UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

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Thursday 29 April 2004 2.30 to 4

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Module 4D8

PRESTRESSED CONCRETE

*Answer* ***one*** *question from Section A and* ***two*** *questions from Section B.*

*Questions from Section A carry* ***twice*** *as many marks as questions from Section B.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

SECTION A

*Answer **one** question*

1 A 20 m long concrete beam of uniform cross-section is simply supported at one end and 6 m in from the other end. It is to carry a live working load of 60 kN/m, which may be placed over any length of the beam. The beam is to be prestressed, with compressive stress in the concrete at working load to be in the range 0 to 22 MPa.

(a) Determine the required magnitude of section modulus for the beam, and suggest initial choices of overall depth and flange and web dimensions. (You may assume that the critical position in the beam is 1.29 m from the middle of the 14 m span.) [20%]

(b) The beam adopted has cross-sectional area $A = 0.40\ \text{m}^2$ and section moduli $Z_t = -0.17\ \text{m}^3$, $Z_b = +0.14\ \text{m}^3$. Show that the moment due to dead load (for concrete weight $24\ \text{kN/m}^3$) at the critical position given in (a) is about 125 kN m, and that the maximum and minimum total moments there are about +1.55 and − 0.51 MN m respectively. Draw a Magnel diagram for the critical section, and determine the maximum and minimum values of the required prestressing force $P$. [50%]

(c) A cable force of 3 MN is adopted. Sketch the allowable range of eccentricity $e$ along the beam, giving values at the ends, at the internal support, and at the critical position. Could the beam sensibly be pre-tensioned? [15%]

(d) The dead load comes to act on the beam at transfer, when the permissible stresses on the concrete are between −1 and +13 MPa. Extend your Magnel diagram for the critical position to cover this situation. Would you expect these further considerations to affect the beam design, given that 25% of the prestressing force at transfer will be lost by the time working loads are added? If the design is indeed affected, what changes would you advocate to meet all the criteria? [15%]

2 A precast concrete beam for a simply-supported bridge of 15 m span has a uniform cross-section in the form of an inverted Tee with the following properties:

overall depth 800 mm, area $0.21 \text{ m}^2$ depth to centroid 500 mm

$Z_t = -0.018 \text{ m}^3$, $Z_b = +0.030 \text{ m}^3$.

The beam is pretensioned, the effective force in the wires in service being 1.5 MN at an eccentricity of 170 mm below the centroid. The concrete weighs $24 \text{ kN/m}^3$.

(a) Find the stresses at midspan in the beam after erection without props, under prestress plus its own weight. [20%]

(b) A top flange 700 mm wide by 200 mm deep is now cast above the precast beam, on lightweight formwork supported by the beam, with shear connectors to ensure composite action after the in-situ concrete has set. The effective Young's modulus of the precast concrete is 35 GPa, and that of the in-situ concrete is 25 GPa . Find the maximum uniformly-distributed live load which can be applied to the whole length of the beam without causing tension in the concrete. Plot the distribution of stress in the concrete at midspan under this loading. [30%]

(c) An engineer points out that although the live load may come on infrequently, creep and shrinkage under the permanent loads and prestress may well alter the stress distribution within the beam at midspan over time. Discuss carefully how you would predict the stress distribution after a very long period of time, writing equations where appropriate and outlining principles, but not necessarily carrying through to a full solution. Assume that after the in-situ flange has set at time $t_o$ the precast unit will shrink a further $\varepsilon_{sp}$ while the in-situ concrete shrinks $\varepsilon_{si}$ where $\varepsilon_{si} > \varepsilon_{sp}$. Also assume that the given values of Young's modulus apply up to time $t_o$, but that between $t = t_o$ and $t \to \infty$ there will be creep factors $\varphi_p$ and $\varphi_i$ in the precast and in-situ respectively, where $\varphi_i > \varphi_p$. Also assume that any change in the prestressing force will be insignificant. [50%]

SECTION B

*Answer **two** questions*

3 Explain why it is desirable to use high strength concrete in prestressed concrete structures. If steel is the material selected for the prestressing tendon, why should it also be of high strength? Discuss the relative advantages and disadvantages of using bonded and unbonded tendons. Would your comments be different if the tendon were of high-strength fibres such as carbon or aramid rather than steel? [100%]

4 A rectangular beam of breadth 200 mm and overall depth 450 mm is made of concrete of characteristic cube strength $f_{cu} = 90$ MPa. It has a prestressing tendon of area 900 $mm^2$ at eccentricity 75 mm below the centre of the cross-section. There is also unprestressed reinforcement of area 800 $mm^2$ and design yield strength 400 MPa, centred 150 mm below the beam centre.

If the tendon has been prestressed to 900 MPa and fully bonded, determine the ultimate (sagging) moment of resistance of the beam. Assume that at failure the maximum compressive strain in the concrete is 0.0032, and the concrete compression zone then has uniform stress $0.4\ f_{cu}$ (including a material safety factor). Neglect any concrete strains during the prestressing operation, and take the tendon stress-strain curve (again with a safety factor) to be tri-linear from zero to 1200 MPa and 0.006 strain, then to 1523 MPa and 0.012 strain, then at constant stress. [80%]

Comment on the advantages and disadvantages of using unprestressed reinforcement in prestressed concrete. [20%]

5 A concrete beam of uniform cross-section has two spans each of length $L$, and is simply-supported at the outer ends and continuous over the central support. Its own weight holds it in contact with all three supports. The beam is prestressed by a tendon of uniform force $P$ and uniform eccentricity $+e_o$ below the beam centroidal axis.

By treating one span as a cantilever of length $L$, and considering how much the end would deflect during the prestressing operation if not supported, or otherwise, determine the line of thrust due to the tendon. [60%]

Find one concordant profile (with non-zero eccentricity) for this beam. What would be the shear force on the concrete, just to one side of the central support, for another tendon with bilinear profile, eccentricity $-e_o$ over the central support, and $+e_o$ at the outer ends? [40%]

**END OF PAPER**