UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Tuesday 4 May 2004 2.30 to 4

Module 4D5

FOUNDATION ENGINEERING

Answer not more than ***three*** *questions*

All questions carry the same number of marks

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

i) 4D5 datasheet (17 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) By considering a sliding block, show that in a purely frictional material obeying the normality principle there is no internal energy dissipation along a slip surface.

[30%]

The same result holds for regions of shear.

(b) The bearing capacity factor N_q of a smooth surface strip footing is defined as the ratio of the vertical bearing pressure at failure, q_f, divided by the surcharge acting alongside the footing, σ'_{vo}. By considering the work dissipated in the upper bound mechanism shown in Fig. 1, show that for a weightless frictional (Coulomb) soil obeying normality,

$$N_q = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi \tan\phi)$$

Hint: since BC is a log spiral, velocities v_1 and v_2 are related by:

$$v_2 = v_1 \exp\left(\frac{\pi \tan\phi}{2}\right)$$

[70%]

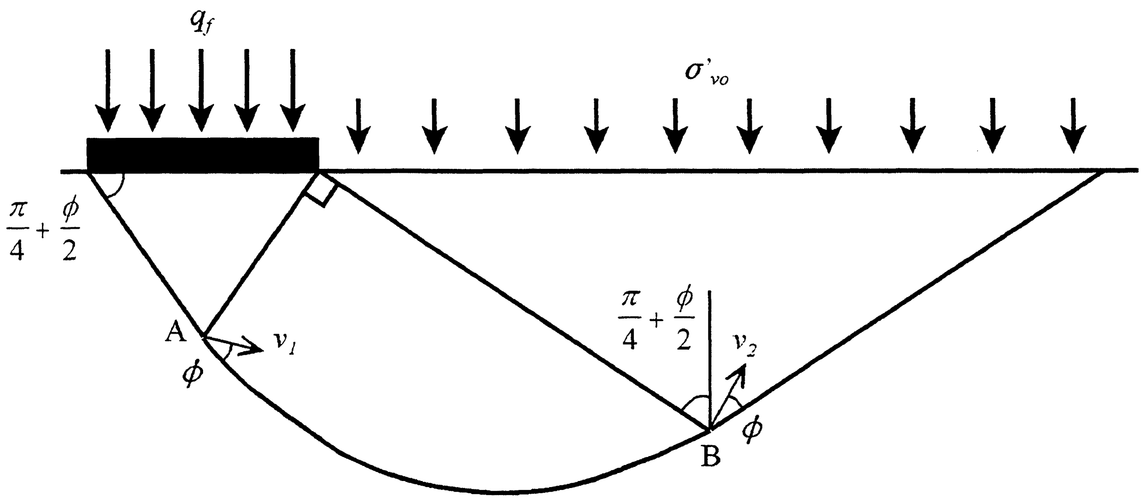

Fig. 1. Strip footing upper bound mechanism.

2 A plan view of a proposed building and the corresponding ground conditions are shown in Fig. 3. The building will be constructed on a raft foundation. The base of the raft will lie 2 m below the original ground level. The weight of the building and the raft foundation will be 30000 kN. The water table is at the top of the clay layer. The compressibility of the sand and bedrock can be ignored. The one dimensional stiffness of the clay was found from drained one-dimensional compression tests to be:

$$E_0 = 500(\sigma'_v)^{0.7} \qquad \text{(both in units of kPa)}$$

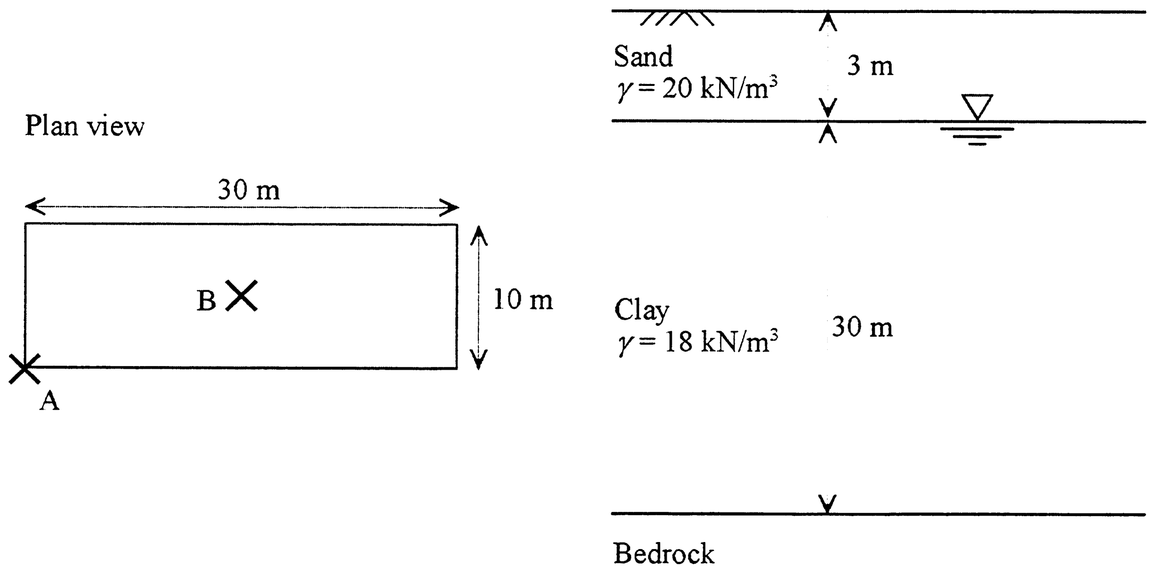

Fig. 3. Building and ground conditions

(a) Define and calculate the net bearing pressure exerted by the building. [10%]

(b) Divide the clay stratum into 3 layers and calculate the differential settlement of points A and B after consolidation of the clay stratum, assuming that the raft exerts a uniform bearing pressure. [70%]

(c) Suggest two methods by which the foundation design could be changed in order to reduce the differential settlement between A and B. [20%]

3 A building is being designed for an urban site, bounded by existing structures. The building footprint is to be 15 m × 15 m. The ground conditions are shown in Fig. 2.

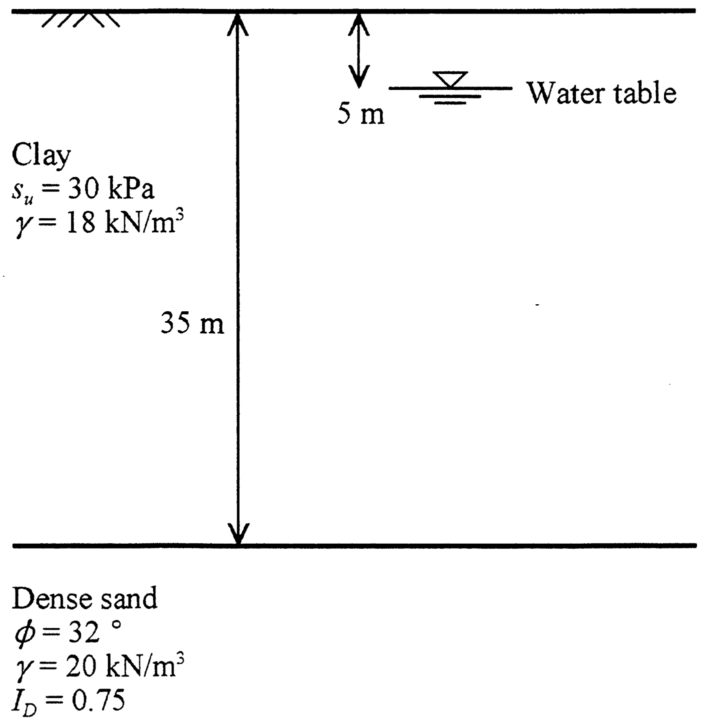

Fig. 2. Ground conditions

(a) The number of storeys of the building has not been chosen. Each storey will exert a dead load of 20 kPa and a live load of 10 kPa. Calculate the design ultimate load per storey if partial load factors of 1.35 and 1.5 are to be applied to dead and live loads respectively. [10%]

(b) Calculate the maximum number of storeys that could be supported by a raft foundation cast at ground level. Apply a partial resistance factor of 1.4 to the bearing capacity to limit settlements and account for uncertainty in the derived capacity. [30%]

(c) Calculate the maximum number of storeys that could be supported by 1 m diameter tubular piles bearing fully into the sand layer. Assume a uniform rectangular grid of piles at a spacing of 3 diameters to avoid interaction effects. Assume that shaft adhesion α= 0.8 and use the Fleming et al. (1992) design charts (see databook) to estimate base resistance. Apply a partial resistance factor of 3 to the pile capacity. Demonstrate the weak influence of pile diameter on this calculation by repeating the design for piles of diameters 0.5 m. [40%]

(d) Would pile diameter have a greater influence on the results if the piles were founded entirely in clay? [10%]

(e) Give one advantage and one disadvantage of using few large diameter piles compared to many small diameter piles at this site [10%]

4 An offshore structure is to be installed at a site comprising 30 m of loose sand overlying dense sand. Assume both strata have a submerged unit weight of 10 kN/m^3. Each leg of the structure will be supported on a group of identical 1 m diameter closed-ended tubular driven piles. The design storm compression and tension loads that each leg must sustain are 68 MN and 34 MN respectively.

(a) Using the API (2000) guidelines (see databook), find the variation in unit end bearing and shaft friction over the depth range 0-50 m. Sketch your results. [25%]

(b) Derive expressions for the ultimate compression and tension capacity of a 1 m diameter closed-ended tubular pile as a function of pile length, L, for the range $30 < L < 50$ m. [25%]

(c) Select the optimum number and length (to nearest 1 m) of piles (i.e. minimum total pile length). Ignore pile interaction effects and apply a safety factor of 2 to your calculated pile capacity. [40%]

(d) The API (2000) guidelines were validated against experience with relatively short onshore piles. Suggest two reasons why these guidelines may overestimate unit shaft friction in sand on long offshore piles. [10%]

END OF PAPER

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Cambridge University Engineering Department

Supplementary Databook

Module 4D5: Foundation Engineering

DJW. February 2004

Section 1: Plasticity theory

This section is common with the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by s_u rather than c_u.

Plasticity: Cohesive material $\tau_{max} = s_u$

– Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2s_u^2$

where q_u is the undrained triaxial compression strength, and s_u is the undrained plane shear strength.

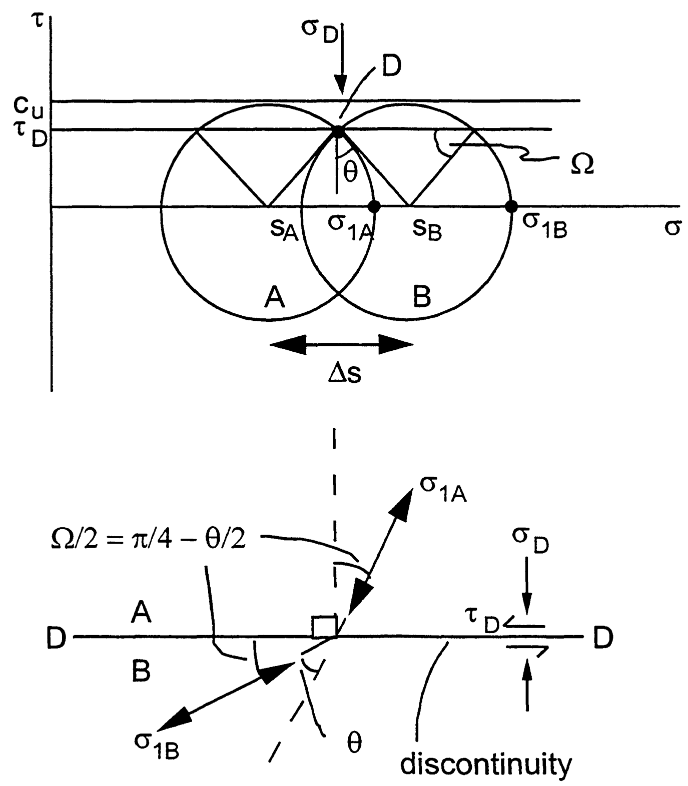
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength s_u, this becomes

$$D = A s_u x$$

– Stress conditions across a discontinuity



Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin\theta$$
$$\sigma_{1(B)} - \sigma_{1(A)} = 2s_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2s_u \, d\theta$$

Useful example:

$$\theta = 30°$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

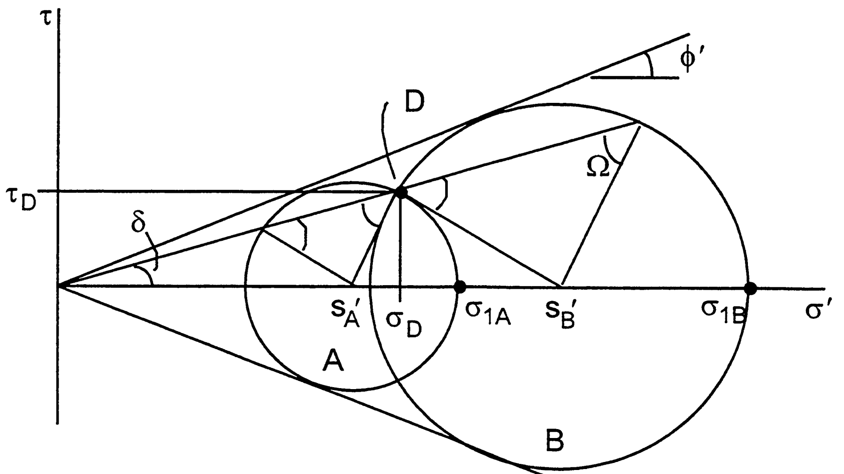
Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi'$

– Limiting stresses

$$\sin\phi' = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principal total stresses at failure, and u_s is the steady state pore pressure.

– Stress conditions across a discontinuity



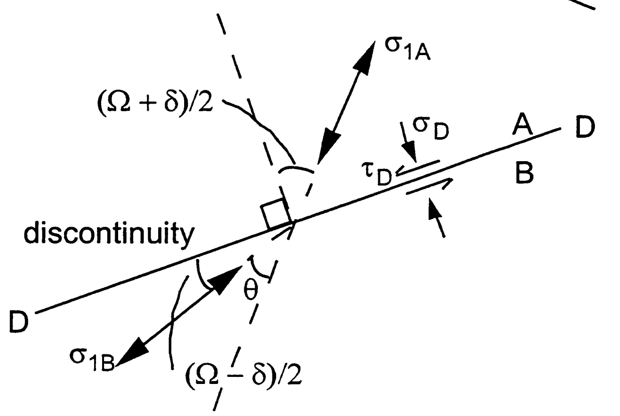
Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega + \delta)/\sin(\Omega - \delta)$

In limit, $\delta\theta \rightarrow 0$ and $\delta \rightarrow \phi'$,

$ds' = 2s' \,.\, \delta\theta \tan\phi'$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi')$

Section 2: Bearing capacity of shallow foundations

2.1 Cohesive (Tresca) soil, with undrained strength s_u.

The general equation for the bearing capacity, q_f, of a shallow foundation on cohesive soil is:

$q_f = s_c\, d_c\, N_c\, s_u + \gamma D$

D is the embedment of the footing base and γ is the density of the overburden.

Exact values of bearing capacity factor N_c for surface footings (zero embedment) from plasticity analysis:

Strip footing (rough or smooth):	$N_c = 2 + \pi$	(Prandtl, 1921)
Circular footing (rough):	$N_c = 6.05$	(Cox *et al.*, 1961)
Circular footing (smooth):	$N_c = 5.69$	(Cox *et al.*, 1961)

Shape correction factor:

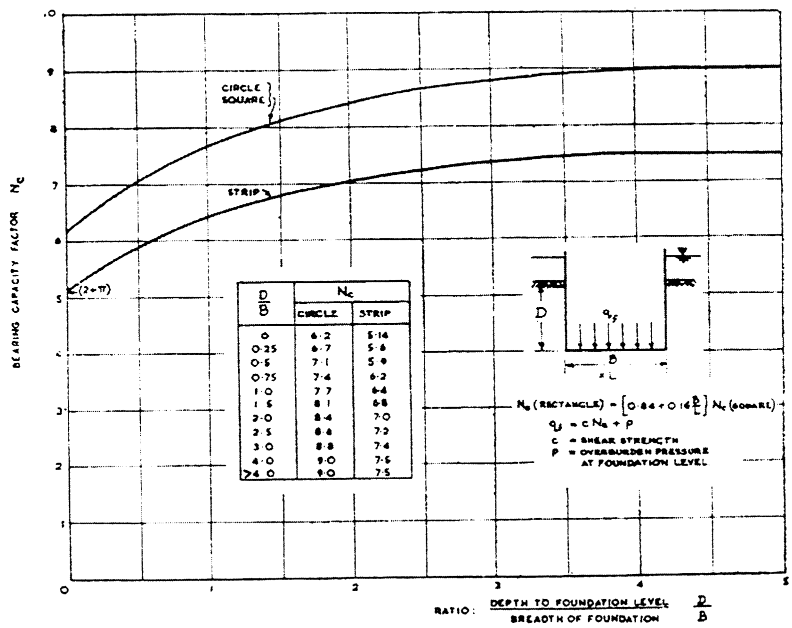
N_c for a strip should be multiplied by an empirical shape factor, s_c, for rectangular and square footings. Eurocode 7 suggests that for a rectangular footing of length L and breadth B:

$s_c = 1 + 0.2\, B / L$

Embedment correction factor:

Skempton (1951) proposed empirical curves for the increase in N_c with normalised footing embedment D/B. B is the width of a strip footing, the diameter of a circular footing, or the side length of a square footing. Note that Skempton's value of N_c at zero embedment for a circular footing slightly exceeds the exact plasticity solution. Skempton's curves can be approximated as a depth modification factor, d_c:

$d_c = 1 + 0.33 \tan^{-1}(D/B)$



D/B	Nc CIRCLE	Nc STRIP
0	6·2	5·14
0·25	6·7	5·6
0·5	7·1	5·9
0·75	7·4	6·2
1·0	7·7	6·4
1·5	8·1	6·8
2·0	8·4	7·0
2·5	8·6	7·2
3·0	8·8	7·4
4·0	9·0	7·5
>4·0	9·0	7·5

Empirical correction factors for shallow foundation embedment in cohesive soil (Skempton 1951)

References:

Skempton A.W. 1951. The bearing capacity of clays. Proc. Building Research Congress, London. 1:180-189
Prandtl L. 1921. Uber die Eindringungs-festigkeit (Harte) plastischer Baustoffe und die Festigkeit von Schneiden, Zeitung Angew. Math. Mech 1:15-20.
Cox A.D., Eason G., Hopkins H.G. 1961. Axially symmetric plastic deformation in soils. Proc. R. Soc. London (Ser. A) 254:1-45

2.2 Frictional (Coulomb) soil, with friction angle ϕ.

The general equation for the bearing capacity, q_f, of a shallow foundation on frictional soil is:

$$q_f = s_q N_q \sigma'_{v0} + 0.5 s_\gamma N_\gamma \gamma' B$$

The bearing capacity factors N_q and N_γ account for the strength arising from surcharge and self-weight of the foundation soil respectively. D is the embedment of the foundation base. γ' is the effective density of the soil beneath the foundation. σ'_{v0} is the in situ effective stress acting at the level of the foundation base. B is the footing width.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2 (N_q - 1) \tan \phi$$

Shape correction factors:

Empirical shape factors, s_q and s_γ should be applied for circular and square footings respectively. Eurocode 7 suggests that for a rectangular footing of length L and breadth B:

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3 B / L$$

For circular footings assume L = B.

Embedment correction factor:

Treat embedment as a surcharge in the N_q term.

Other empirical relationships for N_γ are widely used throughout Europe. Refer to Sieffert & Bay-Gress (2000).

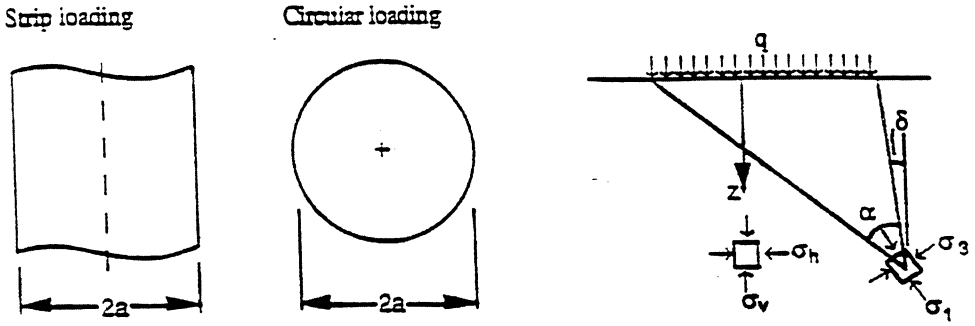
Reference:

Sieffert J-G. & Bay-Gress Ch. 2000. Comparison of European bearing capacity calculation methods for shallow foundations. Proc. Instn. Civ. Enrs Geotech. Engng. 143 (Apr.) 65-74

Section 3: Settlement of shallow foundations

3.1 Elastic stress distribution below strip and circular footings

The following closed-form elastic solutions are available for estimating the stress beneath foundations. They are derived from integration of the Boussinesq solution for a point load on an elastic half-space. More details can be found in Chapter 3 of 'Elastic Solutions for Soil and Rock Mechanics' by Poulos & Davis (1974).



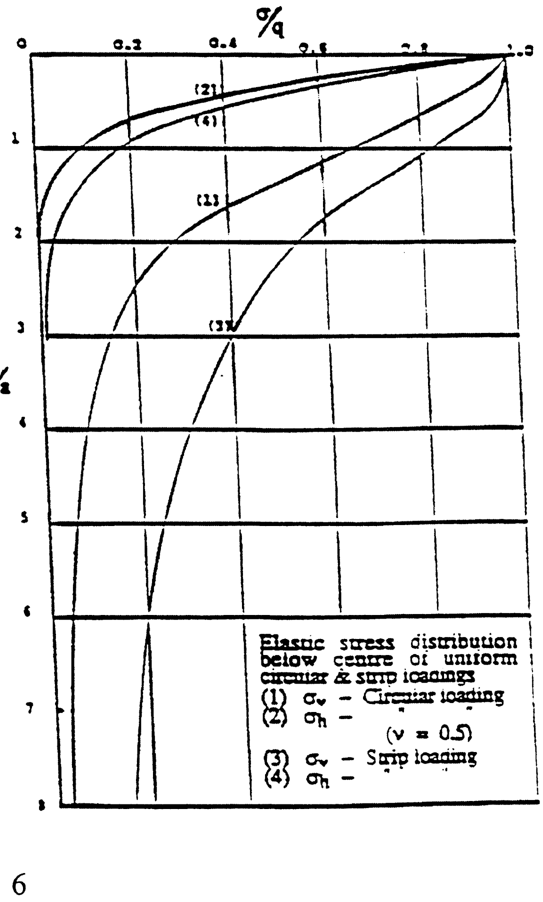
Strip: $\sigma_v = \frac{q}{\pi}[\alpha + \sin\alpha \cos(\alpha + 2\delta)]$; $\sigma_h = \frac{q}{\pi}[\alpha - \sin\alpha \cos(\alpha + 2\delta)]$

$\sigma_1 = \frac{q}{\pi}[\alpha + \sin\alpha]$; $\sigma_3 = \frac{q}{\pi}[\alpha - \sin\alpha]$

Circle: (centre-line only)

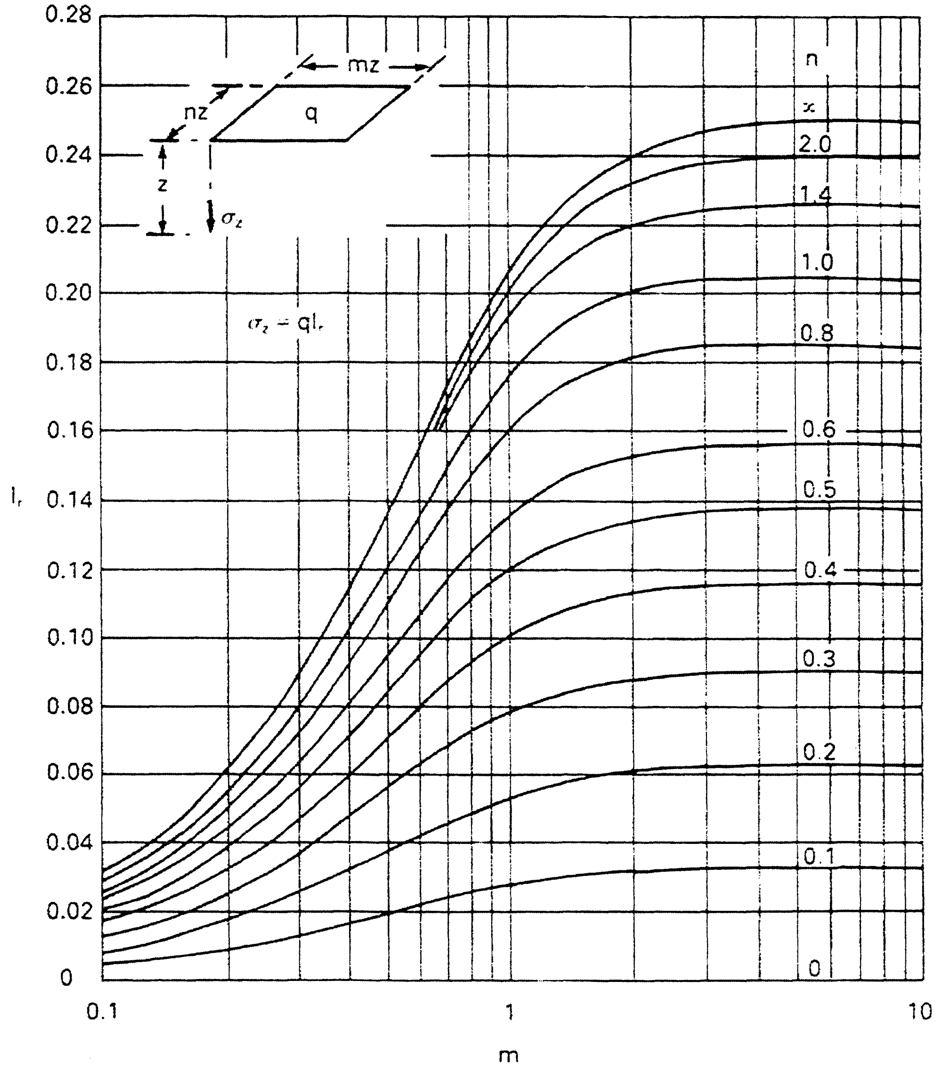
$$\sigma_v = q\left[1 - \left\{\frac{1}{1+(a/z)^2}\right\}^{\frac{3}{2}}\right]$$

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



3.2 Elastic solutions for vertical stress below uniform rectangular surface loads

The vertical settlement below a footing can be calculated by integrating the vertical compression of each sub-surface layer. If the vertical stiffness of each layer is known (or can be found from the vertical stress level), the settlement of that layer under the action of the foundation load can be found. The fraction, I_r, of the applied foundation load, q, which reaches a depth z below the corner of a rectangular loaded area can be found using Fadum's chart (Fadum 1948). This chart is derived from integration of the Boussinesq solution for a point load on an elastic half-space. Since this is an elastic solution, superposition of multiple rectangular areas is permitted. Further details are given by Poulos & Davis (1974), pp 54-57.



Vertical stress under the corner of a rectangular area carrying uniform vertical pressure
(after Fadum, 1948, reproduced from Craig, 1996)

References:

Craig R.F. 1992. Soil Mechanics. 5th edition. Chapman & Hall, London
Fadum R.E. 1948. Influence values for estimating stresses in elstic foundations. Proc. 2nd Int. Conf. Soil Mech. & Foundations. 3:77-84
Poulos H.G. & Davis E.H. 1974. Elastic solutions for soil and rock mechanics. Wiley, New York.

3.3 Elastic solutions for surface settlement

3.3.1 Infinite half-space

The following closed-form elastic solutions can be used to estimate the settlement of a circular or rectangular footing on an infinite half-space. Further details are given in Poulos & Davis (1974).

Notation:

G	shear modulus [$= E / 2 (1 + \nu)$ where E is Young's modulus]		
q	applied vertical stress	ν	Poisson's ratio
a	radius of loaded area	L, B	foundation length and breadth

Circular Area on Homogeneous Soil

central settlement is $w_0 = (1-\nu)\frac{qa}{G}$

edge settlement is $w_e = \frac{2}{\pi}(1-\nu)\frac{qa}{G}$

rigid punch ($q_{avg} = P/\pi a^2$) $w_r = \frac{\pi}{4}(1-\nu)\frac{qa}{G}$

Surface settlement profile under point load, Q

$$w(s) = \frac{1}{\pi}\frac{(1-\nu)}{2G}\frac{Q}{s}$$

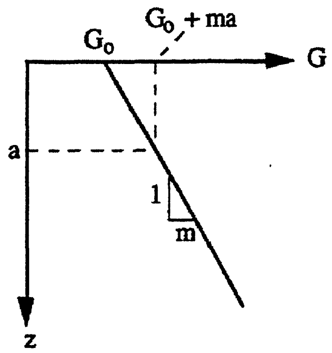
Circular Area on Non-Homogeneous Soil

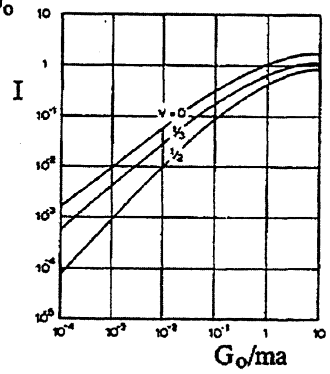
Consider case where shear modulus varies linearly with depth as $G = G_0 + mz$:

(1) $G_0 = 0$, $\nu = 0.5$ $\quad w = \frac{q}{2m}$ under loaded area of any shape

$w = 0$ outside loaded area.

(2) $G_0 > 0$, central settlement is $\quad w_0 = \frac{qa}{2G_0} I$





For $\nu = 0.5$, central settlement may be approximated by $w_0 = \frac{qa}{2(G_0 + ma)}$

Rectangular Area on Homogeneous Soil

corner settlement is $w_c = (1-\nu)\frac{qB}{2G} I$

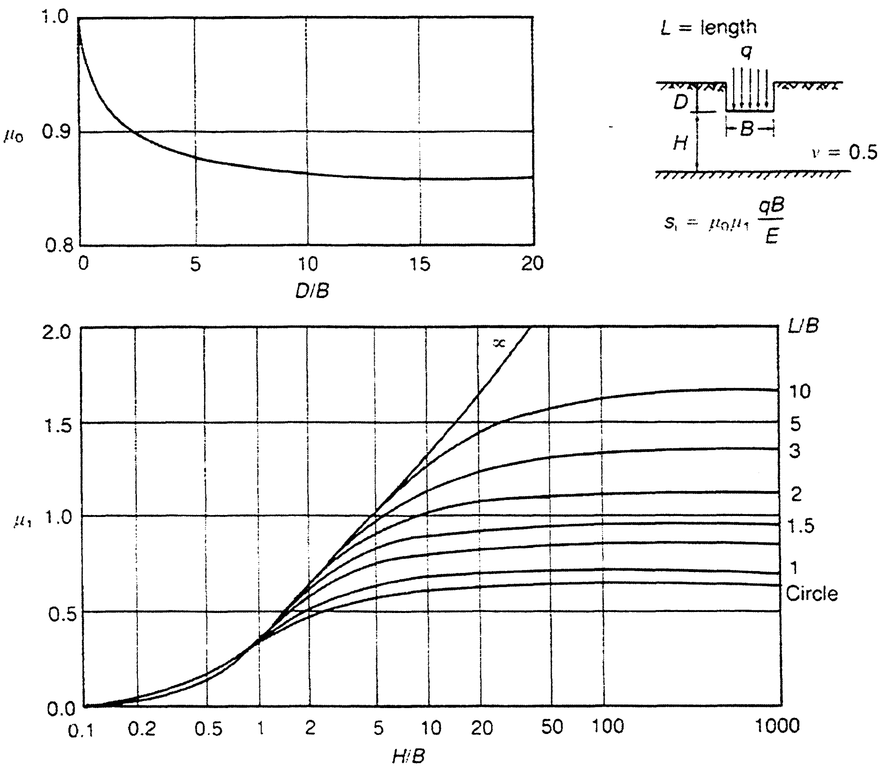
where:

L/B	I	L/B	I	L/B	I	L/B	I
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2.0	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = (1-\nu)\frac{q_{avg}\sqrt{BL}}{2G} I_{rgd}$ where I_{rgd} varies from 0.9 to 0.7 for L/B = 1 - 10

3.3.2 Layer of finite thickness, immediate (undrained) settlement

The following graphical solution for the average immediate settlement of a uniformly loaded footing, s_i, on a layer of finite thickness was proposed by Christian & Carrier (1978). The solution is derived by integrating an elastic stress field over the region above bedrock and is for ν= 0.5 (undrained, or immediate settlement). If sub-surface layers of different undrained stiffness are present, superposition can be used. The component of footing settlement due to an individual layer can be found by calculating the footing settlement with the bedrock above and below that layer.



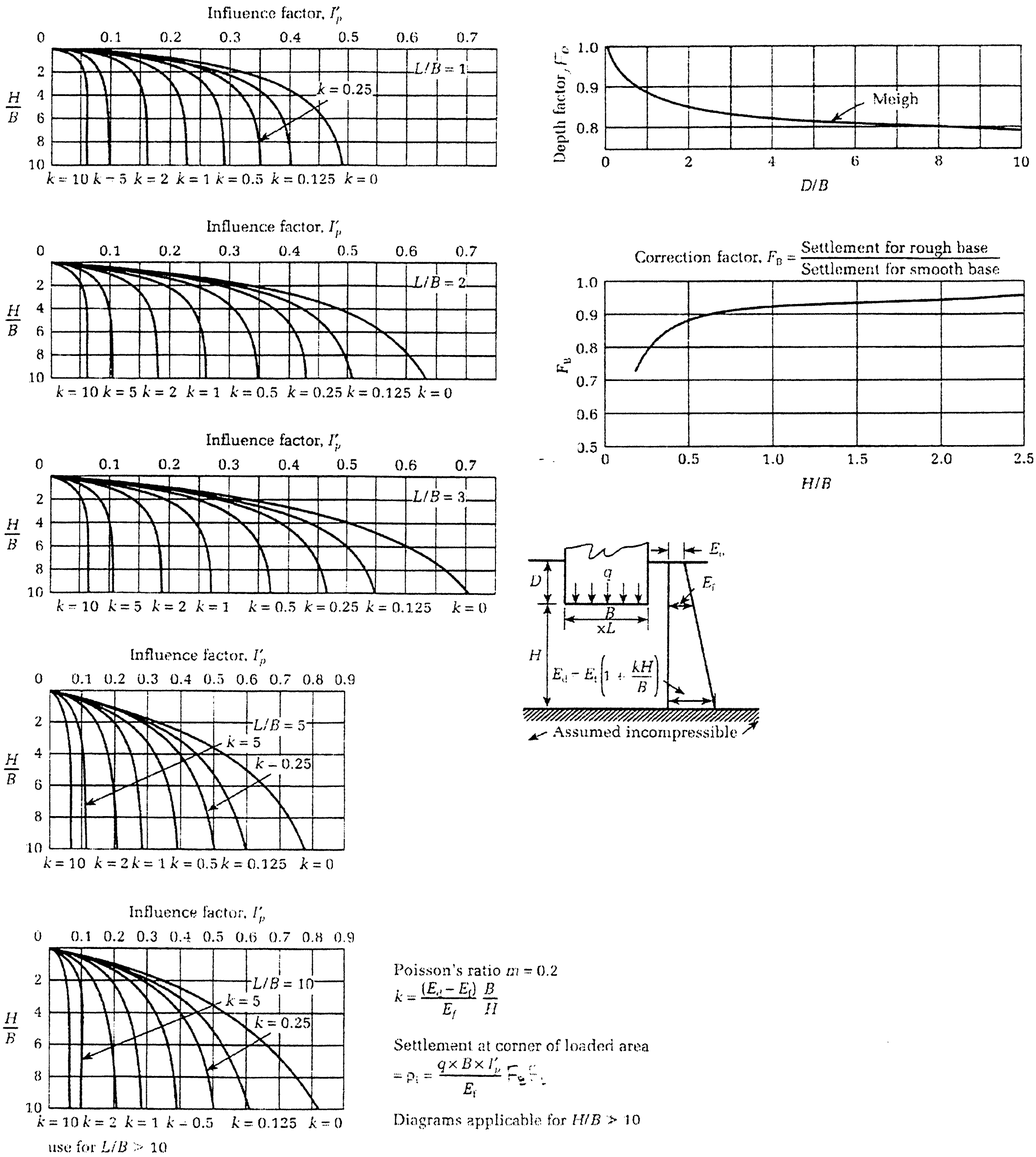
Average immediate settlement of a uniformly loaded finite thickness layer (after Christian & Carrier, 1978)

Christian J.T & Carrier III W.D. 1978. Janbu, Bjerrum and Kjaernli's chart reinterpreted. Canadian Geotechnical Journal 15:123-128 and 15:436-437

3.3.2 Layer of finite thickness, total (drained) settlement

The following graphical solution for the total settlement *at the corner* of a uniformly loaded footing, ρ_i, on a layer of finite thickness was proposed by Meigh (1976). Since it is based on elastic solutions, superposition is permitted to allow settlement at the centre of a footing to be found. Solutions for the case of increasing stiffness with depth (k > 0) are included. A Poisson's ratio of 0.2 has been used, for drained conditions. The base correction factor, F_B, should be applied for rough footings.

Meigh A.C. 1976. The Triassic rocks, with particular reference to predicted and observed performance of some major foundations. Geotechnique 26(3):391-452



Total settlement at the corner of a uniformly loaded finite thickness layer
(after Meigh, 1976, reproduced from Tomlinson, 2001)

Section 4: Bearing capacity of deep foundations

4.1 Axial capacity in sand

American Petroleum Institute (API) (2000) guidelines for driven piles in sand.

Unit shaft resistance: $\tau_s = K\,\sigma'_{vo} \tan\delta < \tau_{s,limit}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

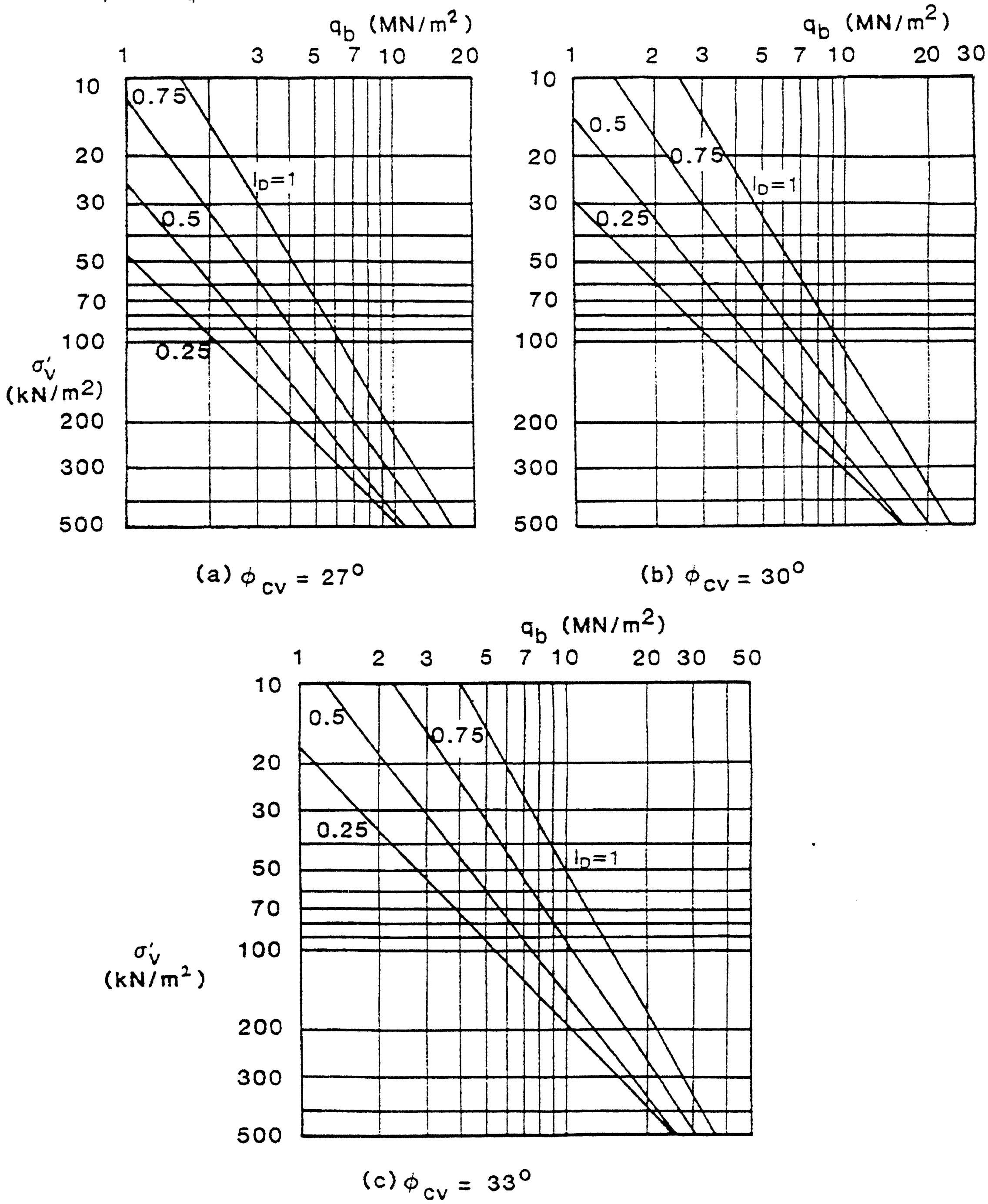
Soil density	*Soil type*	*Soil-pile friction angle, δ*	*Limiting value, $\tau_{s,limit}$ (kPa)*
Very loose Loose Medium	Sand Sand-silt Silt	15	48
Loose Medium Dense	Sand Sand-silt Silt	20	67
Medium Dense	Sand Sand-silt	25	81
Dense Very Dense	Sand Sand-silt	30	96
Dense Very Dense	Gravel Sand	35	115

Unit base resistance: $q_b = N_q\,\sigma'_{vo} < q_{b,limit}$

Soil density	*Soil type*	*Bearing capacity factor, N_q*	*Limiting value, q_{limit} (MPa)*
Very loose Loose Medium	Sand Sand-silt Silt	8	1.9
Loose Medium Dense	Sand Sand-silt Silt	12	2.9
Medium Dense	Sand Sand-silt	20	4.8
Dense Very Dense	Sand Sand-silt	40	9.6
Dense Very Dense	Gravel Sand	50	12

American Petroleum Institute (API) (2000) RP2A: Recommended practice of planning, designing and constructing fixed offshore platforms- working stress design, 21st edition, Washington 59-61

Randolph (1985) and Fleming et al's (1992) tables for base resistance, based on Bolton's (1986) correlations for friction angle, combined with Berezantzev's (1961) relationship between ϕ and N_q:



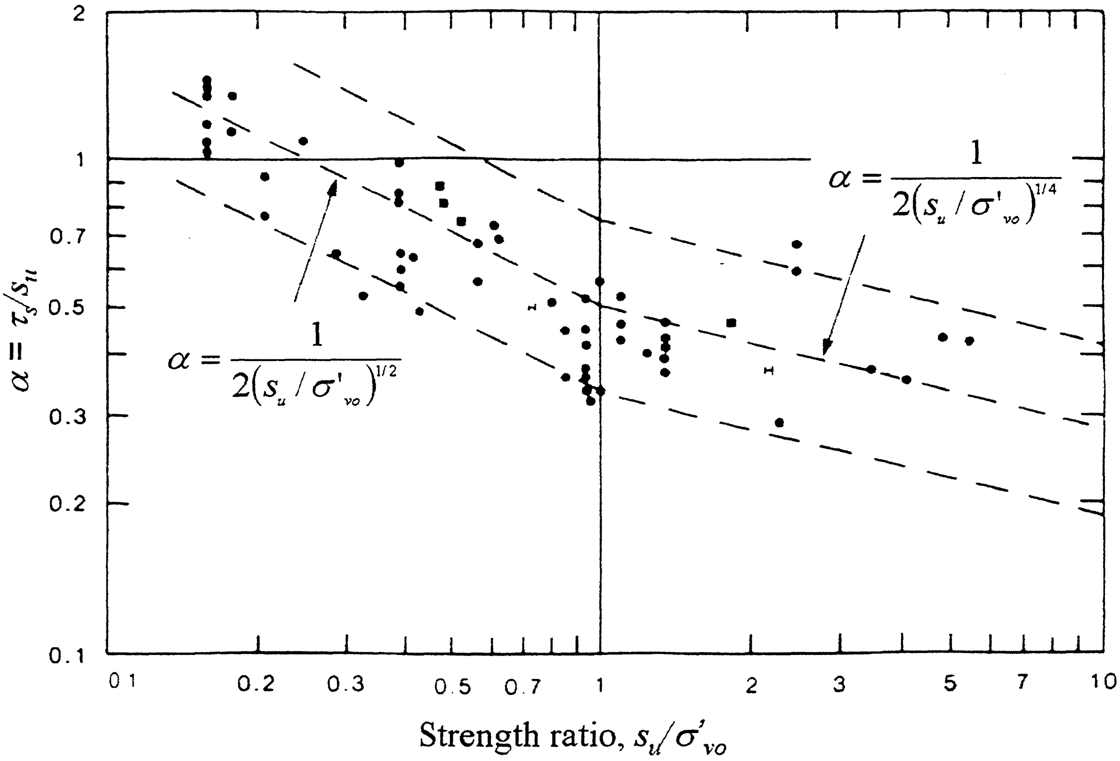
Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

Berezantzev V.C., Kristoforov V. & Golubkov V. 1961. Load-bearing capacity and deformation of piled foundations Proc. 5th Int. Conf. Soil Mechanics & Foundation Engineering, Paris 2:11-12
Bolton M.D. 1986. Strength and Dilatancy of Sands, Geotechnique, Vol. 36, No.1, 65-78, 1986.
Randolph M.F. 1985. Capacity of piles driven into dense sand. Cambridge Univ. Technical Report CUED/D-SOILS TR171
Fleming W.G.K., Weltman A.J., Ranolph M.F. & Elson W.K. 1992. Piling Engineering.

4.2 Axial capacity in clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

Unit shaft resistance: $\tau_s = \alpha\, s_u$



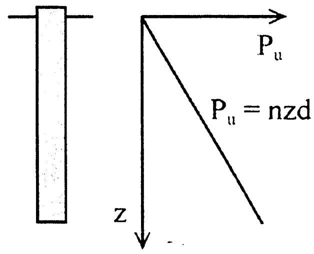
Shaft friction on driven piles in clay
(Randolph & Murphy, 1985; Fleming et al, 1992; API, 2000)

Unit base resistance: $q_b = N_c\, s_u$ $\qquad$ $N_c = 9$.

Randolph M.F. & Murphy B.S. 1985. Shaft capacity of driven piles in clay. Proc. 17th Annual Offshore Technology Conference, Houston. 1:371-378

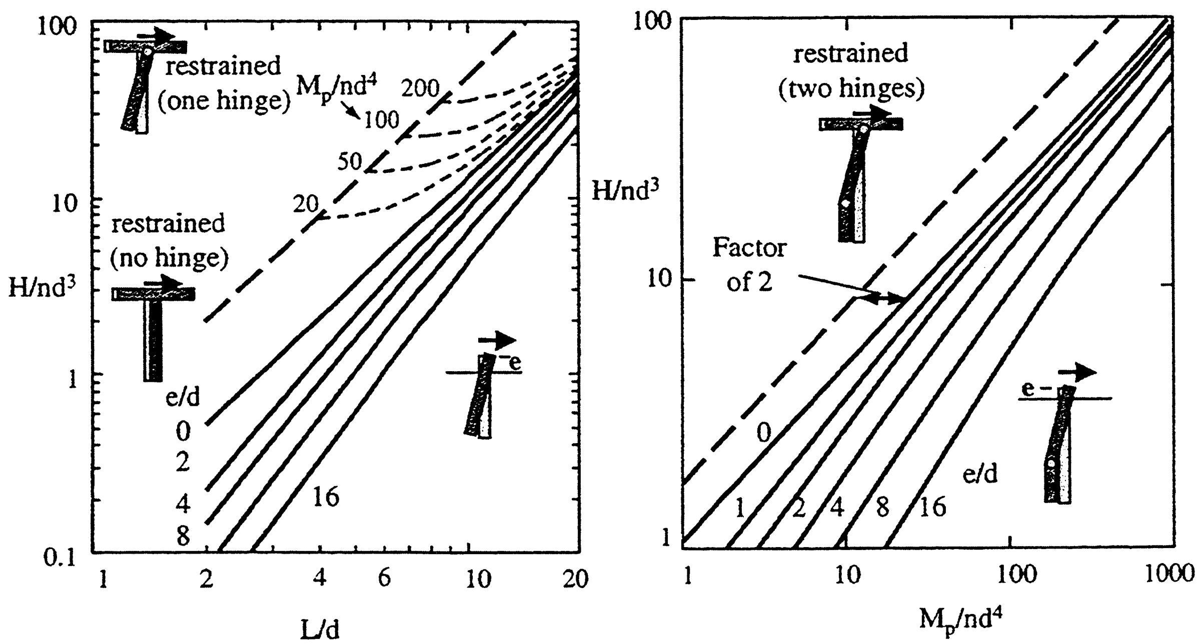
4.3 Lateral capacity in sand (or n.c. clay) (linearly increasing lateral resistance with depth)

Lateral soil resistance (force per unit length), $P_u = nzd$
In sand, $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient k; $s_u = kz$; $n=9k$



Distribution of lateral resistance

H	horizontal load on pile head
M_p	plastic moment capacity of pile
d	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
γ'	effective unit weight
K_p	passive earth pressure coefficient, $(1+ \sin \phi)/(1- \sin \phi)$

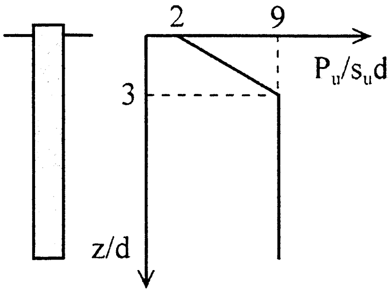


Short piles | Long piles

Lateral pile capacity
(linearly increasing lateral resistance with depth)

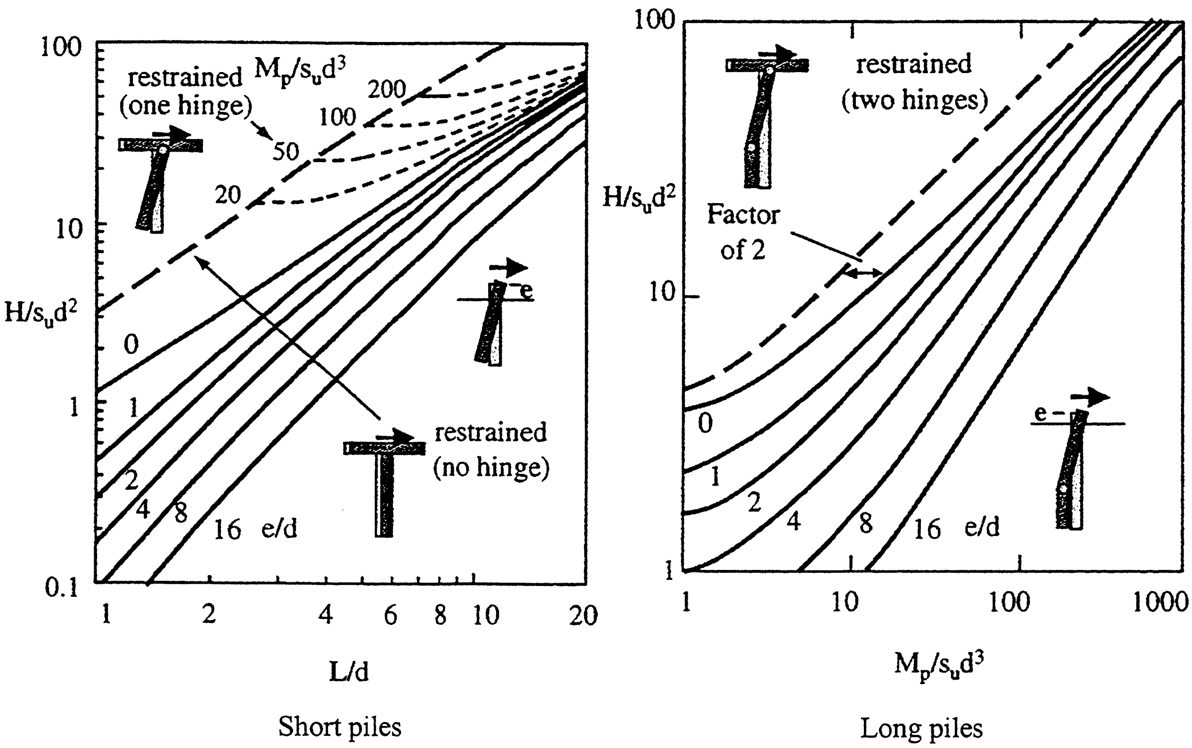
4.4 Lateral capacity in uniform clay

Lateral soil resistance (force per unit length), P_u, increases from $2s_ud$ at surface to $9s_ud$ at 3d depth then remains constant.



Distribution of lateral resistance

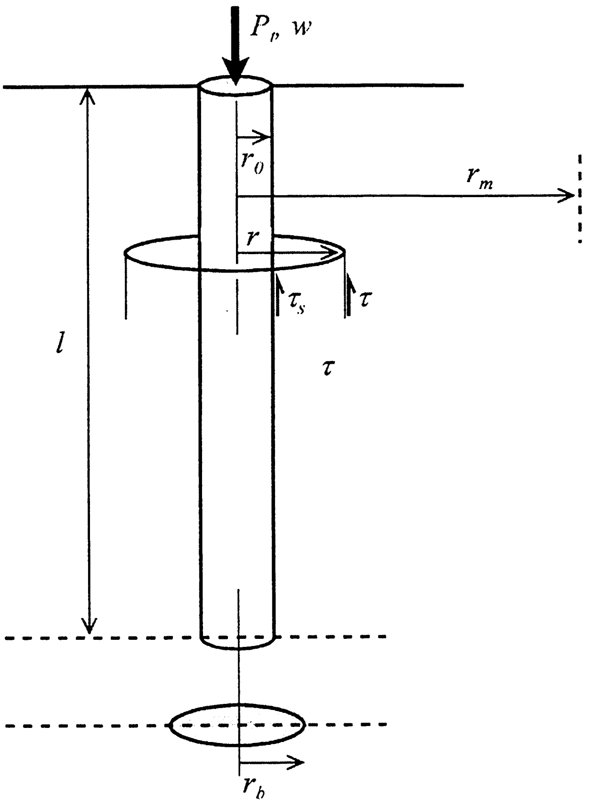
H	horizontal load on pile head
M_p	plastic moment capacity of pile
d	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
s_u	undrained shear strength



Lateral pile capacity
(in uniform clay)

Section 5: Settlement of deep foundations

5.1 Settlement of a rigid pile



Vertical eq[m] on circumferential element

$$\tau = \frac{\tau_s r_0}{r}$$

Compatibility

$$\gamma \approx \frac{dw}{dr}$$

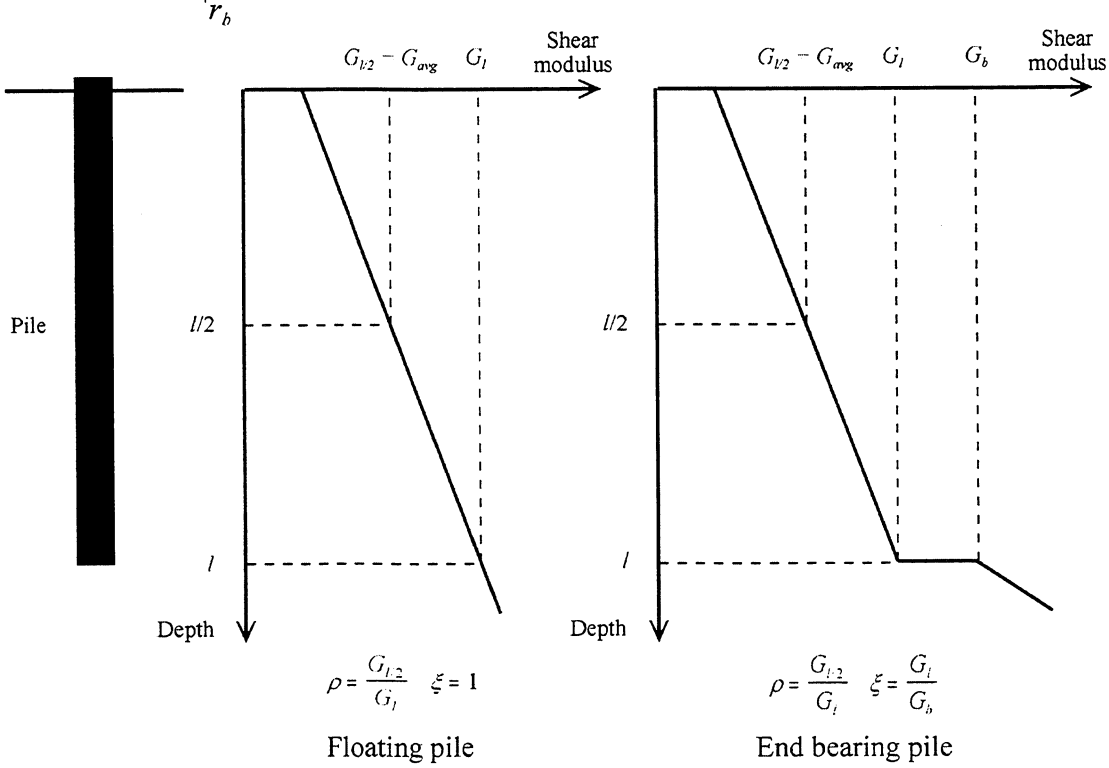
Hence $\tau_s = \dfrac{G w_s}{\zeta r_0}$ where $\zeta = \ln(r_m / r_0)$

Vertical eq[m] of pile

$$P_s = 2\pi r_0 l \tau_{s,avg}$$

Rigid punch

$$P_b = \frac{4}{(1-\nu)} r_b G_b w_b$$



Nomenclature for settlement analysis of single piles

5.1 Settlement of a rigid pile (continued)

$$\frac{P}{w} = \frac{P_b}{w_b} + \frac{P_s}{w_s}$$

$$\frac{P}{w} = \frac{4r_b G_b}{1-\nu} + \frac{2\pi l G_{avg}}{\zeta}$$

$$\frac{P}{w r_0 G_l} = \frac{4}{1-\nu}\frac{G_b}{G_l}\frac{r_b}{r_0} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_l}\frac{l}{r_0}$$

$$\frac{P}{w r_0 G_l} = \frac{4}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{l}{r_0}$$

where:	$\eta = r_b/r_0$	base enlargement ratio
	$\xi = G_l/G_b$	base stiffness ratio
	$\rho = G_{avg}/G_l$	soil stiffness gradient
	$\lambda = E_p/G_l$	pile-soil stiffness ratio
	$\zeta = \ln(r_m/r_0)$	dimensionless influence zone

and:

$$\zeta = \ln\left\{[0.25 + (2.5\rho(1-\nu) - 0.25)\xi]\frac{l}{r_0}\right\}$$

for ξ=1:

$$\zeta = \ln\left\{2.5\rho(1-\nu)\frac{l}{r_0}\right\}$$

5.2 Settlement of a compressible pile

$$\frac{P}{G_l r_0 w} = \frac{\dfrac{4\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh \mu l}{\mu l}\dfrac{l}{r_0}}{1 + \dfrac{1}{\pi\lambda}\dfrac{4\eta}{(1-\nu)\xi}\dfrac{\tanh \mu l}{\mu l}\dfrac{l}{r_0}}$$

where $\mu l = (2/\lambda\zeta)^{1/2}\,(l/r_0)$ measure of pile compressibility