UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Thursday 22 April 2004 9 to 10.30

Module 4C4

DESIGN METHODS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
i) Special data book for 4C4 (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 Methotrexate is an antimetabolite used in the treatment of certain neoplastic diseases, severe psoriasis and adult rheumatoid arthritis. Methotrexate can cause a decrease in the number of blood cells in the bone marrow, severe skin reactions, infections such as pneumonia, bone and soft tissue damage, and severe damage to the liver, kidneys, lungs, and gastrointestinal tract (some of which can be fatal). As a result it is one of only five prescription-only medicines that can be prescribed to be taken weekly. Inadvertent daily use has been known to result in patient death.

Methotrexate may be prescribed to adults and is available in tablet form in two doses, 2.5 mg and 10 mg, where prescriptions range from 10 mg to 40 mg weekly. The tablets are delivered to the pharmacist in bottles of 100, to be dispensed in child-proof containers to the patient in quantities sufficient for one month's treatment.

A number of patients have died in recent years as a direct result of methotrexate misuse. The UK Department of Health wish to commission new packaging for methotrexate in an effort to reduce the incidence of patient harm.

(a) Use a solution-neutral problem statement to describe the overall function of the new packaging. [10%]

(b) List the key requirements for the new packaging. [20%]

(c) Define a process function structure for the safe use of methotrexate. [20%]

(d) Identify solution principles for the critical functions identified in (c), and describe a packaging concept that will ensure the safe administration of methotrexate. [50%]

2 Describe the role and importance in design of:

(a) *design process models* with reference to the key stages found in most process models; [50%]

(b) *evaluation* with reference to verification, validation and review activities; [30%]

(c) *risk management*. [20%]

3 A small mechanical assembly is shown in its assembled and disassembled states in Fig 1. The bush and washer are to be assembled onto the pin and retained in place by the sprung clip. Tolerances on all dimensions are ± 0.1 mm and all parts are supplied by different manufacturers. The sprung clip has a nominal thickness of 1.0 mm in its compressed state (i.e. it has been manufactured from 1.0 mm sheet) and has a nominal thickness of 2.0 ± 0.5 mm in its manufactured state (i.e. after it has been pressed to form a spring). You should assume that 2σ represents the variation from minimum to maximum dimension on all components, where σ is the standard deviation..

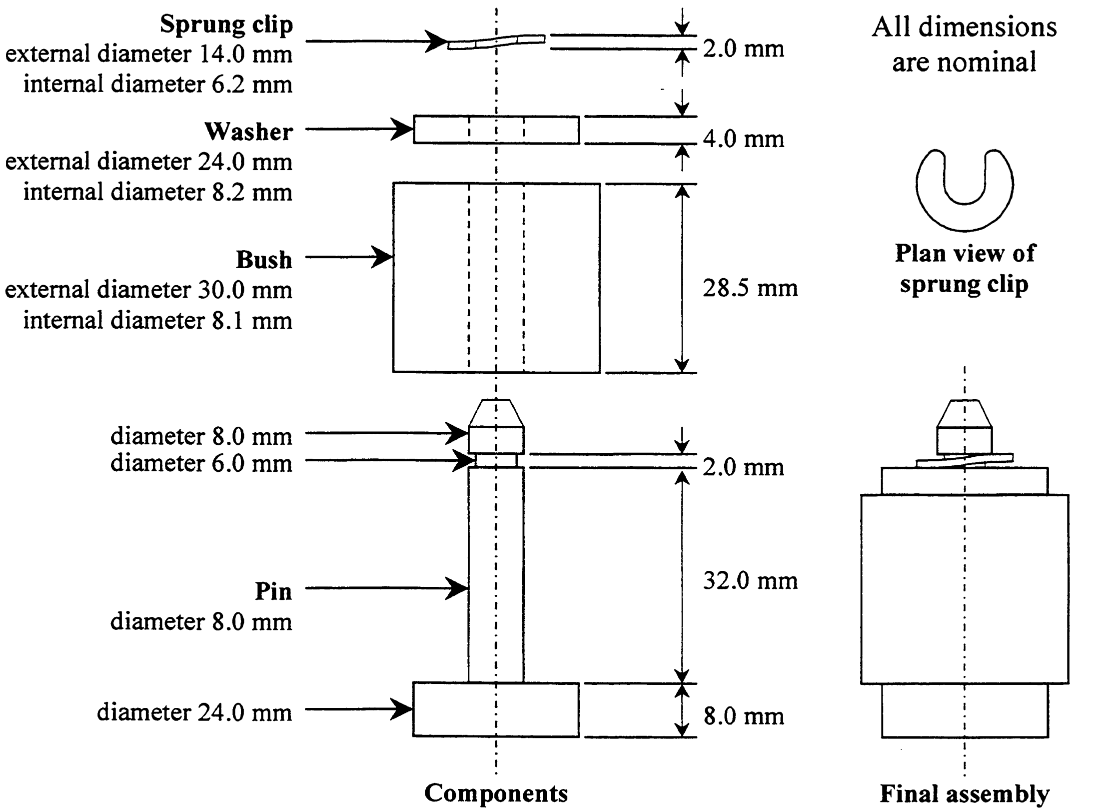


Fig 1.

(a) Draw a fault tree to identify all possible assembly failure modes. [20%]

(b) Calculate the likely percentage of assemblies that will be able to be assembled correctly. [40%]

(c) Suggest how changes to the design or improvements to the manufacture of the components could improve this percentage to 99%. Justify your answer. [40%]

(TURN OVER

4 The inside structure of the bonnet of a car can be modelled as a network of beams that creates a structural frame. You have been asked to minimise the material weight of this structural frame subject to a constraint on the maximum vertical displacement for each beam under the applied loads.

(a) There exist n beam elements in the bonnet structural model, each of which has a defined length and a rectangular cross section with variable width and height, as shown in Fig. 2. Describe a formal optimisation model that maps the engineering design task to a mathematical model. Define all constants, parameters, variables, an objective function, and constraints. List your assumptions. [30%]

(b) One beam element in the car bonnet can be modelled as a simply supported beam of defined length l with a rectangular cross section of variable width b and height h; see Fig. 2. Show that for this simplified model the objective function to minimise mass is not convex. Using an *unconstrained* model, carry out one step of Newton's method, starting from the point $b = h = 0.01$, to find a stationary point. Why does Newton's method converge to a stationary point in one step? Is this point the global optimum? [35%]

(c) For the single beam model under a uniformly distributed load, $W = wl$, the maximum displacement, δ_{max}, occurs in the centre of the beam, and is given by:

$$\delta_{max} = \frac{5Wl^3}{384EI}$$

For a steel beam with $l = 1$ m, $w = 3$ kNm^{-1}, $b = 0.01$ m and $\delta_{max} = 1$ mm, given that the constraint on δ_{max} is active at the optimum, find the corresponding value for h. Thus find the minimum mass. For this point to be an optimum what other constraint must be active and why? [35%]

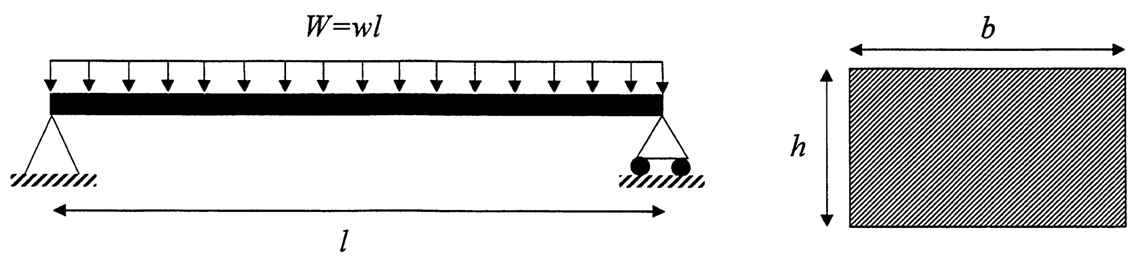


Fig 2.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

S/40

1995

(Revised 2001)

(Revised 2002)

(Revised 2003)

MODULE 4C4

DATA BOOK

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k + \delta) = f(x_k) + \delta f'(x_k) + \frac{1}{2}\delta^2 f''(x_k) + \ldots \quad \text{where} \quad x_{k+1} = x_k + \delta$$

For a function of several variables:

$$f(\underline{x}_k + \underline{\delta x}) = f(\underline{x}_k) + \{\nabla f(\underline{x}_k)\}^t \underline{\delta x} + \frac{1}{2}\underline{\delta x}^t \mathbf{H}(\underline{x}_k)\underline{\delta x} + \ldots \quad \text{where} \quad \underline{x}_{k+1} = \underline{x}_k + \underline{\delta x}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \frac{\partial f(\underline{x}_k)}{\partial x_1} & \frac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \frac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \frac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note: 1. $\nabla f(\underline{x}_k)$ is defined as a column vector.

2. The Hessian is symmetric.

3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio = $\dfrac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f(x_k)\}/\{f'(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right)f(x_1) + \left(x_3^2 - x_1^2\right)f(x_2) + \left(x_1^2 - x_2^2\right)f(x_3)}{\left(x_2 - x_3\right)f(x_1) + \left(x_3 - x_1\right)f(x_2) + \left(x_1 - x_2\right)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t\, \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t \mathbf{H} \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\alpha_k = \dfrac{-\underline{s}_k^{\,t} \nabla f(\underline{x}_k)}{\underline{s}_k^{\,t} \mathrm{H} \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{\nabla f(\underline{x}_{k+1})^t \mathbf{H} \underline{s}_k}{\underline{s}_k^{\,t} \mathbf{H} \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

To find the minimum of the function f($\underline{x}$) where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that f($\underline{x}$) is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu, \underline{x}) = f(\underline{x}) + \frac{1}{\mu} \sum_{i=1}^{p} \left(\max[0, g_{i(\underline{x})}]\right)^2$$

where f($\underline{x}$) is subject to the constraints $g_1(\underline{x}) \le 0, \ldots, g_p(\underline{x}) \le 0$

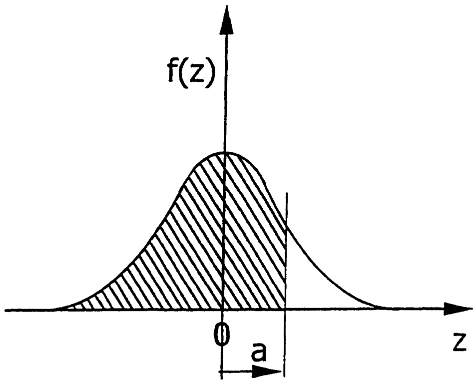
A typical Barrier function for the same problem is:

$$q(\mu, \underline{x}) = f(\underline{x}) - \mu \sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

UNIVERSITY OF CAMBRIDGE

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Combining distributed variables

For the function

$$y = f(x_1, x_2, \ldots x_n)$$

where x_1, x_2 etc. are independent and defined by their respective distributions:

	y	μ_y	σ_y^2
1	$x + a$	$\mu_x + a$	σ_x^2
2	ax	$a\mu_x$	$a^2\sigma_x^2$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^2\sigma_1^2 + a_2^2\sigma_2^2$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$
5	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^4}\left(\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2\right)$

Where: μ = mean; σ = standard deviation; a = constant.