UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 19 April 2004 9 to 10.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) In general, a filter obtained by inverse Fourier transforming some desired zero-phase two-dimensional (2D) frequency response will not have finite support. Explain how the *windowing method* is used for creating finite support digital filters and indicate how the actual frequency response then differs from the ideal frequency response. Discuss the *product* and *rotation* methods of forming a 2D window function from 1D window functions. [25%]

(b) Consider the following Hamming-type 1D windows

$$w_k(u) = \begin{cases} \alpha + \beta \cos\left(\dfrac{\pi u}{U_k}\right) & \text{if } |u| < U_k \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } k = 1, 2$$

Show that the spectrum of the 2D Hamming window formed by the product of two 1D Hamming windows, $w_1(u_1)$ and $w_2(u_2)$, of the form given above, takes the following form

$$W(\omega_1, \omega_2) = f(\omega_1, U_1)\ \text{sinc}(\omega_1 U_1)\ f(\omega_2, U_2)\ \text{sinc}(\omega_2 U_2)$$

Find the functional form of f and sketch the spectrum for values of $\alpha = 0.54$ and $\beta = 0.46$ that are used in the standard Hamming window. [60%]

(c) Discuss the properties we look for in a good windowing function. In forming 2D window functions from 1D window functions, what are the relative merits of the two methods described in part (a) of this question? [15%]

Note:

$$\int_{-U}^{U} \cos\left(\frac{\pi u}{U}\right) e^{-j\omega u}\, du = \frac{2\,\omega\, U^2}{\pi^2 - \omega^2 U^2} \sin(\omega U)$$

2 (a) An observed image, $y(u_1, u_2)$, may often be expressed as a linear distortion of the original image, $x(u_1, u_2)$, plus noise, $d(u_1, u_2)$, as follows

$$y(u_1, u_2) = \int\int h(v_1, v_2)\, x(u_1 - v_1, u_2 - v_2)\, dv_1\, dv_2 \; + \; d(u_1, u_2) \tag{1}$$

where $h(v_1, v_2)$ is the point-spread function of the distorting system.

Rewrite the above equation in discrete form and explain how, in the absence of noise, one can use the Fourier transform to act as an *inverse filter*, i.e. extract the original image, given knowledge of x and h. Discuss the performance of such inverse filters, commenting also on their behaviour in the presence of noise in the image. [35%]

(b) The discrete form of equation (1) can be written in vector notation, e.g. $x(\mathbf{n}) = x(n_1, n_2)$ etc. It is desired to find the best estimate, $\hat{x}(\mathbf{n})$, in a least squares sense, of $x(\mathbf{n})$, under the assumption that $\hat{x}(\mathbf{n})$ can be calculated as

$$\hat{x}(\mathbf{n}) = \sum_{\mathbf{q} \in \mathbf{Z}^2} g(\mathbf{q})\, y(\mathbf{n} - \mathbf{q})$$

where $g(\mathbf{q})$ is the impulse response of a suitable 2D filter. Show, by differentiating with respect to g, that the Fourier transform, $\hat{G}(\boldsymbol{\omega})$, of the optimum filter, $\hat{g}(\mathbf{q})$, is given by

$$\hat{G}(\boldsymbol{\omega}) = \frac{P_{yx}(\boldsymbol{\omega})}{P_{yy}(\boldsymbol{\omega})}$$

where $P_{yy}(\boldsymbol{\omega})$ is the power spectrum of $y(\mathbf{n})$ and $P_{yx}(\boldsymbol{\omega})$ is the cross power spectrum of $x(\mathbf{n})$ and $y(\mathbf{n})$. State clearly any assumptions you make during the derivation. [50%]

(c) Hence obtain, without detailed derivation, the Wiener filter form of $\hat{G}(\boldsymbol{\omega})$ in terms of the frequency response of h, $H(\boldsymbol{\omega})$, and the power spectra of x and d, $P_{xx}(\boldsymbol{\omega})$ and $P_{dd}(\boldsymbol{\omega})$; and indicate when the Wiener filter tends to the inverse filter. [15%]

3 (a) The transform matrix for a one-dimensional 4-point discrete cosine transform (DCT) may be expressed in the form

$$\mathbf{T} = \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix}$$

where $a = 0.5$, $b = 0.6533$ and $c = 0.2706$. Show that this is an orthonormal matrix.

Explain how $\mathbf{T}$ may be used to transform a 4×4 matrix of pixels $\mathbf{X}$ into $\mathbf{Y}$, the two-dimensional DCT of $\mathbf{X}$. [25%]

(b) An image contains a vertical edge, and a 4×4 pixel subimage containing the edge is given by

$$\mathbf{X} = \begin{bmatrix} p & p & p & q \\ p & p & p & q \\ p & p & p & q \\ p & p & p & q \end{bmatrix}$$

where the pixel values are $p = 20$ and $q = 80$. Calculate the 2-D DCT $\mathbf{Y}$ for this subimage in terms of a, b, c, p, q. [25%]

(c) To compress this subimage, it is desired to select only two coefficients from $\mathbf{Y}$ for encoding. Discuss, with reasons, which coefficients should be selected to achieve this with minimum squared error in the reconstructed image, and explain why $\mathbf{T}$ being orthonormal assists in this choice. [25%]

(d) Calculate the rms error that will arise between the input subimage $\mathbf{X}$ and the subimage that would be reconstructed from just the two retained coefficients, if these two coefficients are quantised with a uniform quantiser whose reconstruction levels are $25n$, for integer n. [25%]

4 (a) The basis of wavelet transforms is the two-band filter bank, shown in Fig. 1. Explain how these filters may be used in a two-level wavelet transform and inverse transform for one-dimensional signals, and show how this concept may be extended for processing two-dimensional images. [25%]

(b) Explain the meaning of the term *perfect reconstruction* and show that the system of Fig. 1 can achieve this property if

$$G_0(z)H_0(z) + G_1(z)H_1(z) = 2$$

where $\quad H_1(z) = z^{-1}G_0(-z) \quad$ and $\quad G_1(z) = zH_0(-z)\,.$ [25%]

(c) If the analysis and reconstruction lowpass filters are given by

$$H_0(z) = -\tfrac{1}{4}z^2 + \tfrac{1}{2}z + \tfrac{3}{2} + \tfrac{1}{2}z^{-1} - \tfrac{1}{4}z^{-2} \qquad \text{and} \qquad G_0(z) = az + b + cz^{-1}\,,$$

calculate a, b and c to obtain perfect reconstruction. [25%]

(d) The designer of an image compression system is able to choose whether or not to swap the H and G filters, calculated in part (c). Discuss the effect of this choice on the reconstructed image quality and determine, for this case, whether the filters should be swapped. [25%]

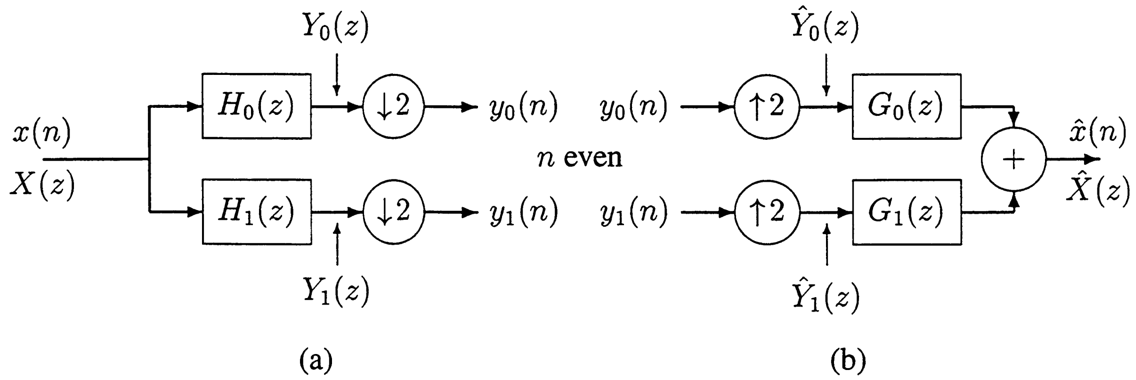

Fig. 1

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