UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Friday 23 April 2004 2.30 to 4

Module 4C8

APPLICATIONS OF DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (3 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(TURN OVER

1 A wheel rolls at yaw angle δ to the direction of motion of its axle. It is assumed that the contact conditions can be modelled using the 'brush' model with lateral stiffness K_y and a rectangular contact area of length $2L$ and width $2h$. The coefficient of friction is μ, the total normal force is Z and the contact pressure may be assumed constant over the contact area.

(a) Determine the realigning moment N, assuming that δ is small and that no micro-slip occurs. Explain what is meant by the 'pneumatic trail' and determine its value for small δ. [30%]

(b) For larger δ when micro-slip occurs, the realigning moment can be written in non-dimensional form as:

$$\frac{N}{\mu Z L} = \frac{1}{4\lambda}\left(\frac{1}{3\lambda} - 1\right), \quad \text{where } \lambda = \frac{4L^2 h K_y}{\mu Z}\delta.$$

Explain how this formula is derived and show that it is consistent with the value of N for small δ from Part (a). [20%]

(c) Sketch the variation of N with δ, showing important features. Find the maximum value of N and the corresponding value of δ. Explain why this maximum occurs. [30%]

(d) Discuss briefly the relative importance of the realigning moment in simple models of vehicle dynamics. [20%]

2 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The car moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients of C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = C_f\delta$$

$$I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = aC_f\delta$$

State your assumptions. [40%]

(b) The steer angle is set to a constant angle δ and the vehicle follows a steady circular path of radius R. Use the equations of motion to derive an expression for the steady state curvature response $(1/R)$. Sketch a handling diagram, showing responses for under-steering, neutral steering and over-steering vehicles. State the parameters that determine the type of response obtained. (Recall that a handling diagram is a graph of u^2/R vs $\delta - L/R$, where $L = a+b$.) [40%]

(c) Show the response of a typical real vehicle on your handling diagram. Explain the main reasons for the differences between the handling of a real vehicle and that of the bicycle model. [20%]

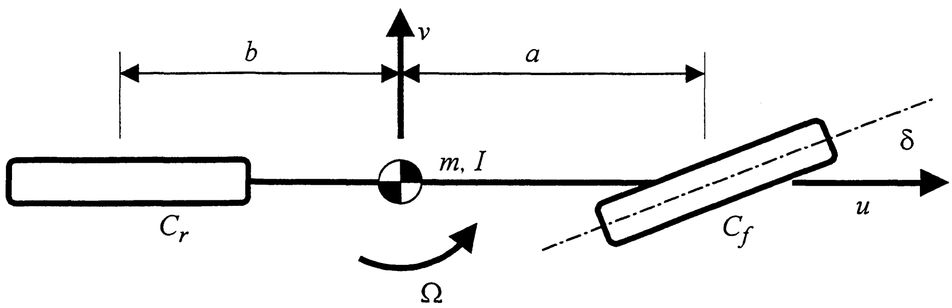

Fig. 1

3 (a) The vertical profiles of the left and right hand wheel tracks z_L and z_R on a randomly rough road surface can each be considered to be the combination of an average vertical displacement z_V and a roll displacement z_ϕ, as shown in Fig. 2.

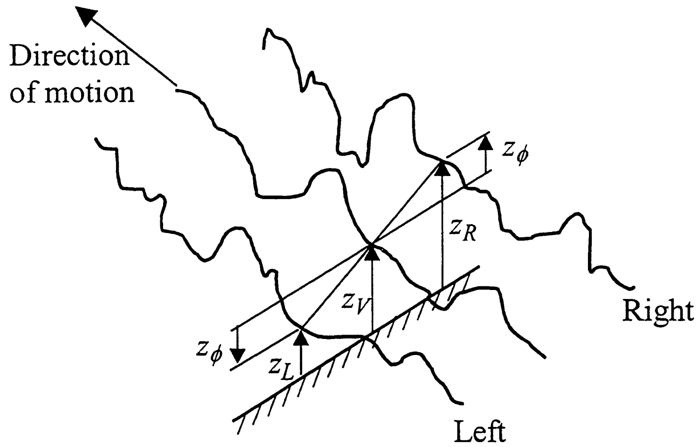

Fig. 2

The mean square spectral densities of z_V and z_ϕ are related by $S_{z_\phi}(n) = |G(n)|^2 S_{z_V}(n)$, where

$$|G(n)|^2 = \frac{n^2}{n_c^2 + n^2}$$

and n_c is a 'cut-off' wavenumber. Derive an expression for $S_{z_\phi}(n)/S_z(n)$, where $S_z(n)$ is the mean square spectral density of z_L or z_R. [15%]

(b) If the displacement spectral density of a road profile is $S_z(n) = \kappa n^{-2}$ where κ is a roughness coefficient and n is wavenumber (cycles m^{-1}), show that the displacement excitation of a vehicle moving over the road can be given by $S_z(\omega) = 2\pi U \kappa \omega^{-2}$, where U is vehicle speed and ω is frequency (rad s^{-1}). [15%]

(c) Derive an expression for the spectral density of roll displacement $S_{z_\phi}(\omega)$ as a function of frequency ω, vehicle speed U, roughness κ and cut off wavenumber n_c. Using logarithmic axes, sketch a graph of $S_{z_\phi}(\omega)$ against speed U to show how the spectral density at two frequencies ω_1 and ω_2 (with $\omega_1 < \omega_2$) varies with speed. [40%]

(d) With reference to your sketch in (c), discuss the influence of vehicle speed on the roll excitation of sprung mass modes (natural frequency ω_1) and unsprung mass modes (natural frequency ω_2) in a vehicle. Typical values are n_c = 0.1 cycles m^{-1}, $\omega_1 = 5$ rad s^{-1} and $\omega_2 = 50$ rad s^{-1}. [30%]

4 (a) A quarter-car model is shown in Fig. 3. The sprung mass is m_s and the unsprung mass is m_u. Their corresponding displacements are z_s and z_u. The vertical tyre stiffness is k_t, and the suspension stiffness and damping are k and c. The input displacement is z_r. Show that

$$m_s \ddot{z}_s(t) + m_u \ddot{z}_u(t) = k_t(z_r(t) - z_u(t))$$ [15%]

(b) Transfer functions relating suspension working space and tyre force to input road velocity are defined as

$$H_{WS}(j\omega) = \frac{z_s(j\omega) - z_u(j\omega)}{\dot{z}_r(j\omega)} \quad \text{and} \quad H_{TF}(j\omega) = \frac{k_t(z_r(j\omega) - z_u(j\omega))}{\dot{z}_r(j\omega)}$$

Show that

$$m_s \omega^2 H_{WS}(j\omega) + \left(1 - \frac{(m_s + m_u)\omega^2}{k_t}\right) H_{TF}(j\omega) = j\omega(m_s + m_u)$$

[50%]

(c) Discuss the implications for suspension design arising from the equation given in (b). [35%]

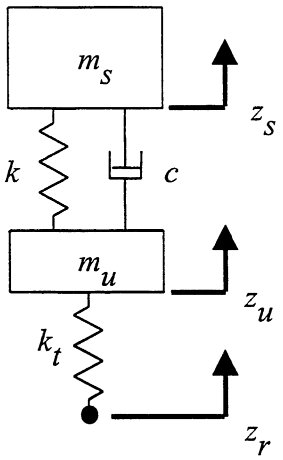

Fig. 3

END OF PAPER

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Engineering Tripos Part IIB

Data sheet for Module 4C8: Applications of Dynamics

DATA ON VEHICLE VIBRATION

Random Vibration

$$E\left[x(t)^2\right] = \frac{1}{T}\int_{t=0}^{t=T} x^2(t)\,dt = \int_{\omega=-\infty}^{\omega=\infty} S_x(\omega)\,d\omega \qquad \left(\text{or } \int_{\omega=0}^{\omega=\infty} S_x(\omega)\,d\omega \quad \text{if } S_x(\omega) \text{ is single sided}\right)$$

$$S_{\dot{x}}(\omega) = \omega^2 S_x(\omega)$$

Single Input – Single Output

$$S_y(\omega) = \left|H_{yx}(\omega)\right|^2 S_x(\omega)$$

$$y(\omega) = H_{yx}(\omega)x(\omega)$$

Two Input – Two Output

$$\begin{Bmatrix} y_1(\omega) \\ y_2(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \begin{Bmatrix} x_1(\omega) \\ x_2(\omega) \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^y(\omega) & S_{12}^y(\omega) \\ S_{21}^y(\omega) & S_{22}^y(\omega) \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^* \begin{bmatrix} S_{11}^x(\omega) & S_{12}^x(\omega) \\ S_{21}^x(\omega) & S_{22}^x(\omega) \end{bmatrix} \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^T$$

* means complex conjugate, T means transpose

If x_1 and x_2 are uncorrelated:

$$S_{(x_1+x_2)}(\omega) = S_{x_1}(\omega) + S_{x_2}(\omega)$$

$$S_{12}^x(\omega) = S_{21}^x(\omega) = 0$$

$$E\left[(x_1(t) + x_2(t))^2\right] = E\left[x_1(t)^2\right] + E\left[x_2(t)^2\right]$$

DATA ON VEHICLE DYNAMICS

1. Creep Forces In Rolling Contact

1.1 Surface tractors

Longitudinal force $X = \iint_A \sigma_x \, dA$

Lateral force $Y = \iint_A \sigma_y \, dA$

Realigning Moment $N = \iint_A \left(x\,\sigma_y - y\,\sigma_x\right) dA$

where

σ_x, σ_y = longitudinal, lateral surface tractions

x, y = coordinates along, across contact patch

A = area of contact patch

1.2 Brush model

$\sigma_x = K_x q_x$, $\sigma_y = K_y q_y$ for $\sqrt{\sigma_x^2 + \sigma_y^2} \le \mu p$

where

q_x, q_y = longitudinal, lateral displacements of 'bristles' relative to wheel rim

K_x, K_y = longitudinal, lateral stiffness per unit area

μ = coefficient of friction

p = local contact pressure

1.3 Linear creep equations

$X = -C_{11}\xi$

$Y = -C_{22}\alpha - C_{23}\psi$

$N = C_{32}\alpha - C_{33}\psi$

where X, Y, N, are defined as in 1.1 above.

C_{ij} = coefficients of linear creep

ξ = longitudinal creep ratio = longitudinal creep speed/forward speed

α = lateral creep ratio = (lateral speed /forward speed) - steer angle

ψ = spin creep ratio = spin angular velocity/forward speed

2. Plane Motion in a Moving Coordinate Frame

$$\ddot{\mathbf{R}}_{O_1} = (\dot{u} - v\Omega)\mathbf{i} + (\dot{v} + u\Omega)\mathbf{j}$$

(i, j, k) axis system fixed to body at point O_1

where

u = speed of point O_1 in **i** direction

v = speed of point O_1 in **j** direction

$\Omega\mathbf{k}$ = absolute angular velocity of body

3. Routh-Hurwitz stability criteria

$$\left(a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if all $a_i > 0$

$$\left(a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 > a_0 a_3$

$$\left(a_4\frac{d^4}{dt^4} + a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 a_3 > a_0 a_3^2 + a_4 a_1^2$