

ENGINEERING TRIPOS PART IIB

Monday 3 May 2004

2.30 to 4

Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments: (i) Concrete and Masonry Structures formula and Data Sheet (4 pages).
(ii) The Cumulative Normal Distribution Function (1 page).*

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you may
do so by the Invigilator**

(TURN OVER

1 (a) Describe the main limit states you would consider when designing a concrete off-shore oil platform in deep water and explain the importance of each. [15%]

(b) Name three examples of major structural failures of concrete structures, identifying the primary mechanism of failure, and suggest actions that would prevent such failures in future. [25%]

(c) A designer specifies that the concrete for a reinforced concrete beam is to have a *characteristic* cube strength f_{ck} of 45 MPa. The quality control at the batching plant is such that the compressive strength of concrete from the plant, for any grade of concrete, may be assumed to be normally distributed with a standard deviation of 7 MPa.

(i) Determine the mean compressive strength of the specified mix and the design strength f_{cd} . [10%]

(ii) Calculate the probability that any cube sample chosen at random will have strength less than f_{cd} . [10%]

(iii) The permanent loads on the beam result in a *design* compressive stress in the concrete equal to the *design* value of the compressive strength of the concrete mix calculated in (i) above. Assume that the value of permanent load is normally distributed, with a standard deviation equivalent to an applied stress standard deviation of 6 MPa at the critical position. What is the reliability index β and hence probability of failure in compression of the concrete subjected to this loading? [20%]

(iv) The client modifies the brief and now requires a target reliability index of $\beta = 3.5$ for the structure. For the loading specified in part (iii) above, what characteristic strength should the designer now specify in order to just achieve this target? [20%]

2 (a) Name the key factors which govern the durability of reinforced concrete structures. Briefly explain why they are important. [20%]

(b) Describe three problems that might arise during and after a mass concrete pour for a large dam and suggest what might be included in the specification to address these issues. [20%]

(c) During a bridge inspection 6 years after construction, dust samples are collected by drilling into the deck concrete which is assumed to have had a negligible chloride contamination when first built. The results indicate a chloride content of 3% (by weight of cement) at a depth of 10 mm from the surface and 1% at a depth of 20 mm. A covermeter survey indicates the cover to the reinforcement is 40 mm at this location. In addition, an indicator test using phenolphthalein on the exterior of the drill hole is colourless to a depth of 12 mm from the surface and thereafter pink. Critical thresholds for depassivation of the steel (and hence initiation of corrosion) are assumed to be $Cl^- = 0.4\%$ by weight of cement or $pH = 12$.

(i) Based on these results, how long after the principal inspection would you expect reinforcement corrosion to begin? Would carbonation of the concrete or the presence of chlorides initiate the corrosion? [40%]

(ii) Estimate the value of the diffusion constant D and the effective surface concentration C_o for chloride ions, assuming that both parameters remain constant with time. [20%]

(TURN OVER

3 A 3 m long horizontal reinforced concrete cantilever is to be designed at ULS for a total load (incorporating dead weight and all required safety factors) of 80 kN/m along its entire length. The proposed uniform cross-section of the beam is shown in Fig. 1(a). The three longitudinal bars are of diameter 32 mm in steel of design yield strength 400 MPa, and the concrete has design cube strength 40 MPa.

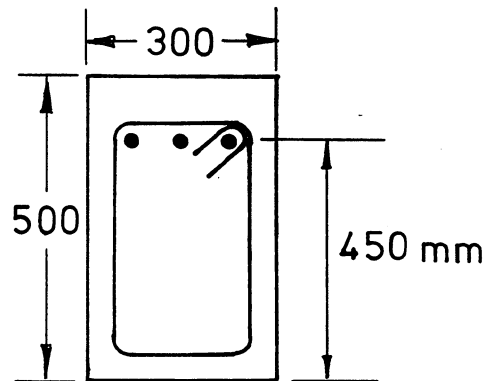


Fig. 1(a)

(a) Show that with this longitudinal steel the section is adequate to carry the maximum applied bending moment. [15%]

(b) Using the standard design method for shear, with $\tau_{Rd} = 0.5 \text{ MPa}$, determine the required spacing of 8 mm diameter double-legged stirrups (in steel of design yield strength 220 MPa) near the root of the cantilever. [25%]

(c) Fig. 1(b) shows schematically an attempt to use a smeared truss analogy for an alternative shear design, giving the required distribution of stirrups (not shown in this Figure) along the cantilever. Assume that bond between concrete and steel is good, so that all necessary transfers of force can occur.

(cont.)

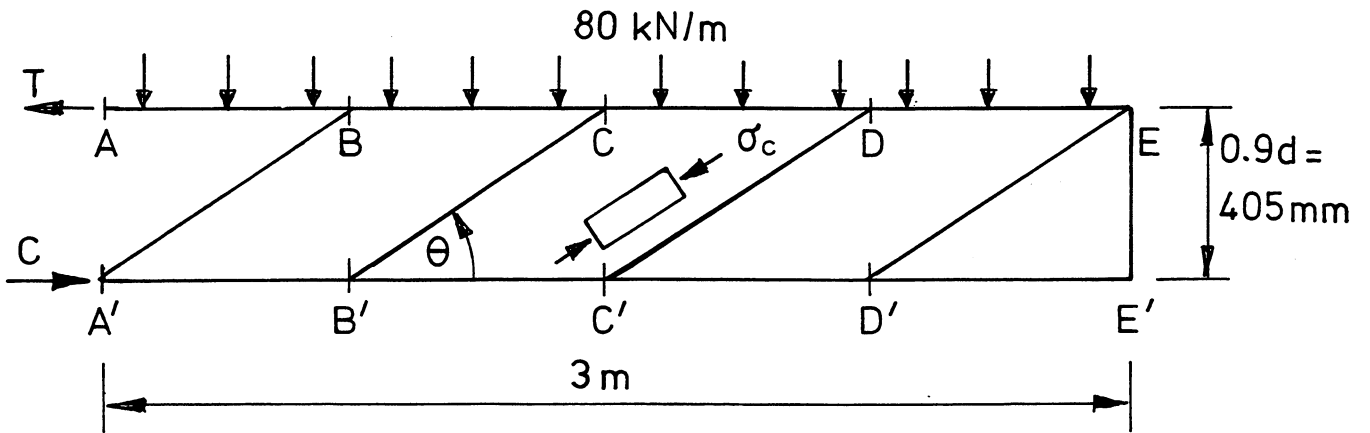


Fig. 1(b)

All the concrete struts are inclined at the same angle θ ; the concrete stress σ_c varies from region to region, but is constant in each skew region.

- (i) By considering the vertical equilibrium of the entire body below $A'B$, find the required spacing of 8 mm diameter vertical stirrups in zone AB (assuming all reach yield). What stirrup spacing is needed in zones BC, CD and DE? [30%]
- (ii) Find the required concrete stress σ_c in zone ABA' and comment on its magnitude. [15%]
- (iii) Determine what force T must be carried, in this analogy, by the top (steel) chord at the root of the cantilever, and comment on its significance. [15%]

(TURN OVER

4 The corner column of a building is 500 mm square in cross-section and is subjected, in some loading cases, to axial compression N combined with bending moment M about one diagonal of the section. At the ultimate limit state the concrete compression zone may be assumed to carry uniform stress $0.6f_{cd}$, where the design cube strength f_{cd} is 30 MPa. The column is reinforced by eight bars of 32 mm diameter, each with cover 40 mm to the bar surface, one in each corner and one in the middle of each side of the cross-section. The steel has design yield strength 400 MPa in tension and compression.

(a) Assume that all steel yields at failure unless the neutral axis goes through the bar centre (when the stress may be anywhere in the range $-f_{yd} \leq \sigma_s \leq +f_{yd}$). Determine the (range of) ultimate values of N and M if the neutral axis at failure goes through

(i) the column centre; [15%]

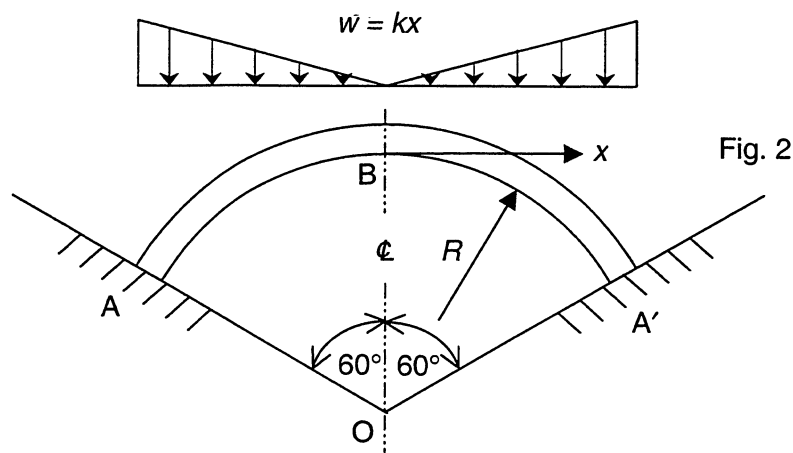
(ii) the centre of two bars at the middle of adjacent sides, with most of the column in compression. [35%]

(b) If the maximum compressive strain in concrete before failure is 0.0035, and the steel behaviour is linear up to yield at strain ± 0.002 , show that N for case (a) (i) above should be adjusted to 2.25 MN and the corresponding eccentricity to 0.25 m. [25%]

(c) The column is loaded through rollers at its ends, parallel to the diagonal, so that the compressive force has eccentricity 150 mm from the column centre. If the column has the properties of (b) above, and is to take a compressive force of 2.25 MN at ULS, what maximum height between rollers may be permitted? [25%]

5 (a) Referring to the Formula and Data Sheet, discuss how it is possible for the overall compressive strength of blockwork to exceed substantially the cube compressive strength of the mortar in the joints. Discuss also the material properties of masonry construction which permit a form of plastic theory to be applied to masonry arches in some circumstances. [20%]

(b) The plane masonry arch sketched in Fig. 2 is to be subjected only to symmetrical vertical total load, whose magnitude per unit horizontal distance varies linearly with distance from the centre of the arch, as shown.



(i) By considering the equilibrium of a large piece of arch from the centre B to position x , or otherwise, show that the depth v of the line of thrust below its level at the crown B varies as $|x|^3$ either side of B. [20%]

(ii) The inner surface of the arch is a circle of radius R , and the arch subtends 120° between springings at A and A' as shown. Assuming that the thrust line reaches the inner surface at B and at the springings, estimate the horizontal thrust H at the springings in terms of k and R . By drawing or calculation find the minimum required thickness of an arch of uniform thickness, as a proportion of R . [40%]

(iii) Describe the collapse mechanism expected if the arch of minimum thickness were constructed. Discuss what thickness would probably be adopted in a practical design, and what other loading cases might be considered. [20%]

END OF PAPER

Module 4D7 : Concrete and masonry structures
Formula and Data Sheet

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

Material variability and partial safety factors

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually γ_m is 1.15 for steel, 1.5 for concrete; and γ_f is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$$\left[3.15 \frac{W}{C} - 1.14h \right] / \left[1 + 3.15 \frac{W}{C} \right] \text{ for hydration degree } h : \text{ and gel/space ratio (gel volume / gel + capillaries) } 2.14h / \left[h + 3.15 \frac{W}{C} + a \right]$$

Mechanical properties of concrete

Cracking strain typically 150×10^{-6} , strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

Durability considerations

Present value of some future good : $S_i / (1 + r)^i$ for stepped, or $S_i / \exp(rct_i)$ for continuous discounting.

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\left\{ 2k(H + h_c) / \Delta n \right\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$

solution $c = c_0(1 - \text{erf}(z))$, $z = x / 2\sqrt{Dt}$

Table of erf (z) :

| | | | | | | | | | |
|---------|------|------|------|------|------|------|------|------|----------|
| z | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | |
| erf (z) | 0 | 0.11 | 0.22 | 0.33 | 0.43 | 0.52 | 0.60 | 0.68 | |
| | | | | | | | | | |
| z | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | ∞ |
| erf(z) | 0.74 | 0.80 | 0.84 | 0.88 | 0.91 | 0.93 | 0.95 | 0.97 | 1.00 |

Passivation for pH > 12 and Cl⁻ < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 $\mu\text{A}/\text{cm}^2$, resistivity > 100 k Ω cm, half-cell potential > -200 mV (but probable for < -350 mV).

SLS : cracking

Steel $A_s > k_c k f_{ct,ef} A_{ct}/f_{yk}$ in tension zone, to produce multiple cracks.

Then, limitation to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = \beta \cdot s_{rm} \cdot \epsilon_{sm}$ (β usually 1.7)

where spacing $s_{rm} = 50\text{mm} + 0.25 k_1 k_2 \phi / \rho_r$

with k_1 0.8 for high bond, 1.6 for plain bars; k_2 1.0 for tension
0.5 for bending.

SLS : deflection

Interpolated curvature $\kappa = (1 - \xi) \kappa_{un} + \xi \kappa_{cr}$

$$\text{where } \xi = 1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2$$

β_1 is 1.0 for high bond, 0.5 for plain bars

β_2 is 1.0 for short-term, 0.5 for sustained load

σ_{sr} is steel stress, for cracked section, but using loads which first cause cracking at the section considered. σ_s is current steel stress, calculated for cracked section.

ULS : moment and axial force

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\epsilon_{cm} = 0.0035$. The yield strain of steel ϵ_y of course depends on strength, as roughly f_y/E .

Initial calculations often use uniform stress of 0.6 f_{cd} on the compression zone at failure.

With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d (1 - 0.5 x/d); \quad x/d = \frac{A_s f_y}{0.6 f_{cd} b d};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$$b_w + \frac{l_o}{5} \leq b_{\text{actual}}, \quad \text{where } l_o \text{ is span between zero-moment points.}$$

For long columns, extra deflection prior to material failure is of order

$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$ where κ_m is curvature at mid-height at failure and l_o is effective length. Eurocode multiplies by further factor K , which is 1 for

$$\frac{l_o}{r} > 35, \quad \text{and } \frac{l_o}{20r} - 0.75 \text{ for } 15 \leq \frac{l_o}{r} \leq 35,$$

r being radius of gyration of gross concrete section.

Shear in reinforced concrete

For unreinforced webs at ULS, shear strength in Code is

$$V_{Rd1} = b_w d \left\{ \tau_{Rd} k (1.2 + 40\rho_1) + 0.15 N/A_c \right\}$$

where ρ_1 is A_s/bd for tension steel, τ_{Rd} is tabulated function of f_{cd} , and $k = 1.6 - d$ (metres) ≥ 1 (and is 1 for more than 50% steel curtailment).

In 'standard' design method, for $V_{sd} > V_{Rd1}$

$$V_{Rd} = V_{Rd1} + V_{Rd3} < V_{Rd2} \quad (\text{tabulated in Eurocode})$$

Stirrup term V_{Rd3} follows from truss analogy with 45° struts and "web" depth 90% of effective;

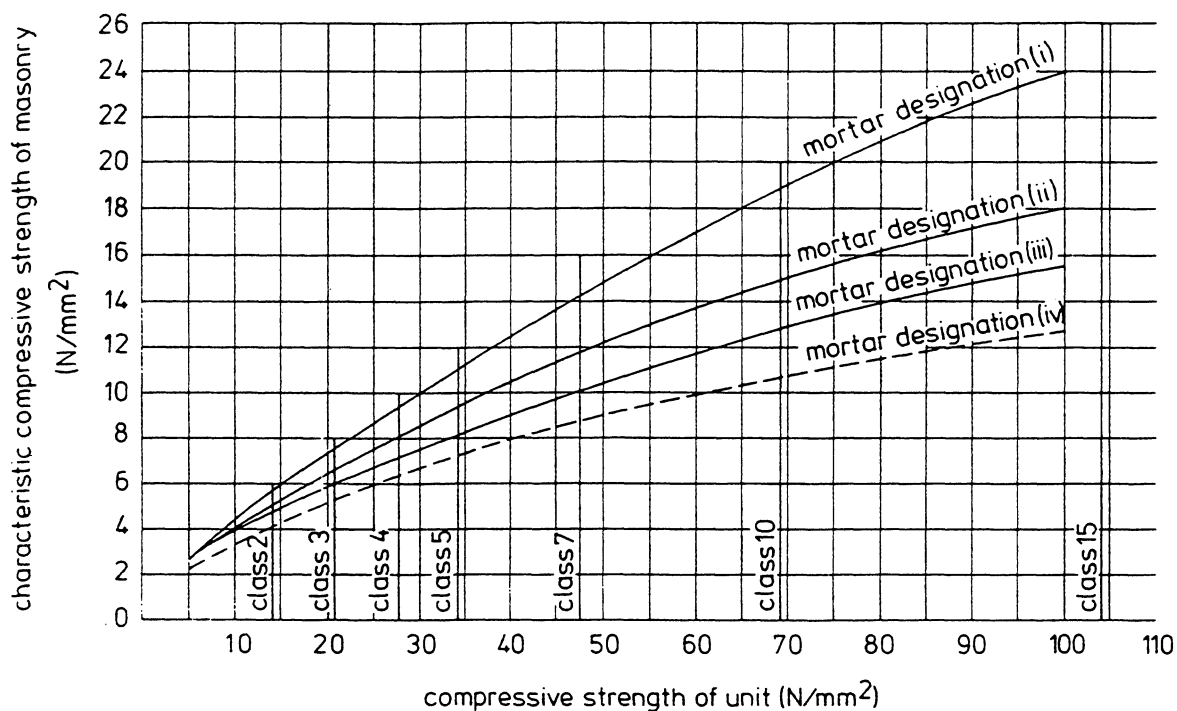
$$V_{Rd3} = A_{sw} f_{wyd} (0.9d)/s.$$

Torsion at ULS

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd} \left\{ (A_w/s) (\sum A_{el}/u) \right\}^{1/2}; \quad \sigma \leq v f_{cd}.$$

Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

Figure 5.6(a) Characteristic compressive strength, f_k , of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\frac{x^2}{2}} dx \quad \text{FOR } 0.00 \leq u \leq 4.99.$$

| <i>u</i> | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7703 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .90147 |
| 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| 1.6 | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| 1.7 | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| 1.8 | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| 1.9 | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| 2.0 | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| 2.1 | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| 2.2 | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| 2.3 | .98928 | .98956 | .98983 | .920097 | .920358 | .920613 | .920863 | .921106 | .921344 | .921576 |
| 2.4 | .921802 | .922024 | .922240 | .922451 | .922656 | .922857 | .923053 | .923244 | .923431 | .923613 |
| 2.5 | .923790 | .923963 | .924132 | .924297 | .924457 | .924614 | .924766 | .924915 | .925060 | .925201 |
| 2.6 | .925339 | .925473 | .925604 | .925731 | .925855 | .925975 | .926093 | .926207 | .926319 | .926427 |
| 2.7 | .926533 | .926636 | .926736 | .926833 | .926928 | .927020 | .927110 | .927197 | .927282 | .927365 |
| 2.8 | .927445 | .927523 | .927599 | .927673 | .927744 | .927814 | .927882 | .927948 | .928012 | .928074 |
| 2.9 | .928134 | .928193 | .928250 | .928305 | .928359 | .928411 | .928462 | .928511 | .928559 | .928605 |
| 3.0 | .928650 | .928694 | .928736 | .928777 | .928817 | .928856 | .928893 | .928930 | .928965 | .928999 |
| 3.1 | .9290324 | .9290646 | .9290957 | .9291260 | .9291553 | .9291836 | .9292112 | .9292378 | .9292636 | .9292886 |
| 3.2 | .9293129 | .9293363 | .9293590 | .9293810 | .9294024 | .9294230 | .9294429 | .9294623 | .9294810 | .9294991 |
| 3.3 | .9295166 | .9295335 | .9295499 | .9295658 | .9295811 | .9295959 | .9296103 | .9296242 | .9296376 | .9296505 |
| 3.4 | .9296631 | .9296752 | .9296869 | .9296982 | .9297091 | .9297197 | .9297299 | .9297398 | .9297493 | .9297585 |
| 3.5 | .9297674 | .9297759 | .9297842 | .9297922 | .9297999 | .9298074 | .9298146 | .9298215 | .9298282 | .9298347 |
| 3.6 | .9298409 | .9298469 | .9298527 | .9298583 | .9298637 | .9298689 | .9298739 | .9298787 | .9298834 | .9298879 |
| 3.7 | .9298922 | .9298964 | .92990039 | .92990426 | .92990799 | .92991158 | .92991504 | .92991838 | .92992159 | .92992468 |
| 3.8 | .92992765 | .92993052 | .92993327 | .92993593 | .92993848 | .92994094 | .92994331 | .92994558 | .92994777 | .92994988 |
| 3.9 | .92995190 | .92995385 | .92995573 | .92995753 | .92995926 | .92996092 | .92996253 | .92996406 | .92996554 | .92996696 |
| 4.0 | .92996833 | .92996964 | .92997090 | .92997211 | .92997327 | .92997439 | .92997546 | .92997649 | .92997748 | .92997843 |
| 4.1 | .92997934 | .92998022 | .92998106 | .92998186 | .92998263 | .92998338 | .92998409 | .92998477 | .92998542 | .92998605 |
| 4.2 | .92998665 | .92998723 | .92998778 | .92998832 | .92998882 | .92998931 | .92998978 | .92999022 | .92999065 | .92999106 |
| 4.3 | .92999146 | .929991837 | .929992199 | .929992545 | .929992876 | .929993193 | .929993497 | .929993788 | .929994066 | .929994332 |
| 4.4 | .929994587 | .929994831 | .929995065 | .929995288 | .929995502 | .929995706 | .929995902 | .929996089 | .929996268 | .929996439 |
| 4.5 | .929996602 | .929996759 | .929996908 | .929997051 | .929997187 | .929997318 | .929997442 | .929997561 | .929997675 | .929997784 |
| 4.6 | .929997888 | .929997987 | .929998081 | .929998172 | .929998258 | .929998340 | .929998419 | .929998494 | .929998566 | .929998634 |
| 4.7 | .929998699 | .929998761 | .929998821 | .929998877 | .929998931 | .929998983 | .9299990320 | .9299990789 | .9299991235 | .9299991661 |
| 4.8 | .9299992067 | .9299992453 | .9299992822 | .9299993173 | .9299993508 | .9299993827 | .9299994131 | .9299994420 | .9299994696 | .9299994958 |
| 4.9 | .9299995208 | .9299995446 | .9299995673 | .9299995889 | .9299996094 | .9299996289 | .9299996475 | .9299996652 | .9299996821 | .9299996981 |

 Example: $\Phi(3.57) = .98215 = 0.9998215.$