UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 19 April 2004 2.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

Answer not more than ***three*** *questions*

All questions carry the same number of marks

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
4D6 Data sheets (4 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 A steel beam is simply supported at its ends A and C, and continuous over the central simple support B, as shown in Figure 1a. The beam is a 457 × 152 × 60 Universal Beam, and its web lies in the plane of the paper.

(a) Show that the fundamental natural frequency of small vertical bending oscillations is around 120 Hz. [20%]

(b) A vertical force $f_1(t)$ is applied to the point P which is 0.8m from A. This force lasts for 20 milliseconds and varies with time in a triangular manner, as shown in Figure 1b. Estimate the maximum dynamic deflection that will occur due to this load. [30%]

(c) A dynamic, uniformly-distributed load is applied to span BC. It is a harmonic force with an amplitude of 130 N/m and a frequency which can vary slowly within the range from 50 Hz to 150 Hz. Assuming that the only damping is viscous damping at a fraction of 1.2 percent of critical damping, estimate the maximum dynamic deflection at any point and the maximum dynamic stress at any point. [50%]

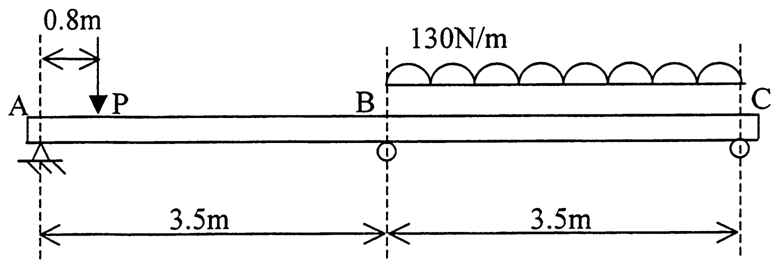

Fig.1a

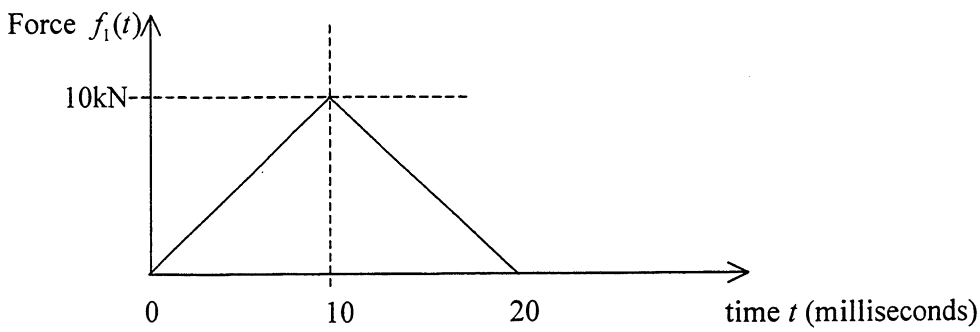

Fig.1b

2 A three-storey, reinforced-concrete framed building is approximated by the lumped mass model shown in Figure 2a. The top storey (i.e. the roof) has a mass of 50 tonne and the lower floors each have a mass of 30 tonne. The floors are supported on columns of circular cross section, each column having flexural rigidity EI = 63 000 kN m^2. Between each floor there are eight columns as shown in the plan view in Figure 2b. Each column is 3m high. The floors may be considered to be much stiffer in bending than the columns. All columns are rigidly connected to the floors, but the bases of all columns at the ground floor level should be considered to be pinned.

(a) The building is anticipated to sway in the direction marked in Figures 2a and 2b. Using an assumed mode shape {0.4, 0.7, 1.0} for the vector of lateral displacements of the floors, show that the fundamental natural frequency of sway oscillations is around 4 Hz and that the corresponding modal participation factor is around 1.2. Explain what a modal participation factor is. [50%]

(b) Using the design response spectrum in Figure 2c, estimate the peak shear force in the base of columns at each floor level when a nearby earthquake causes a peak ground acceleration of $0.15g$ at the site. In Figure 2b, the ordinate is the ratio of spectral acceleration S_{da} to peak ground acceleration a_g. [50%]

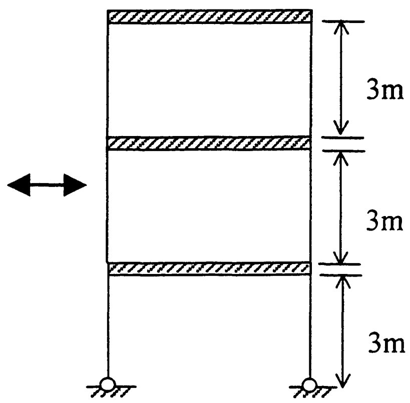

Fig.2a (sectional view)

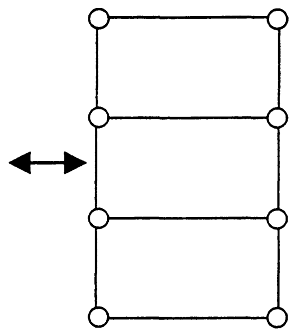

Fig.2b (plan view)

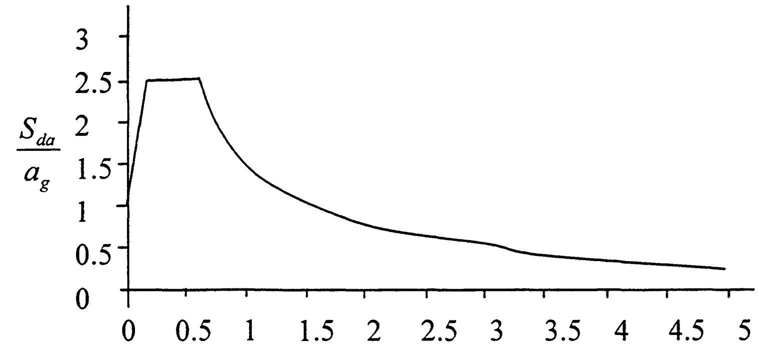

Fig.2c

(TURN OVER

3 (a) Explain the term 'dynamic soil-structure interaction' by considering a structure founded on a deep soil layer overlying bed rock. Consider vertically propagating horizontal shear S_h waves. [10%]

(b) Under what conditions do you expect the stiffness of a saturated soil layer to decrease? [10%]

(c) Explain why excess positive pore pressures develop in saturated soils subjected to earthquake loading. [20%]

(d) The foundations of a single storied building located in the city of Bam in Iran were in a dry sand layer of 10 m thickness overlying bedrock as shown in Figure 3. The unit weight and the void ratio of this soil may be taken as 15 kN/m^3 and 0.6 respectively. The coefficient of earth pressure at rest for this sand is 0.45 and the critical state friction angle is 30°. The building had its foundation located 2 m below the ground surface and applied a bearing pressure of 50 kPa on the soil. Estimate the small-strain shear modulus by considering a point directly below the foundation on a reference plane 4 m below the ground surface. Calculate the time taken by the S_h waves to travel from the bedrock to the foundation of the building. You may take the average dry density of the soil to be 1530 kgm^{-3}. [30%]

(e) The peak shear strain in this soil layer was expected to have reached a value of 0.25×10^{-3} during the Bam earthquake. The number of cycles in this earthquake was estimated to be about 9. How does this affect the shear modulus calculated in part (d)? Also recalculate the time taken by the S_h waves to travel from bedrock to the foundation of the building and compare with your answer in part (d). [30%]

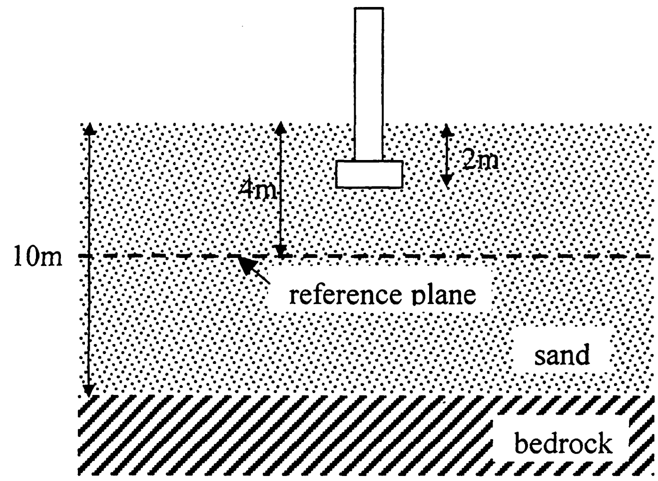

Fig.3

4 (a) Explain why the quasi-static theory of flutter is inappropriate for the analysis of aerodynamic stability of bridge decks. Explain how Theodorsen's theory of flutter is an improvement on the quasi-static theory, and state the limitations of Theodorsen's theory. [20%]

(b) A student applies Rayleigh's Principle to obtain an estimate of the natural frequency of the transverse bending vibrations of the deck of a long-span suspension bridge. Explain, giving reasons, whether you would expect their answer to be an upper or lower bound on the real natural frequency. [20%]

(c) Explain the meaning of the statement "The vibration modes are orthogonal with respect to both structural mass and stiffness". [20%]

(d) Briefly describe and differentiate between the basic wind processes that occur in thunderstorms, hurricanes and tornadoes. [20%]

(e) Explain briefly how the auto-covariance and the covariance of wind velocity components measured at a point can be used to define a representative time scale, and a number of representative length scales of turbulence. [20%]

END OF PAPER

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Engineering Tripos Part IIB/EIST Part II FOURTH YEAR

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

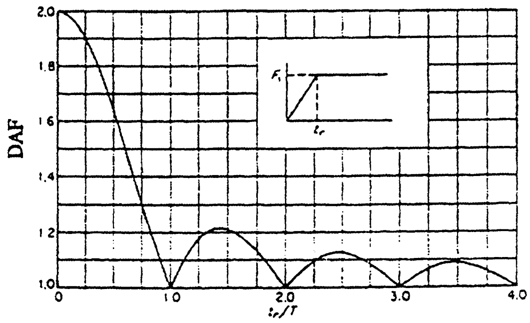
Modal analysis of simply-supported uniform beams

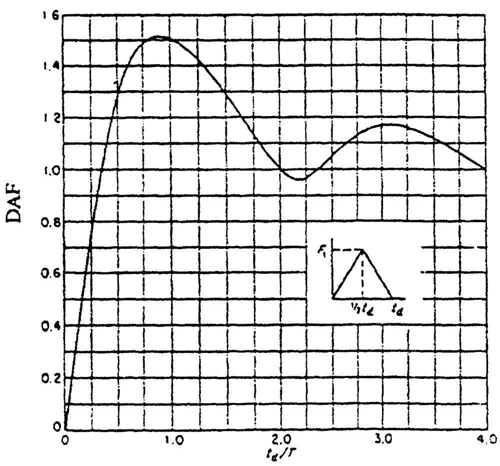
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

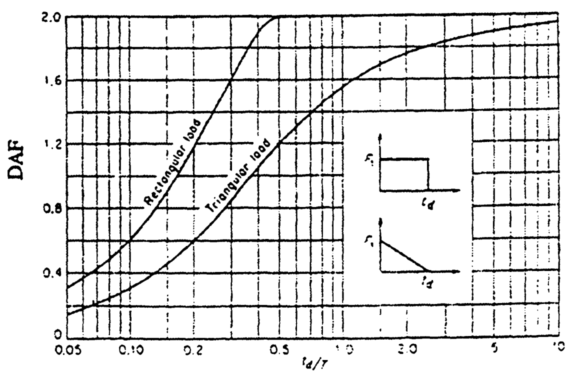
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

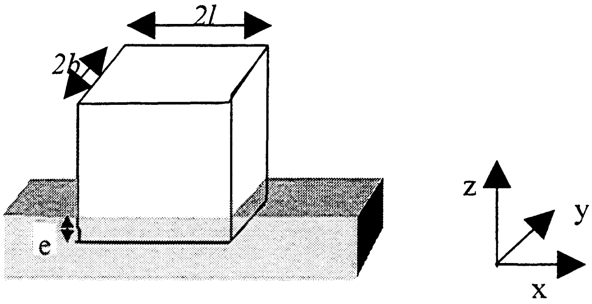
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{\max} = 100 \frac{(3-e)^2}{(1+e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{\max}} = \frac{1}{1+\gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \cdot e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma_v' \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

SPGM
January, 2004