UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 19 April 2004 9 to 10.30

Module 4D12

COASTAL AND OFF-SHORE ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
i) Special datasheets (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 Making use of the solution for velocity potential given in the Data Sheet, show that the kinetic energy per unit surface area of small-amplitude waves of height H is

$$\frac{\rho g H^2}{16},$$

where ρ is the density of water. [35%]

Waves of period 11 s and height 6 m in deep water propagate normally towards a straight coast. How much energy could be obtained using a perfectly efficient wave energy device in water of depth 10 m? [30%]

Why are wave energy devices so rarely used? [35%]

2 (a) Briefly explain the meaning and cause of *wave set up* and *wind set up*. [30%]

(b) A section of breakwater is to be constructed in water of depth 10 m. The bottom contours run North-South. The breakwater is to be designed to withstand waves which in **deep water** are of height 9 m, period 12 s, propagating from the North-East.

(i) Ignoring the effects of wave set up and set down, estimate the wave height at the site of the breakwater. [50%]

(ii) Explain, qualitatively, how the wave height at the breakwater would be changed if wave set up or set down were included in the calculation. [20%]

3 (a) The wind speed (measured 10 m above the water) is 12 m/s. Making use of the wave-prediction curves placed on your desk, determine for deep water and a fetch of 30 km

(i) the maximum significant wave height under these conditions, [10%]

(ii) the corresponding wave period, [10%]

(iii) the minimum length of time for which the wind must blow to achieve this wave height, [10%]

(iv) the significant wave height if the wind only blows for 1 hour. [10%]

(b) What is meant by *significant wave height*? [10%]

(c) Give two examples of situations where use of a significant wave height, rather than the full wave spectrum, might be undesirable. [20%]

(d) Briefly list advantages and disadvantages, compared with other methods, of the use of a pressure transducer on the sea bed to measure wave characteristics. [30%]

(TURN OVER

4 Fig. 1 shows a horizontal seabed (the plane z=0) and an infinite vertical wall (the plane x=0). A cylindrical pile with diameter D stands at a distance from the wall: its axis is the line x=-a, y=0. The still water level is the plane z=d. Small-amplitude waves approach the wall propagating in the positive x-direction. The waves are totally reflected at the wall.

(a) (i) Write down the velocity potential for the incident waves and the reflected waves together, [10%]

(ii) show that the velocity condition at the wall is obeyed, [10%]

(iii) describe the flow qualitatively. [10%]

(b) Assuming that $a=\lambda/4$, where λ is the wavelength, and that D is small in comparison with a, and making use of Morison's equation

(i) find the maximum force on the pile (per unit length of pile) at the still water level, [35%]

(ii) describe qualitatively how this result is altered if D is not small in comparison with a. [15%]

(c) The pile is observed to oscillate in the x-direction. Give two mechanisms that might cause this to happen, explain how you would decide which mechanism is at work, and describe what you would do to reduce the oscillation. [20%]

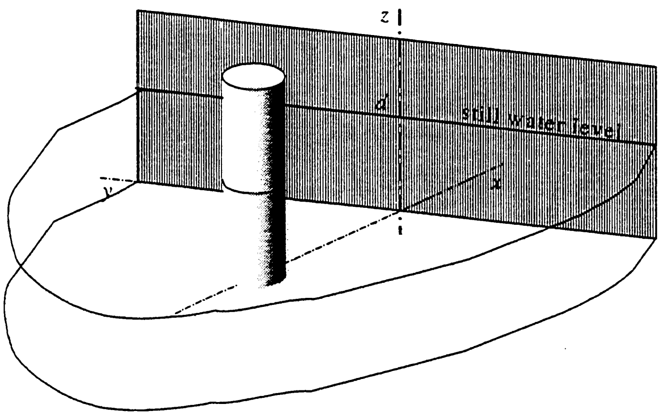
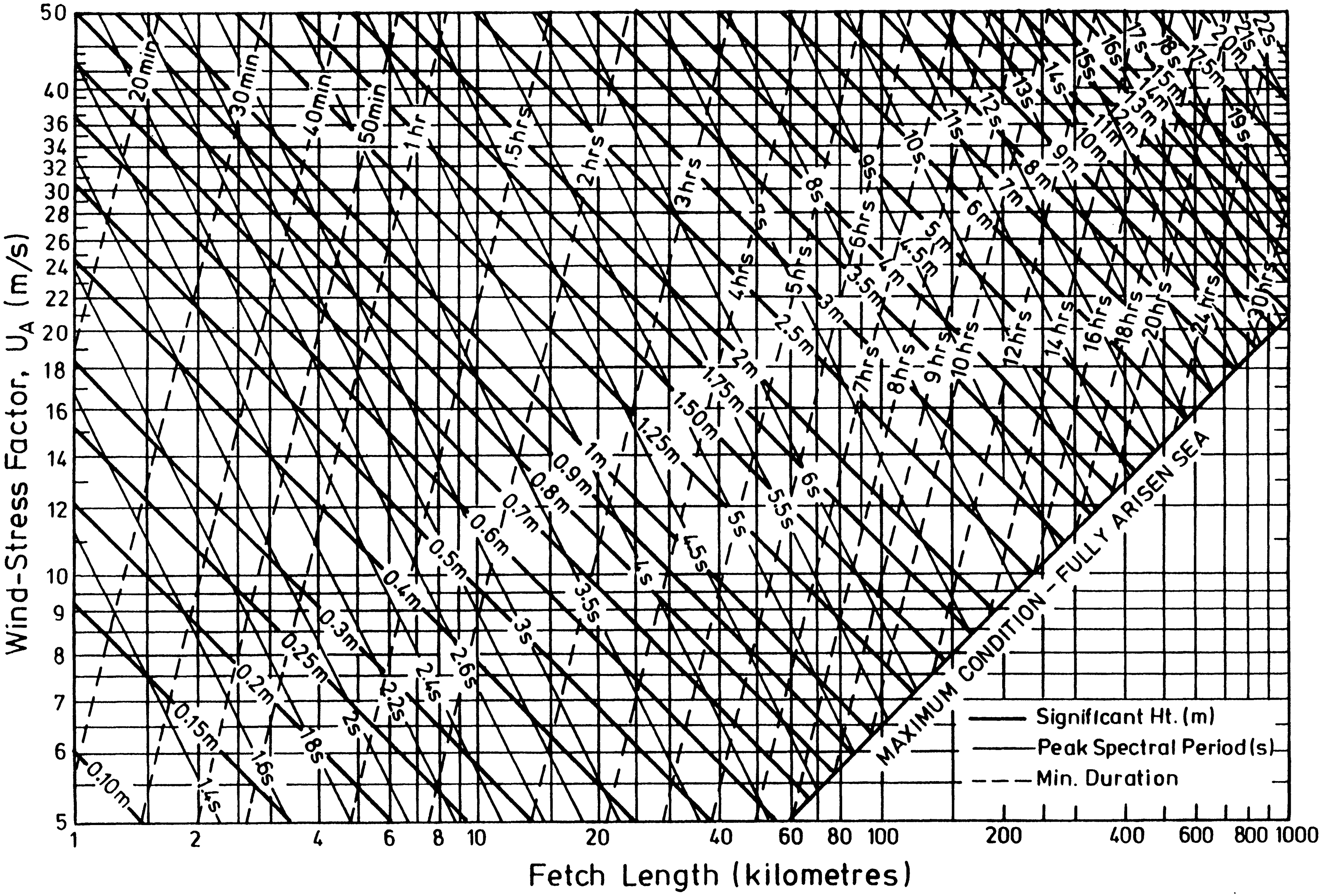


Fig. 1

END OF PAPER

UNIVERSITY OF CAMBRIDGE



Handout n° 8. Deepwater wave prediction curves

Engineering Tripos Part IIB. Module 4D12.

Handout no 4. **WAVE DATA SHEET**

Velocity Potential

$$u = \frac{\partial \phi}{\partial x}$$
$$v = \frac{\partial \phi}{\partial y}$$

Unsteady Bernoulli Equation

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left(u^2 + v^2\right) + gy + \frac{p}{\rho} = const$$

Small-amplitude wave theory

1st order: $$\phi_1 = \frac{\omega H \cosh k(y+d)}{2k \sinh kd}\cos(\omega t - kx)$$

$$p_1 = \frac{\rho g H \cosh k(y+d)}{2\cosh kd}\sin(\omega t - kx) - \rho g y$$

$$c = \frac{\omega}{k} = \left(\frac{g}{k}\tanh kd\right)^{\frac{1}{2}}$$

K.E. (per unit surface area) $= \frac{\rho g H^2}{16}$

P.E. (per unit surface area) $= \frac{\rho g H^2}{16}$

2nd Order: $$\phi_2 = \frac{3H^2 \omega \cosh 2k(y+d) \cos 2(\omega t - kx)}{32 \sinh^4 kd} + (Const)x$$

$$p_2 = \frac{\rho g H^2 k}{8 \sinh 2kd}\left[\left(\frac{3\cosh 2k(y+d)}{\sinh^2 kd} - 1\right)\sin 2(\omega t - kx) + 1 - \cosh 2k(y+d)\right]$$

Group velocity

$$c_g = \frac{d\omega}{dk} = \frac{c}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)$$

Variation of wave height with depth of water

$$\frac{H^2}{k}\left(1 + \frac{2kd}{\sinh 2kd}\right) = \frac{H_0^2}{k_0}$$

$$K_s = \left[\frac{c_0}{c(1 + 2kd / \sinh 2kd)}\right]^{1/2}$$

Refraction

$$\frac{c}{\sin\alpha} = \frac{c_0}{\sin\alpha_0}$$

$$K_r = \left(\frac{\cos\alpha_0}{\cos\alpha}\right)^{1/2}$$

Radiation stress

$$S_{xx} = \frac{\rho g H^2}{8}\left(\frac{1}{2} + \frac{2kd}{\sinh\ 2kd}\right)$$

$$S_{zz} = \frac{\rho g H^2}{8}\left(\frac{kd}{\sinh\ 2kd}\right)$$

Solitary waves

$$\eta = d + H \quad \mathrm{sec}h^2\left[\left(\frac{3H}{4d^3}\right)^{\frac{1}{2}} \quad (x - ct)\right]$$

$$u = H\left(\frac{g}{d}\right)^{\frac{1}{2}} \quad \mathrm{sec}h^2\left[\left(\frac{3H}{4d^3}\right)^{\frac{1}{2}} (x - ct)\right]$$

$$v = \frac{(3H^3 g)^{\frac{1}{2}}\ y}{d^2} \quad \mathrm{sec}h^2\left[\left(\frac{3H}{4d^3}\right)^{\frac{1}{2}} (x - ct)\right] \quad \tanh\left[\left(\frac{3H}{4d^3}\right)^{\frac{1}{2}} (x - ct)\right]$$

$$c = \left(gd + \frac{gH}{2}\right)^{\frac{1}{2}}$$

$$p = \rho g(\eta - y)$$

Wave generation by wind

$$U_A = 0.71\,U_{10}^{1.23}$$

where U is the wind speed in m/s at 10 m above the ground.

For short fetches F and high wind speeds U

$$H_{1/3} = 0.00076\left(U^2F\right)^{\frac{1}{2}}$$

$$T_{1/3} = 0.106\left(U^2F\right)^{\frac{1}{4}}$$

Rayleigh spectrum

The probability p that a given wave will have a height less than H is

$$p = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{H}{\overline{H}}\right)^2\right]$$

$$\overline{H} = 0.625\,H_{1/3} = 0.493\,H_{1/10}$$

Fluid forces on bodies

Morison's equation for a circular cylinder:-

$$F = C_D\,\frac{1}{2}\rho u|u|D + C_M\rho\frac{\pi D^2}{4}\frac{du}{dt}$$

where F is the force/unit length in the flow direction.

Spectral density function

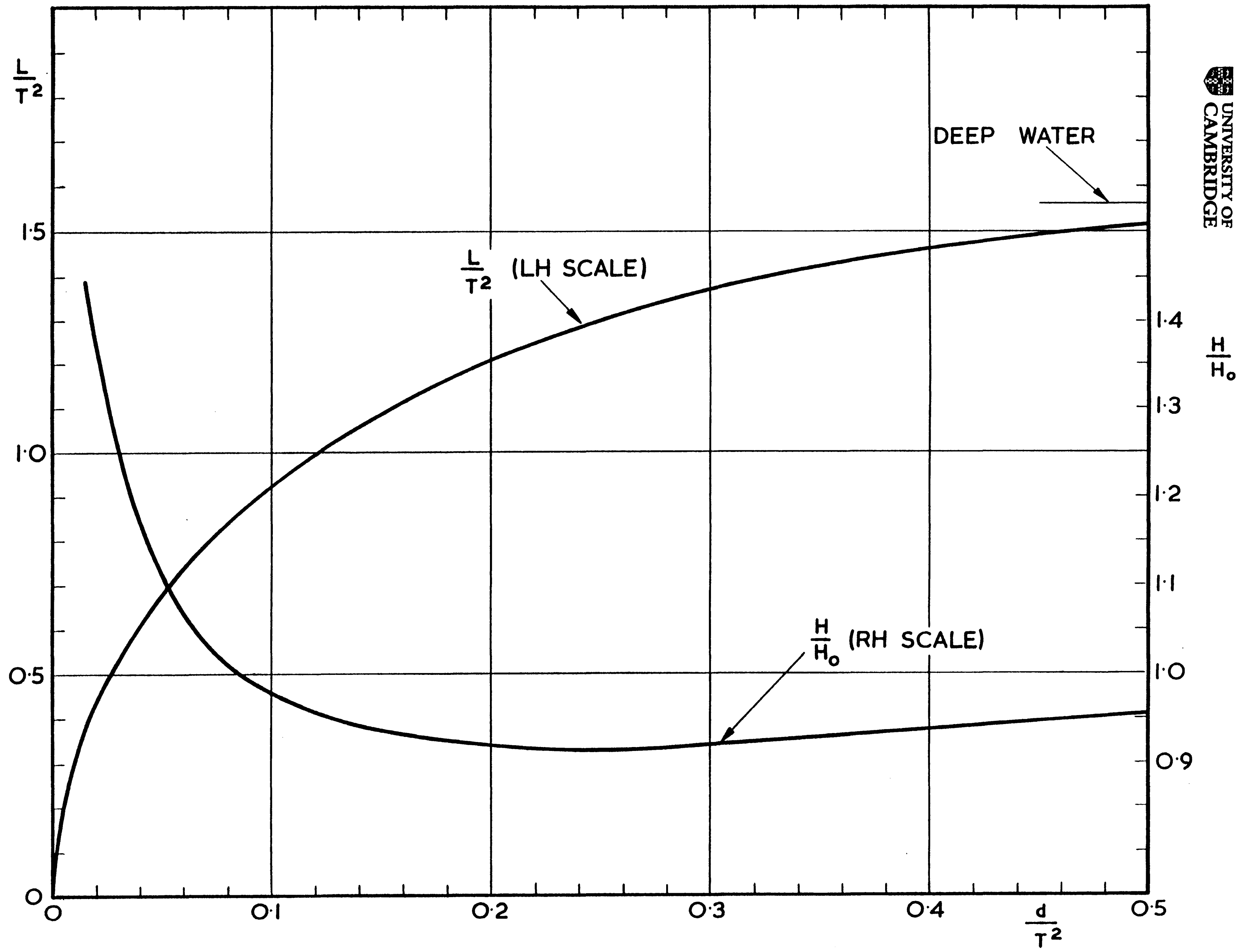
$$S_{uu}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{uu}(\tau)e^{-i\omega\tau}d\tau$$

$$R_{uu}(\tau) = \int_{-\infty}^{\infty} S_{uu}(\omega)e^{i\omega\tau}d\omega$$

S_{uu} is spectral density function, R_{uu} is autocorrelation function.

SYMBOLS

c	wave speed
C_D	drag coefficient
C_M	inertia coefficient
d	depth of water
D	body diameter
D_{65}	grain size for which 65% by weight of grains have a smaller diameter
H	wave height
$H_{1/3}$	mean height of largest 33.3% of waves
k	wave number ($=2\pi/L$).
KC	Keulegan Carpenter number ($=U_{max}T/D$).
K_r	refraction coefficient
K_s	shoaling coefficient
L	wave length
p	pressure
P_l	$=\frac{\rho g}{16}\left(H^2c_g\right)_b \sin 2\theta_b$
t	time
T	wave period
u	horizontal component of fluid velocity
$\bar{u}$	mean velocity
U_A	wind stress factor
v	vertical component of fluid velocity
x,y	co-ordinates (y is vertical)
α	angle between wave crest and bottom contour
ϕ	velocity potential
ρ	density of fluid
ρ_s	density of sediment
τ_o	shear stress on the bed
τ_c	critical value of τ_o for sand movement
ω	wave frequency
<u>subscript</u> o	indicates conditions in deep water
<u>subscript</u> b	indicates conditions at the first row of breakers.



VARIATION OF WAVE LENGTH AND HEIGHT WITH DEPTH