UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 26 April 2004 9 to 10.30

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 A single-degree-of freedom system of mass $M = 2$ kg , stiffness $K = 18$ N m^{-1} and damping rate $C = 0.5$ N s m^{-1} is subjected to a stationary random force for a 3 hr period. The force can be approximated to white noise with (double sided) spectral density $S_0 = 1.0$ N^2 s rad^{-1} . The system will fail if the displacement exceeds 3.2 m, and the probability of such a failure is required to be less than 0.01. Furthermore, the S-N law for fatigue damage of the spring is given by

$$N = 5 \times 10^4 S^{-1}$$

where N is the number of cycles to failure under cyclic loading that produces a spring force of amplitude S.

(a) Calculate the mean squared displacement of the system and the mean squared force in the spring. Also calculate the mean rate at which the spring force crosses zero, and the average height of the peaks in the spring force time history. [30%]

(b) Assess whether the system has an acceptable level of reliability. You should apply a factor of safety of 5 to any calculation of fatigue damage. [40%]

(c) If the loading is applied for a time T, rather than 3 hrs, calculate the maximum value of T for which the system meets the safety requirements. [30%]

2 The joint probability density function of the displacement and velocity of a stationary stochastic process $x(t)$ is written as $p(x,\dot{x})$.

(a) Show from first principles that the average rate at which the process crosses the level $x = b$ with a velocity greater than u is given by

$$\nu_b = \int_u^\infty \dot{x} p(b,\dot{x})\,\mathrm{d}\dot{x}$$ [30%]

(b) If $x(t)$ is Gaussian with zero mean, derive an expression for ν_b in terms of the r.m.s. displacement σ_x and the r.m.s. velocity $\sigma_{\dot{x}}$. [35%]

(c) The deck of an offshore structure is located 14 m above the mean water level. Damage will be caused to the deck if waves impact with a vertical velocity greater than 2 m s^{-1}. In a severe storm of duration 2 hrs, the surface elevation of the sea has r.m.s. vertical displacement 3.75 m and r.m.s. vertical velocity 1.5 m s^{-1}. Calculate the probability of damage to the deck, stating any assumptions you make in your calculation. [35%]

3 A nonlinear undamped vibratory system has the symmetrical force-displacement characteristic shown in Fig. 1. The slope of the force-displacement characteristic for displacement amplitudes less than a is s_1, while the slope for displacement amplitudes greater than a is s_2.

(a) Determine the Describing Function for a sinusoidal motion of amplitude β. Confirm that the Describing Function degenerates to the case of a simple linear spring for $s_1 = s_2$. [50%]

(b) If the system is driven by a force $A\sin\omega t$, determine an approximate relation between the response amplitude β and the force amplitude A. [40%]

(c) Describe how the above force-displacement characteristic, or extensions thereof, might be employed to describe the spring hardening effect that appears in the Duffing equation. [10%]

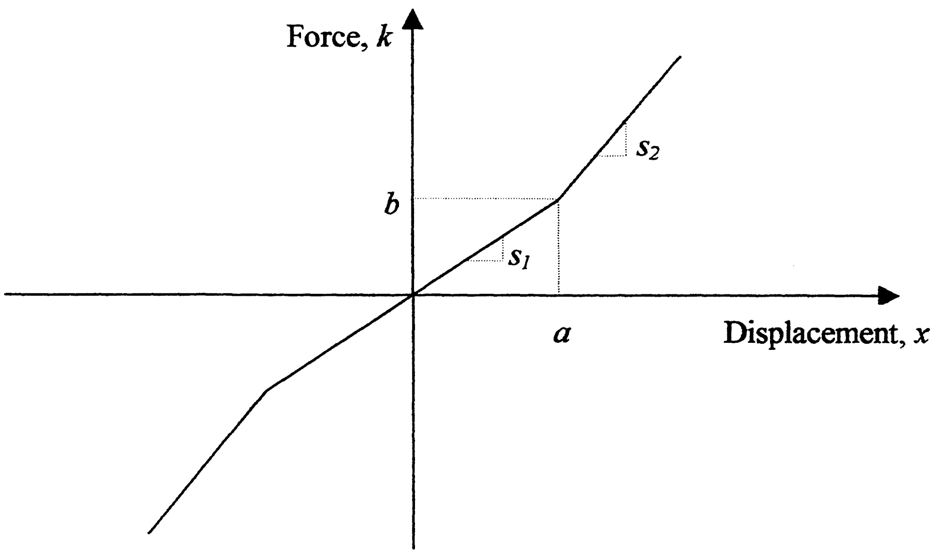

Fig. 1

4 An undamped vibratory system is governed by the following differential equation

$$\ddot{x} + p^2 x + \varepsilon x^2 = a\cos\omega t .$$

(a) Assume a solution in the form

$$x = c_1 \cos\frac{\omega t}{2} + c_2 \cos\omega t .$$

Apply the method of harmonic balance truncated to the frequencies ω and $\omega/2$, and hence find the algebraic equations that govern c_1 and c_2. Assuming that ε is small and that c_1 is non-zero, obtain an approximate solution for c_1 to first order in ε. [50%]

(b) Now assume a solution in the form

$$x = c_3 \cos\omega t + c_4 \cos 2\omega t .$$

Apply the method of harmonic balance truncated to the frequencies ω and 2ω and find the algebraic equations that govern c_3 and c_4. [40%]

(c) Discuss whether it is possible to apply the principle of superposition to produce a solution in the form

$$x = c_1 \cos\frac{\omega t}{2} + c_2 \cos\omega t + \lambda\left(c_3 \cos\omega t + c_4 \cos 2\omega t\right)$$

where c_1, c_2, c_3 and c_4 take values obtained in parts (a) and (b) and λ is an arbitrary real-valued constant. [10%]

END OF PAPER