UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Tuesday 20 April 2004 2.30 to 4

Module 4A6

FLOW INDUCED SOUND AND VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
Data sheet for 4A6 (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 An acoustic point source of frequency ω is located at the origin, and produces a field with unsteady velocity potential

$$\frac{A}{r}\mathrm{e}^{i\omega(t-r/c_o)} \qquad (1)$$

where A is a positive real constant, c_o is the sound speed and r is the distance of the observer from the origin. A second source is placed at the point $(2a, 0, 0)$, and is identical to the first apart from a constant phase difference ψ (so that in (1) A is replaced by $A\mathrm{e}^{i\psi}$).

(a) For the single source at the origin, calculate the acoustic velocity and pressure. Hence, determine the time-averaged energy flux vector far from the source, and calculate the total time-averaged power escaping to infinity. [20%]

(b) For the two sources together, show that for $r >> a$ the amplitude of the acoustic pressure is

$$\frac{2A\rho_o\omega}{r}\left|\cos\left(ka\cos\theta + \frac{\psi}{2}\right)\right|, \qquad (2)$$

[20%]

where $k = \omega / c_o$.

(c) Sketch graphs of the amplitude of the acoustic pressure as a function of θ in the two separate cases $ka << 1$ and $ka >> 1$. Comment on the physical reasons for the appearance of these graphs. [20%]

(d) Calculate the total time-averaged power escaping to infinity, and show that the ratio of this quantity to the result obtained in (a) above for a single source is

$$2\left[1+\frac{\sin(2ka)}{2ka}\cos\psi\right]. \qquad [30\%]$$

Explain the significance of this result when $ka << 1$ and when $ka >> 1$. [10%]

2 (a) Consider a stratified medium in which the sound speed, $c(x)$, is given by

$$c(x) = c_o e^{\alpha x}$$

where c_o and α are positive constants. Sound is emitted from a source at $x = y = 0$ in a direction making an angle θ_o to the x axis.

(i) Using ray theory, show that the equation of the ray is

$$y = \frac{1}{\alpha}\left[\sin^{-1}\left(e^{\alpha x}\sin\theta_o\right) - \theta_o\right]$$

[You may recall, without proof, Snell's Law, which states that $\sin\theta / c$ is constant, where θ is the angle the ray makes to the x axis. You may also find it helpful to use the substitution $u = e^{\alpha x}\sin\theta_o$ when evaluating an integral.] [30%]

(ii) In the case $0 < \theta_o < \frac{\pi}{2}$, find the maximum value of x along the ray. At what value of y does this ray return to $x = 0$? [15%]

(iii) Sketch a graph showing the rays corresponding to

$$\theta_o = 0,\ \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3\pi}{4},\ \pi .$$ [15%]

(b) A plane sound wave of angular frequency ω is normally incident on a wall of mass per unit area m. Show that the ratio of transmitted pressure amplitude to incident pressure amplitude is

$$\frac{2\rho_o c_o}{\sqrt{4\rho_o^2 c_o^2 + m^2\omega^2}},$$

where ρ_o and c_o are the mean density and sound speed of the air surrounding the wall. [35%]

What dimensionless ratio controls the level of attenuation? [5%]

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3 (a) A vibrating string stretched from (0, 0, -L) to (0, 0, L) exerts a force on the surrounding fluid. The force acts in the 1 - direction and at $(0,0,y_3), |y_3| < L$, the force per unit length of the string is equal to $\varepsilon e^{i\omega t}\cos(\pi y_3/2L)$, where ε is a real constant.

Show that the far-field pressure disturbance is given by

$$p(\mathbf{x},t) = -\frac{i\omega\varepsilon L x_1}{r^2 c}\;\frac{\cos\alpha L}{4\alpha^2L^2-\pi^2}e^{i\omega(t-r/c)}$$

where $\alpha = \dfrac{\omega x_3}{cr}$ and $r = |\mathbf{x}|$. You may assume that the density of the fluid is negligible compared with that of the wire, and that any air motion not described by the linearised equations contributes only quadrupole radiation which may be neglected.

Note that $$\int_{-L}^{L} e^{i\alpha y}\cos(\beta y)\,dy = \frac{2}{\alpha^2-\beta^2}\left[\alpha\sin(\alpha L)\cos(\beta L) - \beta\cos(\alpha L)\sin(\beta L)\right]$$ [80%]

(b) What is the main disadvantage of using a vibrating string to generate sound in a musical instrument, and give <u>two</u> examples of ways in which this is overcome. [20%]

4 (a) What variation in the normal surface velocity over a sphere leads to

(i) a monopole sound field
(ii) a dipole sound field

centred on the sphere? [10%]

(b) A spherical air bubble of undisturbed radius a and surface tension T is surrounded by water with mean pressure p_o, density ρ_o and sound speed c. The bubble is irradiated by an incident plane sound wave of amplitude I and frequency ω $\left(<< c/a\right)$.

(i) Determine the scattered sound field if the bubble responds adiabatically. [70%]

(ii) What is the role of surface tension? [5%]

(iii) What controls the response of the bubble at resonance? [5%]

(iv) Describe how the scattered field from a small rigid sphere would differ. (No detailed calculations are required for (iv)). [10%]

END OF PAPER

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Module 4A6 FLOW INDUCED SOUND AND VIBRATION DATA CARD

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins
Speed of sound in sea water, c, is a function of temperature, T

T °C	-4	0	5	10	15	20	25	30
c ms^{-1}	1430.2	1449.5	1471.1	1490.2	1507.1	1521.9	1543.7	1545.9

Units of sound measurement

SPL (sound pressure level) $= 20\log_{10}\left(\dfrac{p'_{rms}}{2.10^{-5}\,\text{Nm}^{-2}}\right)$ dB

IL (intensity level) $= 10\log_{10}\left(\dfrac{\text{intensity}}{10^{-12}\,\text{watts m}^{-2}}\right)$ dB

PWL (power level) $= 10\log_{10}\left(\dfrac{\text{sound power output}}{10^{-12}\,\text{watts}}\right)$ dB

Definitions

Surface impedance, Z_s, relates the pressure perturbation applied to a surface, p', to its normal velocity v_n; $p' = Z_s v_n$.

Characteristic impedance of a fluid $\rho_0 c$

Nondimensional surface impedance of a surface $Z_s/\rho_0 c$

Transmission loss $= 10\log_{10}\left(\dfrac{\text{incident sound power}}{\text{transmitted sound power}}\right)$

Absorption coefficient of a sound absorber $= \dfrac{\text{sound power absorbed}}{\text{incident sound power}}$

Sound absorption (in metric sabins) $= \sum \alpha_i S_i$ where S_i is surface area (in metres^2) with absorption coefficient α_i.

Reverberation time of a room = time taken for the sound intensity level in the room to drop from 60dB to the threshold of hearing.

Wavelength, λ, for sound waves with angular frequency ω, $\lambda = 2\pi c/\omega$

Wave-number $k = 2\pi/\lambda$

Phase speed $= \omega/k$

Group velocity $= \dfrac{\partial \omega}{\partial k}$

Helmholtz number (or compactness ratio) $= kD$ where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω, produced in a flow with speed U, length scale D.

BASIC EQUATIONS FOR LINEAR ACOUSTICS

Conservation of mass $\dfrac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} = 0$

Conservation of momentum $\rho_0 \dfrac{\partial \mathbf{v}}{\partial t} + \nabla p' = 0$

Isentropic $c^2 = \left.\dfrac{dp}{d\rho}\right|_S$

These equations combine to give the **wave equation** $\dfrac{1}{c^2}\dfrac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$

Energy density $e = \frac{1}{2}\rho_0 v^2 + \frac{1}{2} p'^2 / \rho_0 c^2$

Intensity $\mathbf{I} = p'\mathbf{v}$

$\text{div}\,\bar{\mathbf{I}} = 0$ for statistically stationary (in time) sound fields.

Velocity potential $\phi(\mathbf{x},t)$ satisfies the wave equation and $p' = -\rho_0 \dfrac{\partial \phi}{\partial t}$, $\mathbf{v} = \nabla \phi$.

SIMPLE WAVE FIELDS

1D or plane wave

The general solution of the 1D wave equation is

$$p'(x_1,t) = f(x_1 - ct) + g(x_1 + ct)$$

where f and g are arbitrary functions.

In a plane wave propagating to the right $p' = \rho_0 c u$. In a plane wave propagating to the left $p' = -\rho_0 c u$, u being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is

$$p'(r,t) = \frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}$$

where $r = |\mathbf{x}|$; f and g are arbitrary functions.

cos θ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x_1}\left[\frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}\right] = \cos\theta \frac{\partial}{\partial r}\left[\frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}\right].$$

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SOURCES

Point sources

<u>monopole</u> of strength $Q(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t)=\frac{Q(t-|\mathbf{x}|/c)}{4\pi|\mathbf{x}|}.$$

<u>dipole</u> of strength $\mathbf{F}(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t)=-\frac{\partial}{\partial x_i}\left[\frac{F_i(t-|\mathbf{x}|/c)}{4\pi|\mathbf{x}|}\right]=\frac{x_i}{4\pi}\left[\frac{1}{|\mathbf{x}|^3}F_i(t-|\mathbf{x}|/c)+\frac{1}{|\mathbf{x}|^2c}\frac{\partial F_i}{\partial t}(t-|\mathbf{x}|/c)\right]$$

Distributed sources

<u>Monopole</u>, strength $q(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)p'=q$, pressure field $p'(\mathbf{x},t)=\int\frac{q(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$

<u>Dipole</u>, strength $\mathbf{f}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)p'=-\nabla\cdot\mathbf{f}$, $p'(\mathbf{x},t)=-\frac{\partial}{\partial x_i}\int\frac{f_i(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$.

<u>Quadrupole</u>, strength $T_{ij}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)p'=\frac{\partial T_{ij}}{\partial x_i\partial x_j}$, $p'(\mathbf{x},t)=\frac{\partial^2}{\partial x_i\partial x_j}\int\frac{T_{ij}(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$.

<u>Far-field form</u> $|\mathbf{x}|>>|\mathbf{y}|$, $\mathbf{y}$ near origin

$$|\mathbf{x}-\mathbf{y}|\sim|\mathbf{x}|-\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|}+O(|\mathbf{x}|^{-2})$$

$$\frac{1}{|\mathbf{x}-\mathbf{y}|}\sim\frac{1}{|\mathbf{x}|}+O(|\mathbf{x}|^{-2})$$

$$\frac{\partial}{\partial x_i}\sim-\frac{x_i}{|\mathbf{x}|c}\frac{\partial}{\partial t}+O(|\mathbf{x}|^{-1}).$$

Physical sources

<u>Heat addition</u> at a rate $w(\mathbf{x},t)$/unit volume is equivalent to a monopole source of strength $\frac{(\gamma-1)}{c^2}\frac{\partial w}{\partial t}$.

<u>Lighthill's acoustic analogy</u> shows that jet noise is generated by quadrupoles of strength
$T_{ij}=\rho v_i v_j+\left(p'-c^2\rho'\right)\delta_{ij}-\tau_{ij}$.

<u>The Ffowcs-Williams Hawkings equation</u> shows that foreign bodies in linear motion generate far-field sound

$$p'(\mathbf{x},t)=\frac{1}{4\pi|\mathbf{x}|}\frac{\partial}{\partial t}\int_S\rho_0\mathbf{n}\cdot\mathbf{u}\left(\mathbf{y},t-\frac{|\mathbf{x}|}{c}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c}\right)dS+\frac{x_i}{4\pi|\mathbf{x}|^2c}\frac{\partial}{\partial t}\int n_i p\left(\mathbf{y},t-\frac{|\mathbf{x}|}{c}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c}\right)dS$$

USEFUL MATHEMATICAL FORMULAE

In spherical polar coordinates (r,θ,ϕ)

$$\nabla p'=\left(\frac{\partial p'}{\partial r},\frac{1}{r}\frac{\partial p'}{\partial\theta},\frac{1}{r\sin\theta}\frac{\partial p'}{\partial\phi}\right)$$

For $\mathbf{v}=(v_r,v_\theta,v_\phi)$

$$\nabla\cdot\mathbf{v}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta)+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla^2p'=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p'}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2p'}{\partial\phi^2}.$$

Heaviside function $H(t-\tau)=1\ \ t>\tau\ ;\ =0\ \ t<\tau$

δ-functions

<u>Kronecker delta</u> $\delta_{ij}=1$ if $i=j$; 0 if $i\neq j$

<u>1D δ-function</u>: $\delta(t)=0$ for $t\neq0$; $f(t)\delta(t-\tau)=f(\tau)\delta(t-\tau)$

$\int_{-\infty}^{\infty}\delta(t)dt=1$ and $\int_{-\infty}^{\infty}\delta(t-\tau)f(t)dt=f(\tau)$ for any function $f(t)$.

<u>3D δ-function</u>: $\delta(\mathbf{x})=\delta(x_1)\,\delta(x_2)\,\delta(x_3)$;
$\delta(\mathbf{x})=0$ for $|\mathbf{x}|\neq0$;
$f(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})=f(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})$
$\int_V\delta(\mathbf{x})dV=1$ for any volume V that includes the origin

and

$\int_V\delta(\mathbf{x}-\mathbf{y})f(\mathbf{x})d^3\mathbf{y}=f(\mathbf{x})$ for any function $f(\mathbf{x})$ and volume V that includes $\mathbf{x}$.

$$\nabla^2\left(\frac{1}{|\mathbf{x}|}\right)=-4\pi\delta(\mathbf{x}).$$

Autocorrelation

$F(\xi)$, the autocorrelation of $f(y)=\overline{f(y)f(y+\xi)}$

$$F(0)=\overline{f^2}$$

Integral lengthscale ℓ $\quad \ell\overline{f^2}=\int_{-\infty}^{\infty}F(\xi)d\xi.$