UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 19 April 2004 2:30 to 4

Module 4C15

MEMS DESIGN

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1. **COMB DRIVE RESONATOR:** A top view of a comb drive resonator is shown in Fig. 1. The device is implemented in a polysilicon surface micromachining process and the thickness of the structural layer is 6μm. Each beam comprising the suspension for this structure has a length of 150μm and a width of 2μm. The number and layout of electrode fingers in the comb drive are as shown in Fig. 1. The proof mass has an area of 35μm x 60μm and is suspended 3μm above the substrate. The overlap length between the movable and fixed electrode is 10μm and the electrode gap is 1μm. Each beam comprising the suspension for this structure has a length of 150μm and a width of 1.5μm.

(a) Natural Frequency and Damping estimates.

(i) Calculate the spring constant of the suspension and hence the primary drive mode natural frequency for the resonator. Neglect the etch holes and the mass of the comb fingers in your calculation of mass. The spring constant for a single beam comprising the suspension is given by:

$$k = EH\left(\frac{W}{L}\right)^3$$

where E is Young's Modulus, H is the out-of-plane thickness, and W and L are the width and length of the beam respectively.

(ii) Estimate the damping for the resonant structure along the driven mode assuming operation in air. Neglect the effect of the etch holes in your calculations but make a qualitative statement on how these might affect your estimate. The damping constant assuming a Couette flow model for air flow between moving parallel plates is given by:

$$b = \eta\frac{A}{h}$$

where η is the viscosity of air (value: 1.8×10^{-5} kg/m.s at room temperature and atmospheric pressure), A is the overlap area of the plates and h is the gap between the plates. [40%]

(b) Construct a lumped-element equivalent electrical circuit model for this system and hence derive the transfer function relating displacement-to-force and velocity-to-force by the application of Kirchoff's voltage and current laws to the circuit. [30%]

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(c) Static and Dynamic deflection.

(i) Estimate the static deflection for a voltage difference of 1V applied between the moving proof mass and the comb drive.

(ii) Estimate the deflection of the proof mass for the combined application of a sinusoidal AC voltage of 50mV and a DC voltage of 5V and assume forcing at drive mode resonance. For what values of frequency of the applied AC voltage can the structure be driven into resonance? [30%]

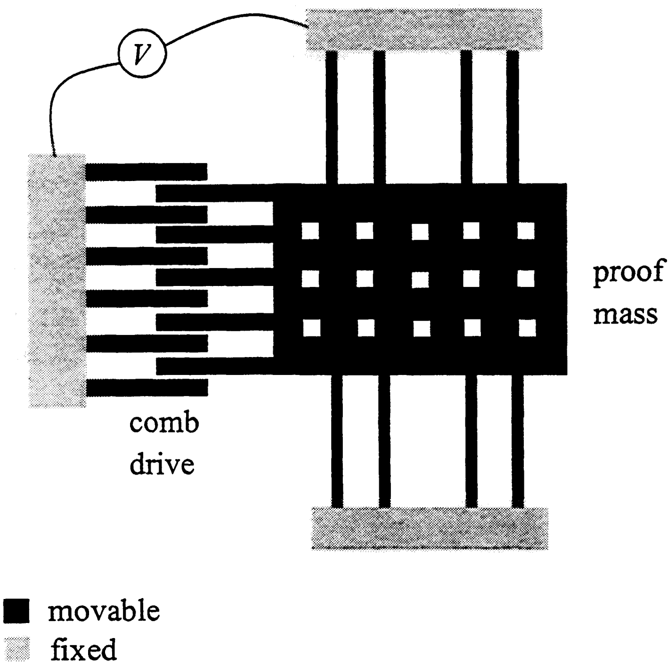

Fig. 1: Top view of a comb drive resonator (not drawn to scale). *V* is the applied potential between the fixed electrodes and the proof mass.

2. **PIEZORESISTIVE PRESSURE SENSOR:** A membrane pressure sensor is fabricated in a silicon bulk micromachining process. A top view and cross-section is shown in Fig. 2. The dimensions of the square diaphragm are 1000μm x 1000μm with a thickness of 10μm. Piezoresistive detection is employed to monitor the deflection of the membrane in response to an applied external pressure. Two rectangular N-type polysilicon piezoresistors are located on the top of the plate. The membrane is modeled as a plate, the deflection (*w(x,y)*) of which in response to an external pressure load (*P*) is given by:

$$w(x,y) = \frac{c_1}{4}\left[1+\cos\left(\frac{2\pi x}{L}\right)\right]\left[1+\cos\left(\frac{2\pi y}{L}\right)\right]$$

$$\text{where } c_1 = \frac{6P(1-v^2)L^4}{\pi^4 EH^3}$$

and P is the applied pressure load on the membrane, L is the diaphragm size, v is Poisson's ratio and E is Young's Modulus for the membrane material and H is the thickness of the membrane.

(a) Show that the polysilicon resistors have to be located at the edge of the diaphragm to maximize the scale factor. Calculate the *x-y* coordinates for the centroids of the two resistors so as to place them at the location of maximum induced strain in the membrane. Would you locate the piezoresistors longitudinally or transversely assuming that the piezoresistive coefficients are π_{xl}= -31.2 x 10^{-11}Pa^{-1} and π_{yl}= -17.6 x 10^{-11}Pa^{-1}? [40%]

(b) Calculate the value for maximum induced stress at the centroid of the resistor. Hence estimate the fractional change in resistance for each piezoresistor for an externally applied pressure of 1kPa. Assume that the dimensions of the resistor are small compared to the dimensions of the plate. [20%]

(c) Assume that the temperature dependence of the piezoresistor can be modeled by the following linear expression over a range 0-100°C:

$$R = R_o(1+\alpha T)$$

where R_o is the nominal resistance, α is 2x10^{-5} °C^{-1} and T is temperature in degrees Celsius. This change in resistance with temperature results in output drift. Estimate the corresponding uncertainity in the pressure reading due this temperature drift assuming that the device has to operate at temperatures between 0-100°C. [20%]

(d) Compare and contrast the advantages and disadvantages of piezoresistive sensing and capacitive sensing for pressure sensors. [20%]

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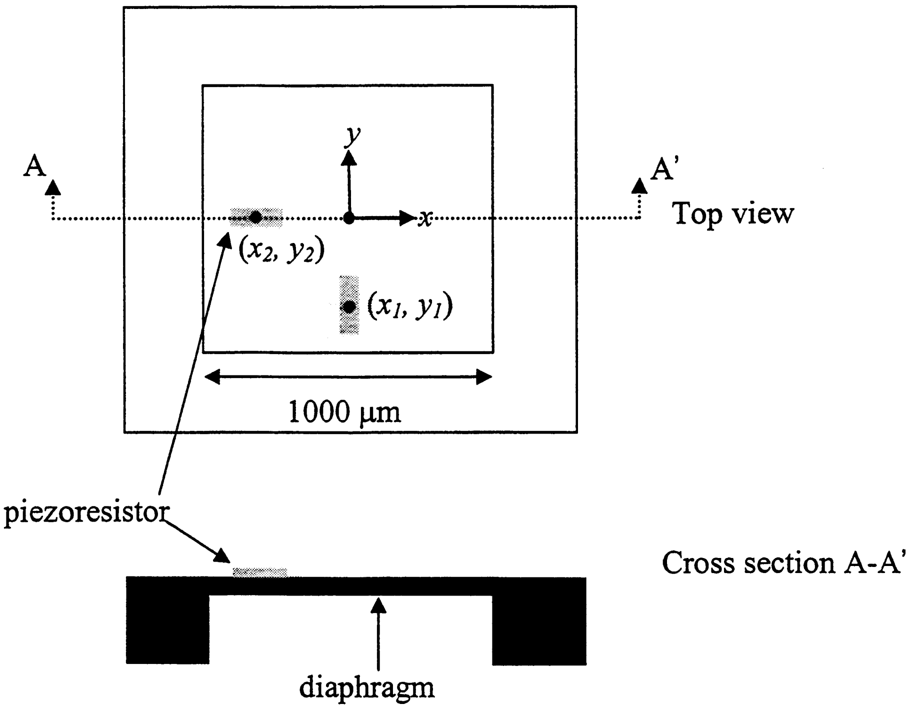

Fig. 2: Top view and cross-section of a micromachined pressure sensor (not drawn to scale).

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3. **CAPACITIVE ACCELEROMETER:** An accelerometer is designed in a silicon micromachining process. A top view (layout) for the accelerometer is shown in Fig. 3. The proof mass is constructed out of silicon 10μm thick. The width of the beams comprising the suspension is 2μm. The proof mass weighs $4\text{x}10^{-9}$kg. Capacitive sensing is employed to detect the motion of the proof mass. The gap between the movable and fixed electrodes is 1μm and the overlap length of the electrodes is 100μm.

(a) Choose a value for the length (L) for the beams comprising the suspension so that the system has a natural frequency of 5 kHz along the sensitive axis. The spring constant for a single beam comprising the suspension is given by:

$$k = EH\left(\frac{W}{L}\right)^3$$

where E is Young's Modulus, H is the out-of-plane thickness, and W and L are the width and length of the beam respectively. [15%]

(b) Estimate the deflection of the accelerometer for a constant input acceleration of 1 g. Hence, estimate the scale-factor defined as the ratio of the change in capacitance to the input acceleration. The overlap length of the electrodes is 100μm. The spacing between fixed electrodes is assumed to be much larger than the nominal gap between the moving and fixed electrodes. [20%]

(c) To measure the change in capacitance, a DC potential difference of 5Volts is applied between the proof mass and the stationary electrodes. The application of this potential results in an electrostatic force and a negative electrostatic spring constant along the sensitive-axis. Calculate this equivalent electrostatic spring constant. The electrostatic spring changes the overall spring constant and hence the scale-factor. Calculate the new scale-factor. [30%]

(d) Damping and Accelerometer Performance.

(i) Calculate the damping constant for the device operating at atmospheric pressure assuming squeeze-film damping between the electrodes dominates and neglecting drag between the substrate and the structure. The squeeze-film damping constant is given by the expression:

$$b = \frac{96\eta L W^3}{\pi^4 h_o^3}$$

(cont.

where η is the viscosity of air (value: $1.8\text{x}10^{-5}$ kg/m.s at room temperature and atmospheric pressure), L is the overlap length, W is the width for the parallel plates and h_o is the nominal gap between the plates.

(ii) Random thermal dissipation from the proof mass results in mechanical Brownian noise that sets a fundamental limit on the minimum detectable signal. The equivalent Brownian noise force ($\overline{F}_n$) for a 1Hz bandwidth is given by:

$$\overline{F}_n^2 = 4k_bTb$$

where k_b is Boltzmann's constant and T is temperature in degrees Kelvin. What is the minimum detectable acceleration (in a 1Hz bandwidth), assuming that this is the dominant noise source in the system? [35%]

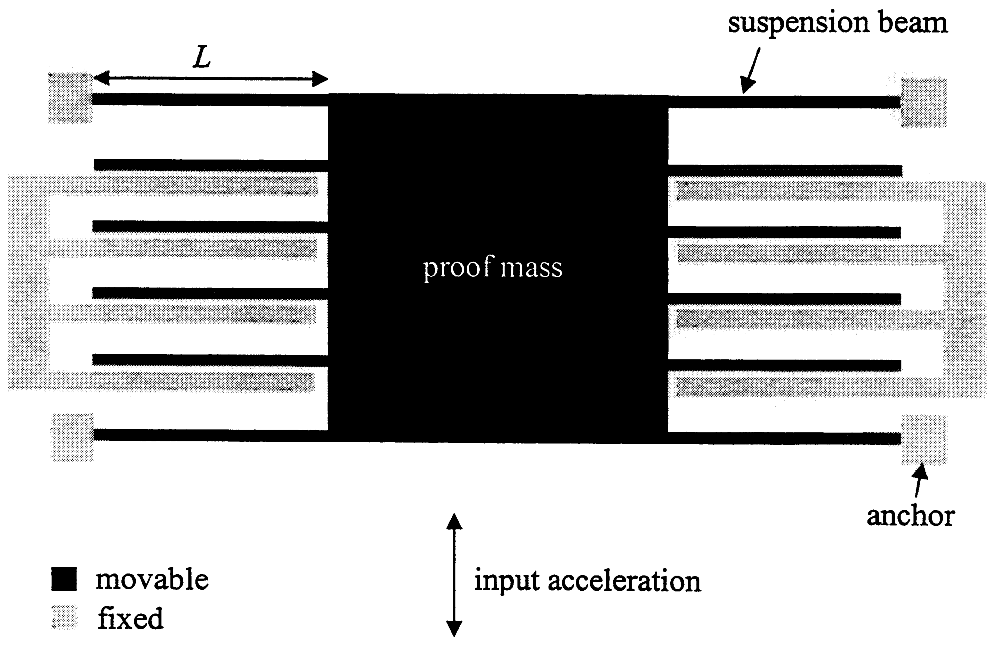

Fig. 3: Top view of an accelerometer (not drawn to scale). The fixed structures and the movable mass are constructed out of silicon.

4. **TORSIONAL ACTUATOR**: A torsional actuator is designed as part of an optical microsystem. The moving element is a plate of area A and length L which rotates about the fixed axis Q as shown in cross-section in Fig. 4. The plate is driven by an electrostatic force when a voltage is applied between an electrode attached to the substrate and the moving plate. The nominal gap between the two electrodes is g, and the plate is initially parallel to the plane of the substrate. The rotation angle θ of the plate relative to its initial position is proportional to the applied torque $\tau = k_\theta \theta$ where k_θ is the torsional spring constant. Assume that the electric field in the gap is uniform throughout and neglect fringing field effects. Also assume that the electrode and the plate have the same dimensions.

(a) Find an expression for the deflection angle θ as a function of voltage difference V between the plates. [30%]

(b) The actuator has an associated pull-in instability. Show that the angle at which pull-in occurs is approximately equal to $0.44g/L$. [40%]

(c) Calculate the value of the pull-in voltage, V_{PI}. In practice, would you expect the value of the measured pull-in voltage for such a device to be higher or lower than the theoretical value? Why? [30%]

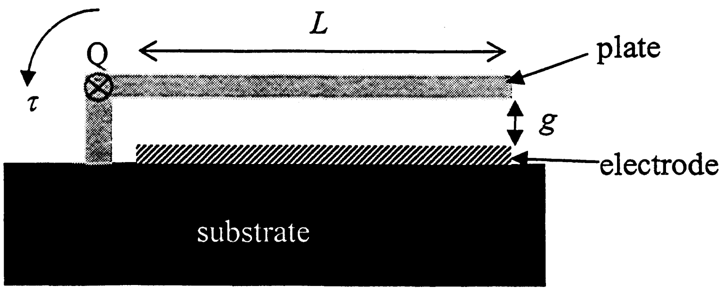

Fig. 4

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