ENGINEERING TRIPOS PART IIB

Wednesday 21 April 2004 2.30 to 4

Module 4A3

TURBOMACHINERY I

Answer not more than two questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of the question is indicated in the right margin.

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) State the definition of the stagnation pressure loss coefficient for a turbine cascade and describe how it varies with exit Mach number for a typical cascade. Also explain how and why the exit flow angle varies with exit Mach number.

The energy loss coefficient of a turbine cascade is defined by

$$\zeta = \frac{V_{2,is}^2 - V_2^2}{V_{2,is}^2}$$

where V_2 is the actual exit velocity from the cascade and $V_{2,is}$ is the velocity that would be obtained in an isentropic expansion from the same inlet conditions to the same exit static pressure. Calculate the energy loss coefficient for a cascade operating at an exit Mach number of 1.0 and with a stagnation pressure loss coefficient of 0.05. [30%]

- (b) The above cascade has an opening to pitch ratio of 0.3 and an exit Mach number of 1.0. It is estimated that $\frac{2}{3}$ of the stagnation pressure loss occurs downstream of the throat. Calculate the exit flow angle at this condition. [20%]
- (c) Describe what is meant by the limit load of a turbine cascade and explain at what condition it occurs. For the above cascade, explain why the stagnation pressure loss upstream of the throat is expected to be unchanged as the exit Mach number is increased from 1.0 to the limit load condition, whilst the stagnation pressure loss downstream of the throat increases. If at the limit load condition the stagnation pressure loss downstream of the throat is double that which occurs when the exit Mach number is 1.0, what are the exit Mach number and the exit flow angle of the cascade? Note a trial and error or a graphical solution is expected. [40%]
- (d) The above cascade forms the stator row of a turbine stage which has an overall stagnation temperature ratio of 0.7. The exit Mach number from the stator is 1.0. Estimate the loss of efficiency of the stage due to the losses which occur in the stator alone. [10%]

Assume the working fluid has the properties of air throughout.

2 (a) Define the isentropic efficiency and polytropic efficiency for a compressor and explain why these two different definitions are used.

A centrifugal compressor is being designed for a stagnation pressure ratio of 4.0 at a rotational speed of 25000 rpm. The working fluid is air with an inlet stagnation temperature of 300 K and inlet stagnation pressure of 1 bar. Assume the total-to-total isentropic efficiency of the compressor is 0.8. Calculate the stagnation temperature ratio and the polytropic efficiency of the compressor based on the stagnation conditions. [20%]

- (b) The impeller blades are to be backswept at 45° at their exit and their slip factor is estimated to be 0.85. The absolute Mach number of the flow leaving the impeller is to be 1.0 and the absolute flow direction is at 70° to the radial. Sketch the velocity triangle at the impeller exit and calculate the impeller tip speed and radius. [30%]
- (c) The Mach number at the leading edge of the diffuser blades is to be 0.8. The velocity leaving the diffuser blades may be assumed to be negligible. The static pressure rise through the diffuser is 70% of the isentropic static pressure rise. Calculate the static pressure at the diffuser blade leading edge. [20%]
- (d) Explain why a pressure rise occurs in the vaneless space between the impeller trailing edge and the diffuser blade leading edge. The compression in the vaneless space may be assumed to have a polytropic efficiency of 0.7 based on the static flow conditions. Calculate the static pressure at the impeller trailing edge. If the mass flow rate through the compressor is to be 10 kgs⁻¹, calculate the axial width of the impeller trailing edge. [20%]
- (e) Calculate the total-to-total efficiency of the impeller alone based on absolute stagnation conditions at its trailing edge. [10%]

- 3 (a) If a propulsion nozzle is connected to the exit of a turbine, explain how the ratio of stator throat area to propulsion nozzle area can be used to fix the operating point of the turbine. [10%]
- (b) A single shaft turbojet has a compressor pressure ratio of 8. The turbine and compressor polytropic efficiencies are both 0.9. The exit of the turbine is connected to an isentropic propulsion nozzle. At design, the exit temperature of the combustion chamber is 1500 K. The turbojet is tested in a test cell at ambient conditions of 1 bar and 15 °C. The turbine's stator blades have a small throat area and at design the turbine stator and propulsion nozzle are both choked. Calculate the ratio of the turbine stator throat area to the propulsion nozzle throat area. [35%]
- (c) The turbine stator and propulsion nozzle both remain choked over the entire operating range of the turbojet. Show that as the turbine inlet temperature T_{03} of the turbojet is altered the compressor non-dimensional mass flow rate is given by

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{P_{01} A_1} = B\sqrt{\frac{T_{01}}{T_{03}}} \left(1 + C\frac{T_{03}}{T_{01}}\right)^{\eta_p \frac{\gamma}{\gamma - 1}}$$

where P_{01} and T_{01} are the turbojet inlet pressure and temperature, A_1 is the compressor inlet area, \dot{m} is the mass flow rate, η_p is the polytropic efficiency of the compressor, c_p is the specific heat capacity at constant pressure and B and C are constants. Determine expressions for the constants B and C.

- (d) Assuming the compressor and turbine have constant efficiency show graphically how the working line of the turbojet on the compressor characteristic can be determined. The efficiency of the real compressor drops as the speed of the turbojet is reduced. How will this change the working line of the turbojet? [20%]
- (e) In an emergency the fuel flow rate to the turbojet is suddenly cut. Explain how the locus of points on the compressor characteristic, through which the turbojet moves, differs from the working line sketched in part (d). [10%]

For air take the specific heat capacity at constant pressure to be 1.005 kJkg⁻¹K⁻¹ and take gamma to be 1.4.

For combustion products take the specific heat capacity at constant pressure to be 1.224 kJkg⁻¹K⁻¹ and gamma to be 1.3.

END OF PAPER