UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Tuesday 20 April 2004 9 to 10.30

Module 4A8

ENVIRONMENTAL FLUID MECHANICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A8 Data Card (5 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Noting the advantages and disadvantages of each approach, explain the differences between computing turbulent flows by

(i) Direct Numerical Simulation;

(ii) Large Eddy Simulation;

(iii) Reynolds Averaged Navier Stokes turbulence modelling. [25%]

(b) Explain the derivation and meaning of the Richardson number. What is its significance in a turbulent flow? [25%]

(c) What is the Dry Adiabatic Lapse Rate? [25%]

(d) *Katabatic* flows run downhill, while *anabatic* flows run uphill. What drives these flows? By consideration of the Richardson number, decide which of these flows is likely to be faster. [25%]

2 A parcel of fluid of mass m is displaced vertically by a distance Δz in a still region where the local density gradient is $d\rho / dz$. Assume that the density of the parcel remains unchanged by the move.

(a) Assuming that $d\rho / dz$ is negative and the displacement is vertically upwards, derive an expression for the restoring force acting on the fluid parcel. [30%]

(b) Assuming that the parcel executes simple harmonic motion, derive an expression for the natural frequency of oscillation. (Hint: Compare with a spring-mass system.) [30%]

(c) Explain the phenomenon of internal waves in the atmosphere. What drives these waves? Explain under what conditions you would expect large internal waves. [40%]

3 (a) The concentration (in kg m^{-3}) c of an inert pollutant in the air above a city is to be calculated with a Box Model. The mixing height is H, the city has dimensions L and W in the directions parallel and normal to the wind respectively, and the wind has velocity U. The pollutant is emitted uniformly across the city at a rate q kg s^{-1} m^{-2} and the incoming air is uncontaminated from the pollutant. Derive an equation for the rate of change of c and find the steady-state value of c as a function of the given parameters. [30%]

(b) Discuss how c may change during the day, given that the wind direction and speed and the emission rate are constant. [20%]

(c) We may assume that aviation traffic throughout the year releases another S kg s^{-1} m^{-3} of pollutant in the Box Model of part (a), where S is distributed according to a normal distribution with mean $\overline{S}$ and variance σ^2.

(i) Find the yearly average $\overline{c}$ and the variance $\sigma_c{}^2$ of the concentration for this case. [40%]

(ii) What is the probability density function of c? A qualitative answer or a mathematical expression are equally sufficient. [10%]

4 (a) A line source of length L emitting Q kg s^{-1} m^{-1} of inert pollutant per unit length is placed *parallel* to the wind of speed U (Fig. 1). Find an expression for the pollutant concentration as a function of x along the direction of the line source (i.e. at $y = z = 0$), where x is measured from the downwind edge of the source. You may assume that the dispersion coefficients in the two cross-wind directions (i.e. y and z) are equal and proportional to downwind distance. [70%]

(b) How would you modify your answer to part (a) above, if the line source was located at a height H above a fully-reflecting solid surface? [30%]

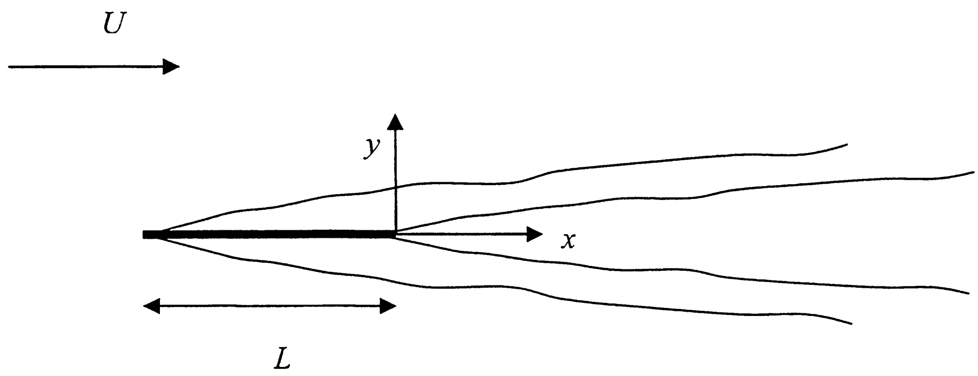

Fig. 1

END OF PAPER

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4A8: Environmental Fluid Mechanics

Part I: Turbulence and Fluid Mechanics

DATA CARD

Rotating Flows

Geostrophic Flow
$$-\frac{1}{\rho}\nabla p = 2\underline{\Omega}\times\underline{u}$$

Ekman Layer Flow
$$-2\Omega_z v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}$$

$$2\Omega_z v = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

OR
$$-2\Omega_z v = \nu\frac{\partial^2 u}{\partial z^2}$$

$$-2\Omega_z v(u_0 - u) = \nu\frac{\partial^2 v}{\partial z^2}$$

GEOSTROPHIC VELOCITY

Solution is

$$u = u_0\left[1 - e^{-z/\Delta}\cos\frac{z}{\Delta}\right]$$

$$v = u_0 e^{-z/\Delta}\sin\frac{z}{\Delta}$$

$$\Delta = \left(\frac{\nu}{\Omega_z}\right)^{1/2}$$

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Turbulent Flows – **Incompressible**

Continuity Equation $\nabla \bullet \underline{U} = \frac{\partial U_i}{\partial x_i} = 0$

Momentum Equation $\rho \frac{D\underline{U}}{Dt} = -\nabla \mathrm{P} + \mu \nabla^2 \underline{U} + \underline{F}$

$$\rho \frac{DU_i}{Dt} = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j^2} + F_i$$

Enthalpy Equation $\rho c_p \frac{DT}{Dt} = -k \frac{\partial^2 T}{\partial x_i^2}$

Reynolds Transformation $U_i = \overline{U_i} + u_i$ etc

Reynolds Stress $= -\rho \overline{u_i u_j}$

Reynolds Heat Flux $= -\rho \overline{c_p u_j \Theta}$

Turbulent Kinetic Energy Equation

$$\frac{D}{Dt}\frac{q^2}{2} = -\overline{u_i u_k}\frac{\partial \overline{U_i}}{\partial x_k} - \nu \overline{\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_k}\right)} + \frac{\overline{f_i u_i}}{\rho} + \text{ transport of kinetic energy forms}$$

In flows with thermally driven motion

$$\frac{f_i u_i}{\rho} = \frac{g}{T} \bullet \overline{\Theta u_i}$$

i = Vertical direction

Dissipation of turbulent kinetic energy $\varepsilon \approx \frac{u'^3}{\ell}$

Kolmogorov microscale $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$

Taylor microscale (λ) $\varepsilon = 15\nu \frac{u'^2}{\lambda^2}$

(ν is the kinematic viscocity)

Density Influenced Flows

Atmospheric Boundary Layer

$$\left.\frac{dT}{dz}\right|_{\text{NEUTRAL STABILITY}} = -\frac{g}{C_p} = \left.\frac{dT}{dz}\right|_{\text{DALR}}$$

$$R_i = \frac{g}{T}\frac{\frac{dT}{dz} - \left.\frac{dT}{dz}\right|_{\text{DALR}}}{\left(\frac{dU}{dz}\right)^2} = \text{RICHARDSON NUMBER}$$

Neutral Stability

$$U = \frac{u_*}{\kappa}\ln\frac{z}{z_0}; \quad \frac{dU}{dz} = \frac{u_*}{\kappa z}$$

$$u_* = \sqrt{\frac{\tau_w}{\rho}} \; ; \quad \kappa = \text{von Karman Constant} = 0.40$$

Non-Neutral Stability

$$\text{L} = \text{Monin-Obukhov length} = -\frac{u_*^3}{\kappa \frac{g}{T}\frac{Q}{\rho c_p}}$$

Q = surface heat flux

$$\frac{dU}{dz} = \frac{u_*}{\kappa z}\left(1 - 15\frac{z}{L}\right)^{-1/4} \qquad \text{Unstable}$$

$$= \frac{u_*}{\kappa z}\left(1 + 4.7\frac{z}{L}\right) \qquad \text{Stable}$$

Buoyant plume for a point source

$$\frac{d}{dz}\pi R^2 w = 2\pi R u_e \qquad \text{(i)}$$

$$\frac{d}{dz}\rho\pi R^2 w = \rho_a 2\pi R u_e \qquad \text{(ii)}$$

$$\frac{d}{dz}\rho\pi R^2 w = g(\rho_a - \rho)\pi R^2 \qquad \text{(iii)}$$

(i) and (ii) give

$$\pi R^2 w \left(\frac{\rho_a - \rho}{\rho_a} \right) g = \text{constant} = F_0 \text{ (buoyancy flux)}$$

$$u_e = \alpha w$$

(α = Entrainment coefficient)

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} = \frac{g}{T}\frac{dT}{dz}$$

Actually $\frac{g}{T}\left(\frac{dT}{dz} - \left.\frac{dT}{dz}\right|_{\text{DALR}} \right)$

N = Brunt – Vaisala Frequency or Buoyancy Frequency

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4A8: Environmental Fluid Mechanics

Part II: Dispersion of Pollution in the Atmospheric Environment

DATA CARD

Transport equation for the mean of the reactive scalar ϕ:

$$\frac{\partial \overline{\phi}}{\partial t} + \overline{u}_j \frac{\partial \overline{\phi}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial \overline{\phi}}{\partial x_j} \right) + \overline{\dot{w}}$$

Transport equation for the variance of the reactive scalar ϕ:

$$\frac{\partial g}{\partial t} + \overline{u}_j \frac{\partial g}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial g}{\partial x_j} \right) + 2K\left(\frac{\partial \overline{\phi}}{\partial x_j} \right)^2 - \frac{2}{T_{turb}} g + 2\overline{\phi' \dot{w}'}$$

Mean concentration of pollutant after instantaneous release of Q kg at t=0:

$$\overline{\phi}(x,y,z,t) = \frac{Q}{8(\pi t)^{3/2}(K_x K_y K_z)^{1/2}} \exp\left[-\frac{1}{4t}\left(\frac{(x-x_0)^2}{K_x} + \frac{(y-y_0)^2}{K_y} + \frac{(z-z_0)^2}{K_z} \right) \right]$$

Gaussian plume spreading in two dimensions from a source at $(0,0,z_0)$ emitting Q kg/s:

$$\overline{\phi}(x,y,z) = \frac{Q}{2\pi} \frac{1}{U\sigma_y \sigma_z} \exp\left[-\left(\frac{y^2}{2\sigma_y^2} + \frac{(z-z_0)^2}{2\sigma_z^2} \right) \right]$$

One-dimensional spreading from line source emitting Q/L kg/s/m :

$$\overline{\phi}(x,y) = \frac{Q}{UL} \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2} \right)$$

Relationship between eddy diffusivity and dispersion coefficient:

$$\sigma^2 = 2\frac{x}{U}K$$