UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 30 April 2004 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 data sheet (8 pages).

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Define the terms (i) α decay, (ii) β decay and (iii) γ decay, and state how the mass number and atomic number of the decaying nucleus change in each case. [20%]

(b) What basic principles are used to minimize exposure when working with sources of ionising radiation? What type of radiological protection would be required when working with each of the three sources of ionising radiation listed in part (a)? [30%]

(c) A cobalt-60 source of mass 1 g is to be used for the on-site examination of pipeline welds. It is to be transported to site in a lead-shielded spherical container of radius 250 mm. It can be assumed that the thickness of the shield is, to a good approximation, equal to its radius.

Use the information given below to estimate (i) the activity of the cobalt-60 in Bq and (ii) the effective surface dose rate from the container in $\mathrm{Sv\,hr^{-1}}$, and comment on the latter result. [50%]

Data:

$$D = 1.6\times10^{-13}\times\frac{E\Sigma\phi t}{\rho}$$

where D is the dose in Gy

E is the average γ-ray energy in MeV

Σ is the macroscopic absorption cross-section in $\mathrm{m^{-1}}$

ϕ is the γ-ray flux in $\mathrm{m^{-2}s^{-1}}$

t is the exposure time in seconds

ρ is the density of human tissue in $\mathrm{kg\,m^{-3}}$

Cobalt-60 has a half-life of 5.272 yr and releases two γ-rays of energies 1.17 MeV and 1.33 MeV, respectively, each decay.

The macroscopic absorption cross-section for γ-rays in human tissue is $3\ \mathrm{m^{-1}}$, and the density of human tissue is $1000\ \mathrm{kg\,m^{-3}}$.

The exponential attenuation coefficient for γ-rays in lead can be taken to be $0.046\ \mathrm{mm^{-1}}$.

2 (a) Starting from the general version of the neutron diffusion equation given in the 4A1 Data Sheet, derive the neutron diffusion equation for a steady-state, source-free system

$$\nabla^2\phi + {B_m}^2\phi = 0$$

stating any assumptions made.

Explain the difference between *material buckling* ${B_m}^2$ and *geometric buckling* ${B_g}^2$. [25%]

(b) A large thermal reactor is to be constructed. From reactor physics measurements the *material buckling* of the proposed fuel-moderator-coolant configuration is found to be 25×10^{-6} cm^{-2}. Stating any assumptions made, estimate the minimum critical volume of an *unreflected* core with this material buckling. What should its shape be?

The flux solutions for certain bare reactor geometries are:

Cube: $\phi \propto \cos\left(B_g x/\sqrt{3}\right)$ Sphere: $\phi \propto \dfrac{\sin\left(B_g r\right)}{r}$ [30%]

(c) The reactor is actually to be built in the form of a rectangular parallelepiped with a graphite reflector. Estimate the new minimum volume of core material. You may assume that the core saving arising from the use of the reflector is equal to the diffusion length $L = \sqrt{D/\Sigma_a}$ in the reflector, for each reflector face. Relevant data for graphite can be found in the 4A1 Data Sheet. [30%]

(d) Given the choice of reflector material, what coolant is likely to have been chosen? What advantages does this choice bring? [15%]

3 An AGR fuel element is rated at 10 MW and is 8 m long. The equivalent bare reactor is 10 m long. The coolant enters at 335 °C and leaves at 635 °C. The effective diameter of the fuel in the channel is 0.8 m and the overall cladding-to-gas heat transfer coefficient is $5\,\mathrm{kW\,m^{-2}\,K^{-1}}$. The power distribution along the fuel element can be assumed to have the form of a "chopped cosine".

(a) Sketch the forms of the variation of the coolant and cladding surface temperatures along the channel. [20%]

(b) Determine the location and magnitude of the maximum cladding surface temperature along the channel. [60%]

(c) Another fuel element in the same reactor, further from the centre of the core, is rated at 5 MW. It is proposed to 'gag' the coolant in the channel, i.e. to reduce the coolant mass flow rate through the channel. Assume that the coolant entry temperature, the coolant specific heat capacity and the overall cladding-to-gas heat transfer coefficient are unchanged by any gagging.

(i) Compared to the 10 MW channel, what is the fractional reduction in coolant mass flow rate needed to achieve the same coolant exit temperature?

(ii) If this condition is achieved, will the maximum cladding surface temperature along the 5 MW channel be greater or less than that along the 10 MW channel? Explain your reasoning. [20%]

4 The equations governing the behaviour of xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - \phi \sigma X$$

(a) Explain the meaning of each symbol in these equations. [15%]

(b) Show that the steady-state loss of reactivity ρ_0 due to xenon poisoning in a high power reactor approaches $-\gamma_i / \nu$, where ν is the mean number of neutrons released per fission. [15%]

(c) Such a reactor is shut down rapidly after a prolonged period of operation at flux level ϕ. Find a general expression for the post-shutdown variation of the xenon-135 concentration.

Hence show that, if the flux level is sufficiently high that $|\lambda_x + \sigma\phi| >> |\lambda_i - \lambda_x|$, the post-shutdown variation of xenon-135 is given approximately by

$$X = \frac{\gamma_i \Sigma_f \phi}{\lambda_i - \lambda_x} [\exp(-\lambda_x t) - \exp(-\lambda_i t)]$$ [40%]

(d) Using this approximate result, find an expression for the time after the shutdown at which the xenon concentration is at its maximum, and show that the loss of reactivity then, ρ_{max}, is given by

$$\frac{\rho_{max}}{\rho_0} = \frac{\sigma\phi}{\lambda_i} \left(\frac{\lambda_x}{\lambda_i} \right)^n$$

where $n = \lambda_x / (\lambda_i - \lambda_x)$. [30%]

END OF PAPER

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Data Sheet S/15

Revised January 2003

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	299.792458×10^6 m s^{-1}
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck constant	h	6.626176×10^{-32} J s
Boltzmann constant	k	1.380662×10^{-23} J K^{-1}
Elementary charge	e	$1.6021892 \times 10^{-19}$ C

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	10^{-28} m^2

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

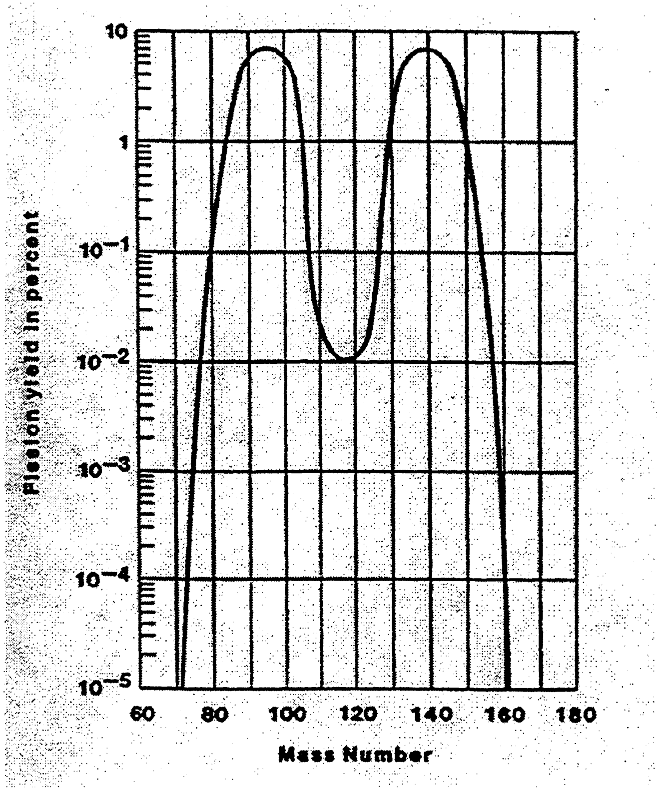
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow{33\text{ s}}$ Rb-90 $\xrightarrow{2.7\text{ min}}$ Sr-90 $\xrightarrow{28\text{ yr}}$ Y-90 $\xrightarrow{64\text{ hr}}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow{23\text{ min}}$ Te-131 $\xrightarrow{24\text{ min}}$ I-131 $\xrightarrow{8\text{ days}}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow{6.7\text{ hr}}$ Xe-135 $\xrightarrow{9.2\text{ hr}}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn .

Pm-149 $\xrightarrow{54\text{ hr}}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn .

Se-87 $\xrightarrow{17\text{ s}}$ Br-87 $\xrightarrow{55\text{ s}}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow{78\text{ min}}$ Rb-87}

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy $\Rightarrow$ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\bar{\mu})}$$

with $\bar{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0^{th} Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $g\,cm^{-3}$	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small }x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L =$ surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \frac{1}{h} + \frac{1}{h_s} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)$$

bulk coolant	scale	thin clad	bond	fuel pellet

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity