UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Tuesday 20 April 2004 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

Candidates may bring their notebooks to the examination.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 An accelerometer is fixed to the wing of an aircraft. An impulse is applied to two grid points on the wing and the transfer functions derived from the impulse-response data so obtained are shown in Fig. 1 as magnitude plots and corresponding Nyquist plots. The data logger sampled 4096 data points per channel at a sampling rate of 200 Hz. Five modes are identified at frequencies 10, 20, 45, 70 and 80 Hz.

(a) For each mode n:

(i) identify its modal circle for each grid point; [10%]

(ii) estimate the quality factor Q_n; [40%]

(iii) estimate the modal amplitude factor $u_n(x)u_n(y)$ for each grid point, where x denotes the position of the driving point and y denotes that of the observing point. [30%]

A larger copy of the two Nyquist plots has been supplied. It should be annotated as part of your answer and handed in with your solution.

(b) Use sketches to illustrate how the position and distribution of points plotted on modal circles are influenced by:

(i) adjacent modes; [10%]

(ii) sampling rate and number of sampled data points. [10%]

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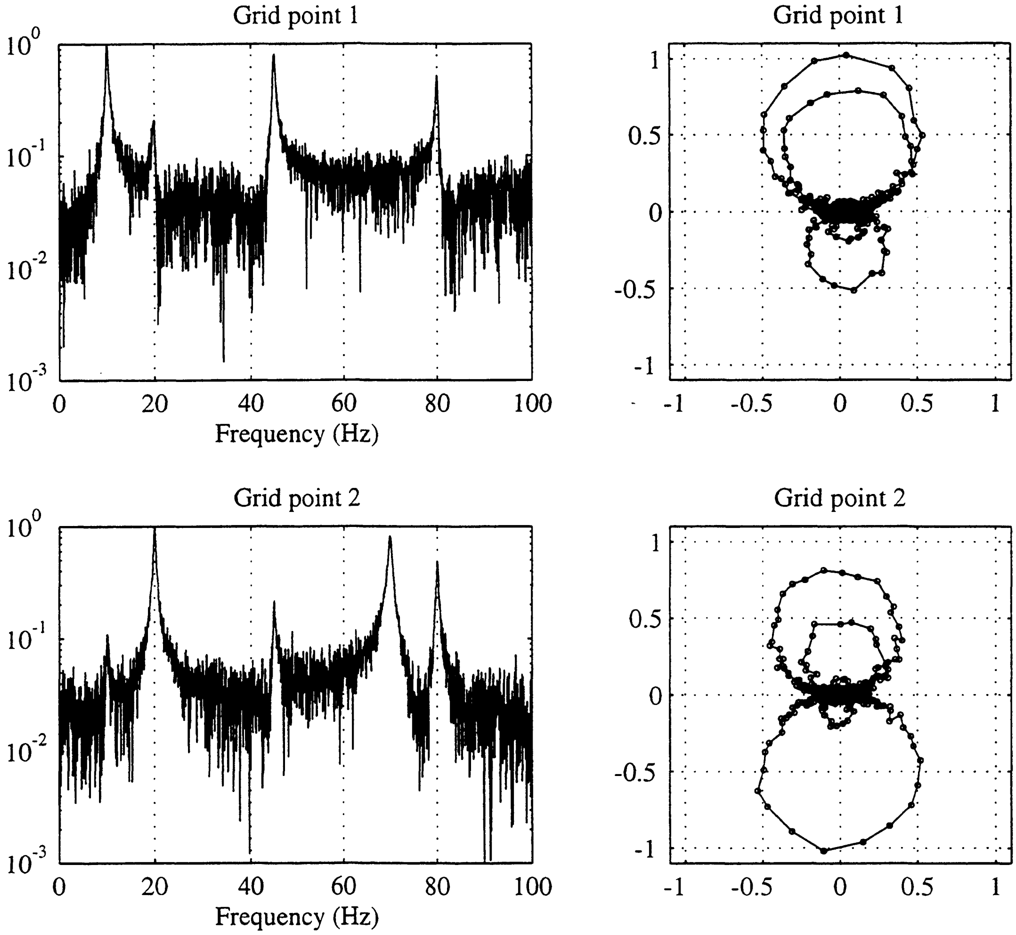

Fig. 1: The magnitude plots show frequency response in units $m\,s^{-2}N^{-1}$.
In the Nyquist plots dots are printed at each frequency point,
at equal intervals of 0.0488 Hz

2 (a) A system comprising two identical oscillators, each of mass m and stiffness k, is shown in Fig. 2(a).

(i) Write down the natural frequencies of the system. Also write down the mode shapes in the vector form (x_1, x_2). [10%]

(ii) The oscillators are now coupled by a spring of stiffness k_c, as shown in Fig. 2(b). Recalculate the system natural frequencies and mode shapes. Discuss the effect of the coupling spring on the natural frequencies in the light of the interlacing theorem. [40%]

(b) A brass tube of uniform cross-section is bent and joined to form a complete circle of radius R, as shown in plan in Fig. 2(c). The air pressure $p(\theta)$ within the tube satisfies the equation

$$\frac{c^2}{R^2}\frac{\partial^2 p}{\partial \theta^2} - \frac{\partial^2 p}{\partial t^2} = 0$$

where θ is the polar angle around the circle, as shown in Fig. 2(c), and c is the speed of sound in air.

(i) Show that the acoustic natural frequencies of the system are $\omega_n = nc/R$ for $n = 0, 1, 2$, etc. Find the associated mode shapes. [30%]

(ii) A small hole is now drilled in the tube. How will this hole influence the acoustic modes in the tube? Discuss the effect on the first two acoustic modes with non-zero frequencies. [20%]

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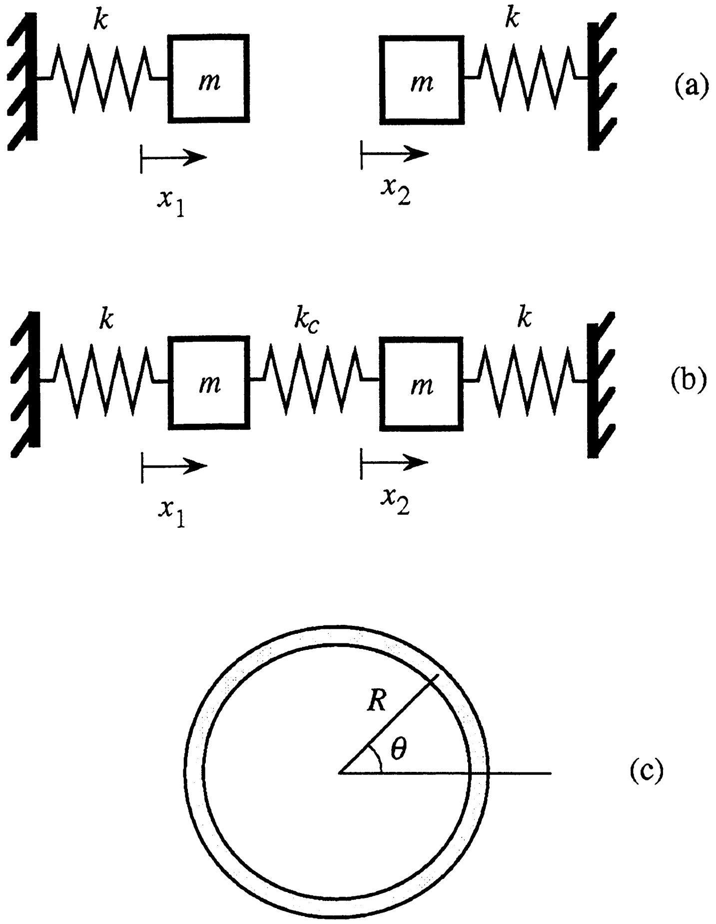

Fig. 2

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3 A cylindrical tank of radius R and height b has the configuration shown in side elevation in Fig. 3. The top plate of the tank is perforated by a circular hole of radius a (labelled A), and an interior circular plate, located at height λb above the base of the tank, is also perforated by a circular hole of radius a (labelled B). The structural components of the tank can be considered to be rigid, and the speed of sound in air is c.

(a) If the system is modelled using the standard approximations and assumptions employed for a Helmholtz resonator, show that the system has two degrees of freedom, and obtain the coupled equations of motion. [30%]

(b) Derive the natural frequencies of the system. Compare your results to: (i) the case where the upper hole (A) is blocked, and (ii) the case where the lower hole (B) is blocked. Discuss these results in terms of the interlacing theorem. [40%]

(c) Explain why your results might break down for $\lambda \to 1$, and suggest a more correct result for $\lambda = 1$. [30%]

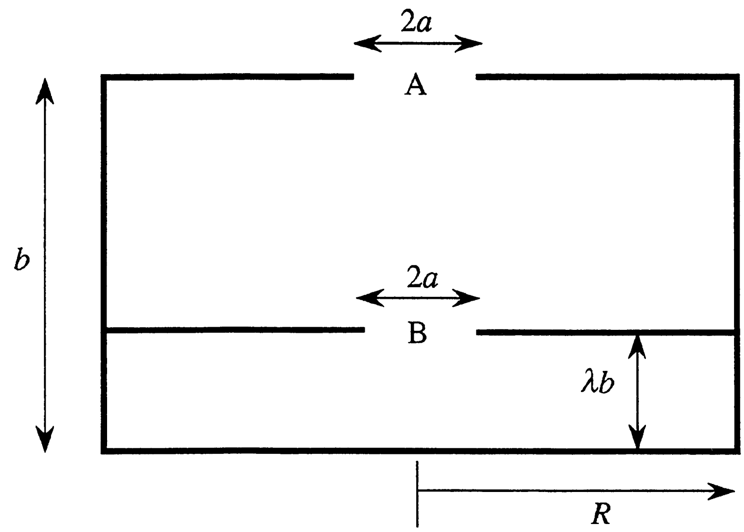

Fig. 3

4 A piece of machinery has a troublesome resonance which is to be treated by the addition of a "tuned vibration absorber". The system can be modelled as shown in Fig. 4: the main resonance as a mass m on a stiffness k, and the absorber as a mass λm and stiffness λk where λ is a constant. The displacement from equilibrium of the main mass is x, and that of the absorber mass relative to the main mass is y.

(a) Write down expressions for the kinetic and potential energy of the system, and hence deduce the mass matrix and stiffness matrix. [10%]

(b) In the undamped system, it may be *assumed* that the two vibration modes have the form

$$\frac{y}{x} \approx \pm \frac{1}{\sqrt{\lambda}},$$

when $\lambda << 1$. Show that the corresponding natural frequencies ω satisfy

$$\omega^2 \approx \frac{k}{m}\left(1 \mp \sqrt{\lambda}\right).$$ [20%]

(c) The spring λk is now modified to include damping, which can be described by a complex value $\lambda k(1+i\eta)$. Explain briefly how Rayleigh's principle can be used to obtain approximate values of the modal damping factors, and apply the method to this problem. [35%]

(d) What characteristics of a practical system make it suitable for adding damping with a tuned absorber? Suggest an example system (different from the examples given in the lecture notes). What considerations would influence the detailed design of a tuned absorber in your chosen example? [35%]

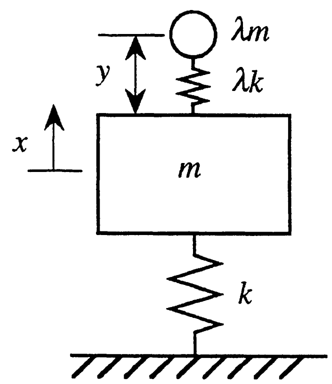

Fig. 4

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