

max total moment $0.125 + 1.42 = 1.55$ MNm sag

min. total $+0.125 - 0.139 = -0.57$ MNm (hog)

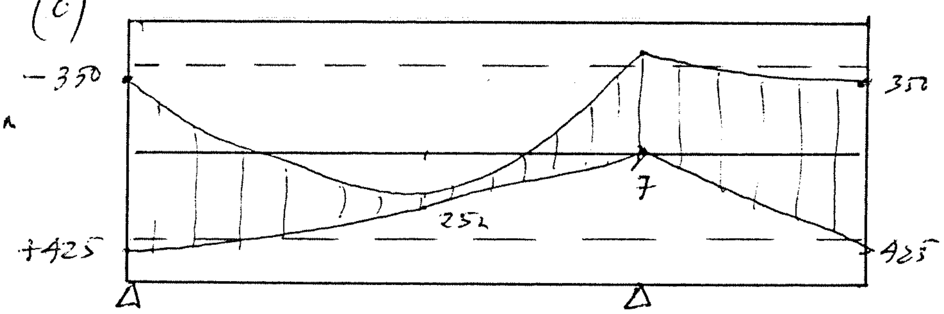
$$-\frac{Z_b}{A} = -0.35 \text{ m} \qquad -\frac{Z_t}{A} = +0.425 \text{ m}$$



$-\frac{Z_b}{A}$
0.2
0.1
0.1
0.2
0.1
0.3
$1/P$
(P in MN)
e
$-\frac{Z_t}{A}$
top, compression
amended, top compression, transfer
transfer e-range
bottom, tension
actual
limit, max P
transfer, bottom compression
e-range
top, tension
bottom, compression
limit min. P
permissible zone
[inc. in 50%]

From Magnel diagram (or by calculation)
min. prestressing force is 2.658 MN
[50%] max. " " 6.142 MN.

(c)



Vertical scale exaggerated

Sketch gives allowable eccentricity — assuming tension governs everywhere (as it does at critical section).

No straight profile possible — so pretensioning would be hard,
[15%] without special measures to debond large parts of the cables.

(d) This can be done in several ways. Here we regard the 1/P axis as being ~~based~~ on the current value of P. There is no point in altering the lines on the Magnel diagram governed by tension — since more stress is allowed, and applied moment at transfer is within the range already covered. But in compression?

top fibre : $\frac{P}{A} + \frac{Pe}{Z_t} - \frac{M_D}{Z_t} \leq +13$

$\therefore e \geq \frac{13 Z_t}{P} + \frac{M_D}{P} - \frac{Z_t}{A}$

for $P = 4.4$ MN
$e \geq -0.049$

bottom fibre : $\frac{P}{A} + \frac{Pe}{Z_b} - \frac{M_D}{Z_b} \leq +13$

and

$\therefore e \leq \frac{13 Z_b}{P} + \frac{M_D}{P} - \frac{Z_b}{A}$

$e \leq 0.092$

If P adopted was 3MN when working loads came on, at transfer P would be 4 MN (25% loss by when working loads added) i.e. $\frac{1}{P} = 0.25$

Range of allowable eccentricities at working load and transfer do not overlap – so design would not work.

To get things to work correctly, might try increasing P (but compressive stress is the problem), or delaying transfer until concrete stronger, or altering Z (e.g. by deepening beam). [15%]

[Note: this question was quite well done, though many candidates went astray on numbers for the Magnel diagram. The final part on conditions at transfer was not well answered.]

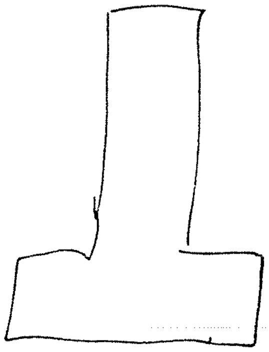
Question 2 on a composite beam and creep

This was another fairly standard question, the first half very similar to a question on an examples paper, the second half more theoretical, on how to predict redistribution of stress in the composite beam due to differential creep and shrinkage. It was answered by 9 undergraduates and all four graduates, the marks ranging from 5 to 80 with a mean of 59 %. There were some very elementary mistakes, e.g. in locating the centroid of a simple area, and one candidate apparently had no concept of the section modulus Z – can he have attended any of the lectures? Some contrived to take the weight of the in-situ concrete on the composite beam instead of on the precast part only – and a few contrived to reduce the weight of the in-situ concrete as well as its effective width, in proportion to the elastic moduli. There were some good answers to the second half, which had been fully covered in the lecture notes, but a few candidates made no serious attempt at this part.

Question 3 on material properties, bond and brittle fibres

An essay question attempted by all but one of the undergraduates and all four graduates, who scored marks from 25 to 90 with mean 61%. A wide range of answers, with some candidates knowing a great deal and some hardly anything. Creep, shrinkage and other prestress losses were not so widely mentioned as I had expected, which is odd given the full coverage of Freysinnet and these topics in the course.

2.



depth 0.8 m $A = 0.21\ m^2$
centroid 0.5 m 15 m span
$Z_t = -0.018$ $Z_b = +0.030\ m^3$
$P = 1.5$ MN $e = 0.17$ m $\rho = 24$
$I = 0.009\ m^4$

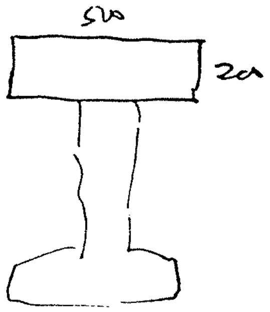
(a) $w = \rho A = 5.04$ kN/m $M_D = 142$ kNm

top $\dfrac{P}{A} + \dfrac{Pe}{Z_t} - \dfrac{M_D}{Z_t} = 7.142 - 14.167 + 7.477$
$= +0.852$ MPa

bottom $\dfrac{P}{A} + \dfrac{Pe}{Z_b} - \dfrac{M_D}{Z_b} = 7.142 + 8.5 - 4.727$
$= 10.915$ MPa

[20%]

(b) $700 \times 200 = 0.14\ m^2$ weight 3.36 kN/m
transform to $700 \times \dfrac{25}{35} = 500$ mm in precast



$A_{new} = 0.1 + 0.21 = 0.31\ m^2$

$\bar{y}$ at $\dfrac{0.1 \times 0.1 + 0.21 \times 0.7}{0.31} = 0.506$ from top

$I = \frac{1}{12} \times 0.5 \times 0.2^3 + 0.1 \times 0.406^2$
$+ 0.009 + 0.21 \times 0.194^2$

$= 0.0337\ m^4$

In-situ flange causes moment $3.36 \times \dfrac{225}{8} = 94.5$ kNm on precast,

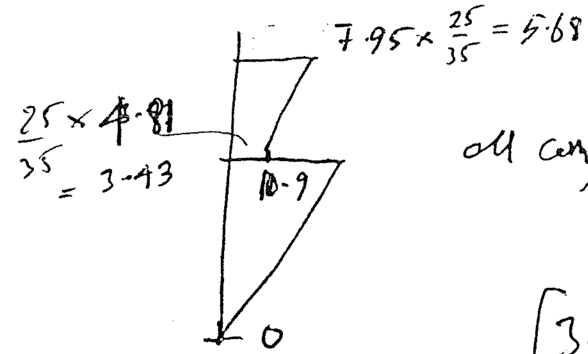
stresses + 5.25 and − 3.15 MPa in precast

leaving + 6.10 and + 7.76 before live load.

So allowable live load moment is $7.76 \times \frac{0.0337}{0.494} = 529$ kNm

i.e. <u>18.8 kN/m</u>

Stress distribution :

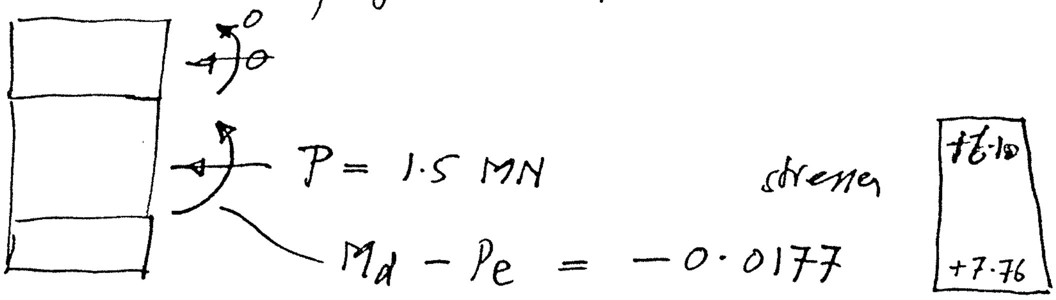


all compression.

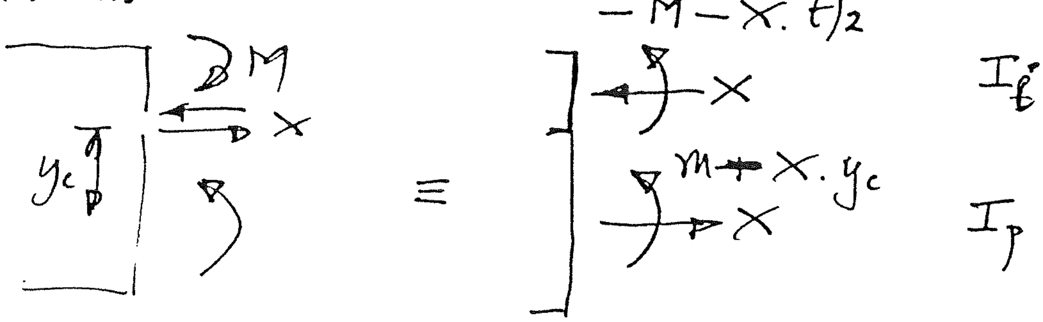
[38%]

(c) Concentrate on outlining principles, rather than numbers.

Immediately after time t_0 we have zero stress in the in-situ flange, prestress plus dead load on precast



Creep and shrinkage will introduce extra self-equilibrating forces/moments



Check, overall force zero ✓

overall moment $M - Xy_c - M - Xt/2 + X(y_c + t/2) =$ ✓

Curvature : flange $(-M - X.t/2)\frac{(1+\phi_i)}{E_i I_i}$

change of curvature in beam.

initial $\left(M_D - Pe\right) \cdot \frac{1}{E_p I_p}$

final. $\left(M_D - Pe + M - X y_c\right) \cdot \frac{(1+\phi_p)}{E_p I_p}$

∴ first equation, equate changes of curvature

$$\left(-M - X \tfrac{t}{2}\right) \cdot \frac{(1+\phi_i)}{E_i I_i} = \frac{\phi_p}{E_p I_p}\left(M_D - Pe\right) + \left(M - X y_c\right) \frac{(1+\phi_p)}{E_p \cdot I_p}. \quad \ldots (1)$$

Then, change of compressive strain at interface, equal.

flange : bottom : $\sigma_f = \frac{X}{A_f} + \frac{1}{Z_f}\left(M + X \tfrac{t}{2}\right)$

$$\Delta\varepsilon_f = \varepsilon_f = \varepsilon_{si} + \frac{(1+\phi_i)}{E_i}\left\{\frac{X}{A_f} + \frac{1}{Z_f}\left(M + X \tfrac{t}{2}\right)\right\} \quad \ldots (2)$$

top of precast : initial stress $+6.10$ strain $\frac{6.10}{E_p}$

final stress $6.10 - \frac{X}{A_p} + \frac{(M - X y_c)}{(-Z_t)}$ —— −ve number

final strain

$$\varepsilon_p = \varepsilon_{sp} + \frac{(1+\phi_p)}{E_p}\left\{6.10 - \frac{X}{A_p} + \frac{X y_c}{Z_t} - \frac{M}{Z_t}\right\}$$

∴ change in strain since in-situ cast is

$$\Delta\varepsilon_p = \varepsilon_{sp} + \frac{(1+\phi_p)}{E_p}\left\{6.10 - \frac{X}{A_p} + \frac{X y_c}{Z_t} - \frac{M}{Z_t}\right\} - \frac{6.10}{E_p} \quad \ldots (3)$$

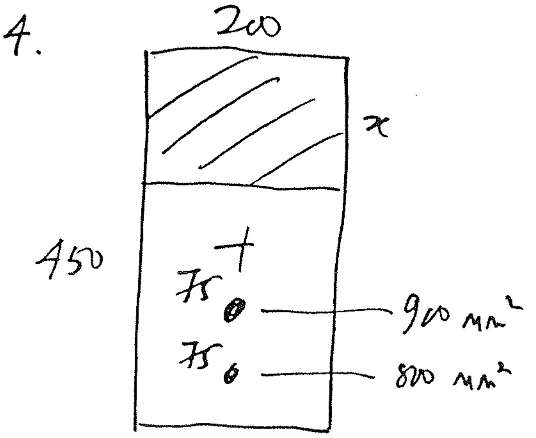
equating (2) and (3) gives another equation for M, X.
Solve, back substitute for stresses.

[50%]

3. This question involves mainly bookwork, and discussion of sources of loss of prestress, especially due to creep and shrinkage. Higher strength concrete has higher elastic modulus, and therefore lower creep strains (even for the same creep factor) — so less loss of prestress. And it probably has lower water content, and so smaller creep factor and shrinkage, than ordinary concrete, etc. Steel needs to be of high strength, so that it can have high initial prestress, so that losses due to the inevitable causes (elastic shortening, pull-in, creep etc) will be of lesser proportional significance, even over long times.

Unbonded tendons can slip relative to concrete — so may not develop full strength at ULS at a critical failing position. But still suffer losses due to creep etc. But can be replaced if found to corrode. Carbon fibres etc — still need high strength to avoid losses due to creep. But maybe use unbonded — must avoid brittle tendon failure at all costs.

4.



tendon force
at prestress 810 kN
at yield 1,371 kN

unprestressed 320 kN
yield strain 0.002

(a) prestress strain $\frac{900}{200} = 0.0045$ (neglect adjacent concrete)

at failure (i) assume all yield, $T = 1{,}691$ kN

$\therefore \quad x = \frac{1{,}691 \times 10^3}{200 \times 36} = 235$ mm — too high

$\therefore$ given $x = 200$ mm $\quad C = 1{,}440$ kN

unprestressed, strain 0.0029 $\therefore$ at yield

prestressed, strain $0.0045 + 0.0032 \times \frac{100}{200} = 0.0061$

$\therefore \quad \sigma = 1200 + \frac{0.0001}{0.006} \times 323 = 1{,}205$ MPa

$\therefore \quad T = 1{,}404$ kN

try x slightly less, say 190 mm $\quad C = 1{,}368$ kN

prestressed strain $0.0045 \times 0.0032 \times \frac{110}{190} = 0.00635$

$\sigma = 1{,}219 \quad \therefore \quad T = 1{,}417$

interpolate to $x = 196$ mm $\quad C = 1{,}411.2$

$\varepsilon = 0.006198 \quad \sigma = 1210.7 \quad T = 1{,}409.6$

OK

$\therefore$ moment $M_u = 320 \times 0.277 + 1089.6 \times .202$

$= \underline{309 \text{ kNm}}$ [80%]

effect of unprestressed reinforcement?

(1) It increases M_u (usually at greater lever arm)

in this case, without extra steel, $M_u \approx 252$ kNm

so not much benefit.

(2) disbenefit is that it attracts some of the prestress force, and more as time goes on. So the effect of prestress in preventing cracking may be vitiated, due to creep and shrinkage causing a high proportional loss of stress in the concrete.

[20%]

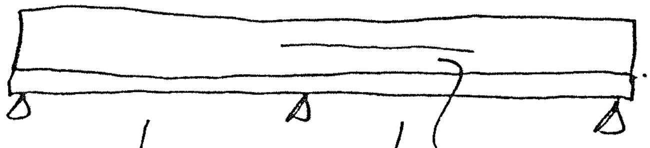
Question 4 on ultimate moment with some unprestressed steel

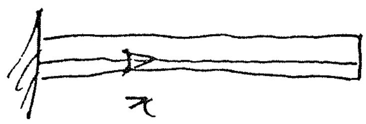
A fairly straightforward question, answered by all but 2 of the undergraduates and all four graduates, who achieved marks from 25 to 100% with mean 73%. Most knew basically what to do, though some omitted the force in the unprestressed steel from the force balance, and some showed surprising uncertainty about finding the moment once they had achieved balance of compressive and tensile forces. One almost perfect answer, and several other very good ones.

Question 5 on line of thrust, concordant profiles etc

A question designed to test understanding of principles, with could be done using simple Data Book coefficients as well as by more elaborate methods with diagrams of secondary moments. Although thought to be fairly simple, it only attracted three undergraduate attempts, only one at all serious, mean mark 36%. The two weak candidates wanted to include the self-weight in the calculations, instead of concentrating (as the lectures and question do) on the line-of-thrust etc *due to the cable*.

5.





$$\kappa = \frac{Pe_0}{EI} \text{ , uniform.}$$

$$\text{end } \Delta = \int_0^L \kappa.(L-x).dx$$

$$= \frac{Pe_0}{EI}.\left[Lx - \frac{x^2}{2}\right]_0^L$$

$$= \frac{P.e_0}{EI}.\frac{L^2}{2} \quad \text{downwards.}$$

so end force carry on : $\frac{R.L^{\cancel{3}}}{3\cancel{EI}} = \frac{Pe_0\cancel{L^2}}{2\cancel{EI}}$

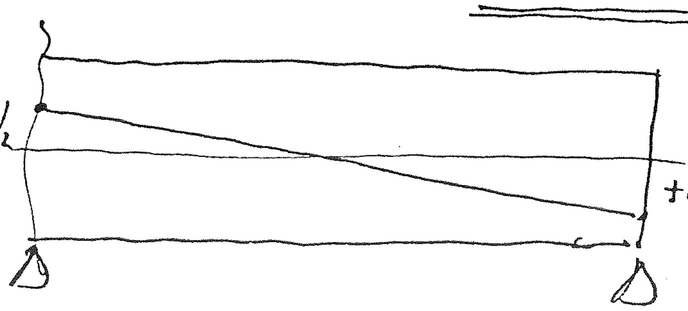
$$\therefore R = \frac{3Pe_0}{2L}$$

$\therefore$ moment at central support is $\frac{3}{2}Pe_0$

Need to add to prestress. $\underset{\text{due to cable}}{-Pe_0} + \frac{3}{2}Pe_0 = \frac{P.e_0}{2}$

$= -P.e_p$

$\therefore$ effective $\underline{\underline{e_p = -e_0/2}}$ at centre



[60%]