UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 19 April 9 to 10:30

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator**

(TURN OVER

1 (a) In Quantum Mechanics, there are situations when one can represent electrons using plane waves, and others when wave-packets should be used. Under what circumstances are each of these representations appropriate? [20%]

(b) Describe the differences between electromagnetic and matter waves in Quantum Mechanics. [20%]

(c) We have a transistor where the current flow is driven by a potential difference across the transistor of 5 V. As we shrink the transistor in all dimensions, a number of factors which have a detrimental effect on the transistor's operation come into play. Discuss the latter statement within the context of Quantum Mechanics. At what size will quantum effects start to play a major role in this transistor? [40%]

(d) Discuss ways in which these Quantum effects might be reduced in the transistor. [20%]

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2 (a) In Quantum Mechanics, what do wave-functions represent? How can we use them to describe a system? State the rules we need to apply when determining the wave-functions in boundary-value problems. What do these rules mean physically? [20%]

(b) In a Television, we have a beam of electrons at 100 V. They are passed through a hollow metal cylinder which is at a potential of 50V. If we can assume that this cylinder is infinitely long, what proportion of the incident beam will be reflected at its entrance? [30%]

(c) If, in fact this cylinder is only 10 nm long, then what proportion of the incident beam will be reflected? [40%]

(d) A careless engineer accidentally swaps the voltages to the beam and the 10 nm long cylinder. Estimate the proportion of the incident beam which passes through the cylinder now. [10%]

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3 (a) Consider an electron in a one-dimensional infinitely deep potential well of width 1 nm. Derive the spectrum of allowed energy eigenvalues. Why is this spectrum discrete rather than continuous? [25%]

(b) Show that if an electron is placed in any symmetric potential well, then the corresponding wave-functions must be either symmetric or anti-symmetric. [15%]

(c) In reality, it is only possible to fabricate *finite* quantum wells. Discuss the techniques used to fabricate them. What are the qualitative differences between the wave-functions of electrons in finite and infinite quantum wells? When is it reasonable to approximate a finite well as an infinite one? [30%]

(d) Now consider a quantum well which is 5 nm wide, and which has electron and hole effective masses of 0.1 $m_e$, and 0.06 $m_e$, respectively, where $m_e$ is the free-electron mass. Given that the band-gap of the semiconductor in the quantum well is 1.5 eV, sketch the approximate form of the optical density of the well in the range 0-1.55 eV. [30%]

4 (a) In conventional optical microscopy, what is the minimum feature size that can be resolved? What is the Rayleigh criterion? Briefly describe how this limit to resolution can be overcome. [15%]

(b) Describe the basic principle of operation of Scanning-Probe Microscopes. [15%]

(c) In the Scanning Tunnelling Microscope (STM), the tunnel current, I, depends on the Quantum-Mechanical transmission probability, T. What is the relationship between I and T? What is the STM actually measuring? How can one arrange the STM so that it is primarily obtaining information about the sample being imaged? [25%]

(d) Sketch the potential as seen by an electron in an STM experiment, stating any assumptions made. For a distance of 0.8 nm between the STM tip and the sample surface, calculate the change in tunnel current due to a reduction of this distance by 0.1 nm, assuming that both tip and sample are made from the same metal with a work-function of 4 eV. [35%]

(e) What is STM useful for? [10%]

(TURN OVER

5 (a) Consider a potential well. For the ideal case at zero Kelvin, and where there are no defects in the semiconductor materials, sketch the kinetic energy versus position of an electron which is incident from the left hand side. After the electron has passed the first interface what term can we use to describe it? [20%]

(b) Now consider the more realistic case that the device is at room temperature, and there are defects present. Sketch the kinetic energy versus position again, clearly marking out any relevant lengthscales. [15%]

(c) Now consider the case where there are no defects. Assuming that the only entities the electron can scatter off are phonons, and that the number of phonons depends on temperature, sketch the dependence of the electron's mean-free path on temperature. [15%]

(d) Assuming that we want to make a fast transistor, what conclusions can we draw about the width and depth of the well? What would happen if the well was ten times deeper? [20%]

(e) What property of the 2-dimensional electron gas (2DEG) makes it so desirable from a device standpoint? Describe how you would create a 2DEG in principle and in practice. How would you actually incorporate a 2DEG in a device to improve its operation? [30%]

**END OF PAPER**