UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS   PART IIB

Monday 19 April 2004   2.30 to 4.00

Module 4F6

SIGNAL DETECTION AND ESTIMATION

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 a) Describe, in detail, how *Maximum Entropy* methods may be used to assign probability distributions. [30%]

b) Given the first moment of a distribution (from experimental measurements, for example) show, using Lagrange multipliers, that the distribution having *Maximum Entropy* is an Exponential distribution. [40%]

c) Show that the entropy of this distribution is:

$$H = \ln \sigma + 1$$

where σ is the standard deviation of the distribution. [30%]

2 a) Define *Fisher Information* and show that for an unbiased estimator, $\hat{\theta}(x)$, of a parameter θ, the variance associated with $\hat{\theta}(x)$ satisfies

$$var(\hat{\theta}(x)) \geq I_\theta^{-1}$$

where I_θ is the *Fisher Information* for the scalar parameter θ. [40%]

b) Derive the following condition for an efficient unbiased estimator:

$$\frac{\partial \ln p(x|\theta)}{\partial \theta} = I_\theta(\hat{\theta}(x) - \theta)$$

where $p(x|\theta)$ is the likelihood function and $\hat{\theta}(x)$ is an estimator for θ. [30%]

c) Describe how this equation leads to the *Neyman-Fisher* factorization theorem. [30%]

3 a) Define the terms *Error of the first kind* and *Error of the second kind* explaining the role they play in Detection Theory and explain the role played by the *log-likelihood ratio.* [40%]

b) In an M-ary digital transmission system the source signal can take one of M possible levels during a symbol period. In each symbol period the detector makes N measurements $\mathbf{y} = [y_1\, y_2 \ldots y_N]^T$ of the channel output.

i) Show that the Maximum A-Posteriori (MAP) decision rule for the detector may be expressed as: [30%]

$$\text{Choose } H_i \text{ if } \max_{H_j}\{p(\mathbf{y}|H_j)\, P(H_j)\} = p(\mathbf{y}|H_i)\, P(H_i)\,.$$

ii) Show that the average error probability P_e for the detector is given by: [30%]

$$P_e = 1 - \sum_{i=1}^{M} P(D_i|H_i)\, P(H_i)$$

where:

$p(\mathbf{y}|H_i)$ is the probability density of the observation vector $\mathbf{y}$ conditional on hypothesis H_i being in force;

$P(H_i)$ is the *a-priori* probability of hypothesis H_i;

$P(D_i|H_i)$ is the probability of deciding in favour of hypothesis H_i when H_i is in force.

4 a) Describe, in detail, the Neyman-Pearson decision rule applied to detection theory and discuss the advantages and disadvantages of this decision rule over the MAP and Bayes criteria. [30%]

b) Show that the value of threshold for the Neyman-Pearson test for a single observation is given by the slope of the receiver operating characteristic (ROC) at the required false alarm probability. [30%]

c) Based on n statistically independent Gaussian samples with variance σ^2, determine the likelihood ratio test to choose between the hypotheses that their mean is zero (H_0) or that their mean is one (H_1). [40%]

END OF PAPER