VHRT IIB . Module 4D12 . Solutions 2004

1/

$$K.E. = \frac{1}{LT}\iiint \rho\left(\frac{u^2+v^2}{2}\right) dy\, dx\, dt$$

Substitute $u = \frac{\partial\phi}{\partial x}$, $v = \frac{\partial\phi}{\partial y}$ and integrate. 35%

$$\text{Energy flux} = C_g \times \frac{\rho g H^2}{8}$$

In deep water $C_g = \frac{1}{2}\frac{g}{\omega} = \frac{9.81 \times 11}{4\pi}$

If we neglect energy dissipation, the energy flux at a depth of 15m is the same as that in deep water 30%

i.e Energy flux $= \frac{1000 \times 9.81 \times 36}{8} \times \frac{9.81 \times 11}{4\pi} = \underline{\underline{379.1}}$ KW/m

Wave energy devices are rarely used because of :-

(1) Capital cost (high)

(2) Maintenance (the sea is <u>very</u> destructive)

(3) Alternative sources of energy, such as coal gas and oil, are cheap, plentiful, and currently unrestricted (largely).

(4) We already have the infrastructure for coal, gas, etc and the technology is well tried and tested.

(5) Energy is not always available when it is wanted (domestic demand not correlated with storms). 35%

2/ (a) Bookwork – See lecture notes or any standard textbook. (30%)

(b) (i) In deep water: $\omega^2 = gK$

$$\therefore \quad K = \frac{4\pi^2}{9.81 \times 144} = 0.02795$$

$$C_0 = \frac{\omega}{K} = 18.736 \text{ m/s}$$

At 10m depth: $\frac{d}{T^2} = 0.0694$

$\therefore$ From curve in Data Sheet $\frac{L}{T^2} = 0.787$

$$\therefore \quad L = 113.3 \text{ m}$$

$$c = L/T = 9.44 \text{ m/s}$$

$$Kd = \frac{2\pi \times 10}{113.3} = 0.5546$$

$$\therefore K_s = \left[\frac{18.736}{9.44(1 + 1.109/1.351)}\right]^{\frac{1}{2}} = 1.044$$

From Snell's Law: $\sin\alpha = \frac{c}{c_0}\sin\alpha_0 = \frac{9.44}{18.736}\sin 45$

$$\therefore \quad \alpha = 20.87^\circ$$

$$\therefore K_r = \left(\frac{\cos\alpha_0}{\cos\alpha}\right)^{\frac{1}{2}} = \left(\frac{1}{0.9344\sqrt{2}}\right)^{\frac{1}{2}} = 0.870$$

$$\therefore H = K_s K_r H_0 = 8.17 \text{ m}$$

But what about wave breaking?

In water of constant depth waves break when $H \doteqdot 0.8d$

This would make the height of waves reaching breakwater $H = \underline{8m}$

However, on a sloping beach the wave height at breaking can be greater than $0.8d$. So, under these conditions, wave height at breakwater could be $\underline{8.17}$ m

50%

2(b)(ii)

Up to the point at which the waves start to break there is a reduction in mean water level (wave set down) but, thereafter, there is a gradual increase (wave set up). In the present problem, the depth of water at the breakwater is not much less than the depth at which the design waves might be expected to break (ie the first line of breakers is not far from the breakwater). So, we might expect the mean water level at the breakwater to be close to the minimum achieved at the first line of breakers. If this is the case, the height of waves reaching the breakwater would be less than that calculated in 2(b)(i).

If the deep-water design wave height were significantly greater the distance between the first line of breakers and the breakwater would also be greater. This would increase the mean water level and, consequently, the height of waves reaching the breakwater.

20%

3 (a) The wind shear stress factor $U_A = 0.71 (12)^{1.23}$

$= 15.09$

Making use of the wave-prediction curves provided:

(i) $(H_s)_{max} = 1.27\,m$ (10 %)

(ii) $T = 4.8\,s$ (10 %)

(iii) Duration = 3.6 hours (10 %)

(iv) If wind only blows for 1 hour $H_s = 0.54\,m$ (10 %)

(b) Significant wave height is the average height of the largest $33\frac{1}{3}\%$ of waves in any given sample. (10 %)

(c) Use of the full spectrum (rather than significant wave height) may be important:

(i) When calculating sediment transport

(ii) When studying the stability of structures exposed to wave action. (There may be resonant frequencies which could be excited by the waves). (20 %)

(d) Advantages

(1) Simple to install. This is particularly important if measurements are required in a hurry

(2) Objective and (subject to the limitations listed below) reasonably accurate.

3(d) Contd

<u>Disadvantages</u>

(1) Best for relatively short periods of observation (due to limited battery life, etc)

(2) No information about wave direction

(3) Not suitable for deep-water observations

(4) Higher frequencies attenuate more rapidly with depth. So, it is relatively easy to calculate the low frequency components of the wave profile but more difficult to measure high frequency terms. The calculation of high frequency terms is also complicated by harmonics of low frequency components and non-linear interactions between components.

30%

(5) Interpretation of measurements may be complex in situations where small-amplitude wave theory does not apply (e.g. when waves are breaking).

4/

(a) (i) the velocity potential for the incident wave is (from the wave data sheet)

$$\phi = \frac{\omega H \cosh k(y+d)}{2k \sinh kd} \cos(\omega t - kx)$$

The presence of the wall adds a reflected wave with the same amplitude, travelling in the negative x-direction. The velocity potential for both waves together is

$$\begin{aligned} \phi &= \frac{\omega H \cosh k(y+d)}{2k \sinh kd} \cos(\omega t - kx) + \frac{\omega H \cosh k(y+d)}{2k \sinh kd} \cos(\omega t + kx) \\ &= \frac{\omega H \cosh k(y+d)}{k \sinh kd} \cos \omega t \cos kx \end{aligned}$$

10%

(ii) and therefore the horizontal velocity is

$$u = \partial\phi / \partial x = -\frac{\omega H \cosh k(y+d)}{\sinh kd} \cos \omega t \sin kx$$

which is zero at $x=0$ for all t, and therefore satisfies the horizontal velocity condition on the wall. The horizontal velocity is also zero for all t wherever

$$\begin{aligned} \sin kx &= 0 \\ kx &= 0, -\pi, -2\pi .... \\ x &= 0, -\lambda/2, -\lambda, -3\lambda/2 .... \end{aligned}$$

since $k=2\pi/\lambda$

10%

(iii) The flow is a standing wave with nodes at multiples of $\lambda/2$ from the wall.

10%

4 (b)(i) At $x=-a=-\lambda/4$

$$u = U\cos\omega t$$
$$\partial u/\partial t = -U\omega\sin\omega t$$
$$\text{where } U = \frac{\omega H\cosh k(y+d)}{\sinh kd}$$

$\therefore$ at $y=0$, $u = \frac{\omega H}{\tanh kd}$

and so

$$F_x = \frac{1}{2}\rho C_D D U^2\cos^2\omega t - \left(\frac{\pi}{4}\right)\rho D^2 C_M U\omega\sin\omega t$$

The form of the expression for F_x is

$$F_x = A\cos^2\theta - B\sin\theta$$

where

$$A = (1/2)\rho C_D D U^2\omega^2$$
$$B = (\pi/4)\rho D^2 C_M U\omega$$

The extreme values of F_x occur when

$$0 = \partial F_x/\partial\theta = -2A\sin\theta\cos\theta - B\cos\theta$$
$$\cos\theta = 0 \text{ or } \sin\theta = -\frac{B}{2A}$$

In the first case, the maximum force per unit length is $\pm\left(\frac{\pi}{4}\right)\rho D^2 C_M U\sin\omega t$

The second case gives a stationary value only if $B/2A \le 1$. In that case

$$\sin\theta = -B/2A$$
$$\cos^2\theta = 1 - B^2/4A^2$$
$$F_x = A - 3B^2/4A$$

35%

(ii) If D is not small by comparison with the quarter-wavelength a, diffraction may be significant. The pile is then large enough to alter the form of the wave and the velocity potential. This particular case has an analytic solution. The effect of diffraction is usually to reduce the wave forces.

15%

(c) The oscillations might be in-line vortex-induced vibrations, or they might be a resonance with the waves. The first step is to measure the frequency. If the frequency is close to the wave frequency, resonance with the wave frequency is probably the cause. In order to investigate the possibility of vortex shedding, calculate the reduced velocity $V_R=U/ND$, where U is the maximum velocity (calculated from the formula above, or measured) and N is the natural frequency of x-direction oscillations

20%

4(c)(Contd)

(calculated, or taken as the same as the observed oscillation frequency). In-line oscillations in steady flows occur when V_R is 1.5 or more.

Oscillations can be reduced by:

1. by raising the natural frequency, for example by linking the pile to the wall by struts, or
2. by increasing damping, for example by filling the pile with sand, or
3. for vortex-excited vibrations, by adding external strakes or shrouds to break up the length-wise correlation of vortex shedding.