UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Wednesday 21 April 2004 9 to 10.30

Module 4F9

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer not more than **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Medical Image Acquisition*

Answer not more than ***one*** *question from this section*

1 (a) What do the acronyms SPECT and PET stand for? List the relative strengths and weaknesses of the two imaging modalities. [20%]

(b) Figure 1 shows axial and side views of a PET detector ring. The ring has diameter 1 m and thickness 0.1 m. An ^{18}F isotropic point source emits photons at an expected rate of 10^7 pairs per second from the point A at the centre of the ring.

(i) Assuming no attenuation, show that the expected number of pairs arriving at the detector ring is approximately 10^6 s^{-1}. [10%]

(ii) Now assume uniform attenuation within the shaded volume, with linear attenuation coefficient 0.1 cm^{-1}. Calculate the expected number of pairs arriving at the detector ring. State any assumptions you make. [20%]

(c) Each detector module subtends an angle of 10° at the centre of the ring, as shown in Fig. 1. Suppose a photon pair hits modules B and C at time t. The pair must be rejected if another photon hits the ring within 10 ns, or another photon hits either B or C within 300 ns.

(i) Assuming no attenuation, calculate the individual probabilities of these two events occurring. Hence, estimate the expected number n_d of successfully detected pairs per second. State any assumptions you make. [20%]

(ii) Sketch a curve showing how n_d varies with the intensity of the radioactive source. [10%]

(d) A PET scanner can operate in 2D or 3D mode. With reference to your answers to parts (b) and (c), discuss the relative advantages and disadvantages of these two modes. [20%]

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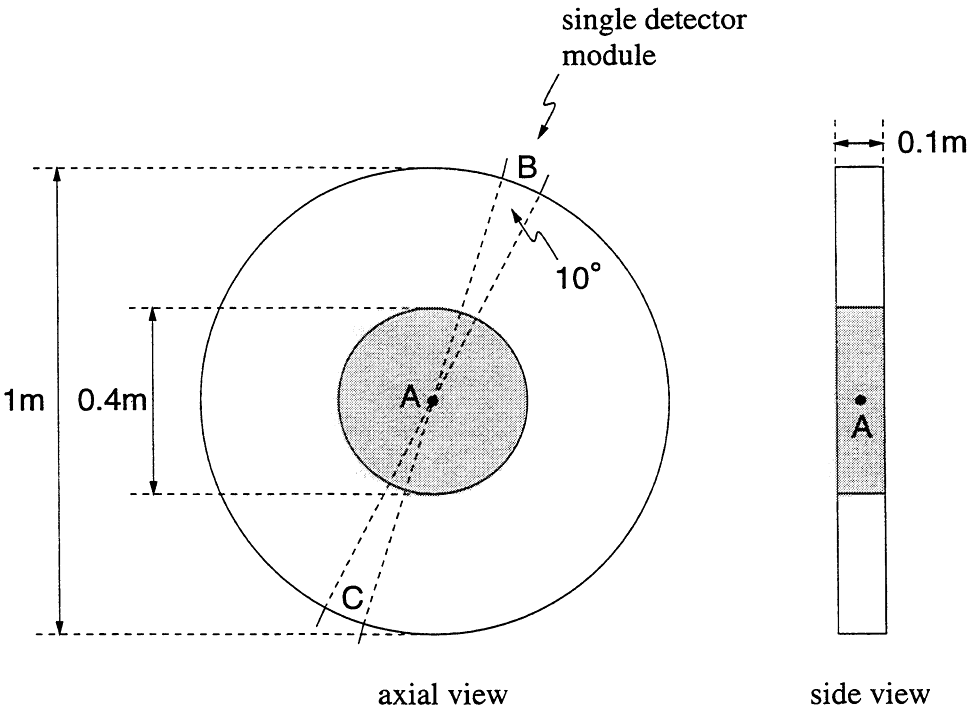

Fig. 1

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2 (a) Define the two-dimensional Radon transform and explain its role in X-ray computed tomography. [20%]

(b) A function $f(x, y)$ is zero everywhere, except at the point $(r\cos\theta, r\sin\theta)$, where it is infinite — see Fig. 2.

(i) Sketch $p(s, \phi)$, the Radon transform of $f(x, y)$. [10%]

(ii) Derive an equation for the curve along which $p(s, \phi)$ is non-zero. [20%]

(c) The inverse Radon transform can be written as

$$f(x,y) = \int_0^{\pi} [p_\phi(s) * q(s)]\, d\phi \,.$$

(i) Write down an expression for the filter kernel $q(s)$. [10%]

(ii) Explain how the inverse Radon transform gives rise to the filtered backprojection reconstruction algorithm. [20%]

(iii) Explain why $q(s)$ is unattainable in practice, and sketch the frequency response of an alternative, band-limited filter. [10%]

(iv) How is the cut-off frequency of the band-limited filter related to the width of the X-ray beam? [10%]

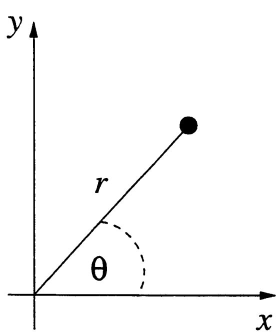

Fig. 2

SECTION B *Curves, Surfaces and Interpolation*

Answer not more than ***one*** *question from this section*

3 (a) A software engineer is trying to choose between Hermite curves, Bézier curves, B-splines and Catmull-Rom splines for a computer graphics application. Discuss the factors which might influence this choice. [40%]

(b) Consider a curve represented by two cubic B-spline segments which are defined by five control points $\mathbf{P}_0 \ldots \mathbf{P}_4$, where $\mathbf{P}_2 = \mathbf{P}_3$ as shown in Fig. 3. The B-spline basis matrix is given by

$$\mathrm{M}_s = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}.$$

Use the convex hull property to show that the join point between the two segments must lie somewhere on the line between $\mathbf{P}_1$ and $\mathbf{P}_2$. Evaluate the spline at the join point to find its precise location in terms of $\mathbf{P}_1$ and $\mathbf{P}_2$. [25%]

(c) Extend this line of reasoning to deduce what happens to the curve when $\mathbf{P}_1 = \mathbf{P}_2 = \mathbf{P}_3$. What continuity is there at the join point? [25%]

(d) Does C^n continuity always imply G^n continuity? Justify your answer. [10%]

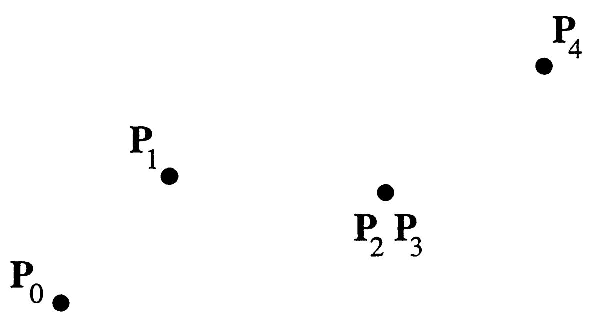

Fig. 3

4 Before treating a patient with radiotherapy, the clinician first has to determine exactly what should be irradiated. In this process, known as *radiotherapy planning*, the patient receives a 3D CT scan, and the clinician delineates the target area in each CT slice. Since the radiotherapy system is not perfectly accurate, a safety margin is added to this initial target volume to ensure sufficient coverage. The following procedure is suggested for determining the enlarged target volume.

- For each parallel CT slice, draw around the target area and then calculate the 2D distance transform of the slice, using city-block distances.
- To account for the large CT slice spacing, interpolate the transformed slices onto closely spaced, parallel planes forming a regular voxel array.
- Extract an isosurface at a non-zero value appropriate for the safety margin.

(a) For one particular slice, the initial target area is a rectangle 4 cm wide and 1 cm high. The safety margin is 2 cm: all points outside the enlarged target area must be at least this far from the initial contour.

(i) For this slice, sketch the enlarged target area which would be generated by the procedure suggested above. On the same sketch, draw the minimum target area required for a 2 cm safety margin. [20%]

(ii) What is the percentage increase in the area which will be treated, over that which is strictly necessary given the safety margin? [20%]

(iii) Repeat (i) using a chamfer-code in place of the city-block distance. Comment on the effect of this modification. [20%]

(b) After interpolation and extraction of the isosurface, the clinician notices that some points *outside* the enlarged target volume lie within 2 cm of the initial contours. The procedure is therefore modified to apply a 3D distance transform *after* interpolation onto the voxel array. The revised procedure is as follows.

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- For each parallel CT slice, draw around the target area.
- To account for the large CT slice spacing, interpolate the contours onto closely spaced, parallel planes forming a regular voxel array.
- Calculate the 3D distance transform of the volume, using city-block distances.
- Extract an isosurface at a non-zero value appropriate for the safety margin.

(i) Other than the city-block estimate of distance, what was wrong with the original procedure, and how does this modification put it right? [20%]

(ii) If shape-based interpolation is used in the interpolation step, comment on the computational expense of the original and revised procedures. [20%]

SECTION C *3D Graphical Rendering*

*Answer not more than **one** question from this section*

5 (a) The basic Phong reflection model can be written as

$$I_\lambda = c_\lambda(I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s \left(\mathbf{R}.\mathbf{V}\right)^n \ .$$

Explain the meanings of the various terms. Show how the model can be extended to account for multiple light sources, depth cueing and shadows. [40%]

(b) An acute triangle has unnormalised vertex normals

$$\begin{bmatrix} -0.3 \\ 0.1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0.3 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0.1 \\ -0.4 \\ 1 \end{bmatrix}.$$

The light source is at infinity in the direction $[0\ -1\ 1]^T$ and the viewer is at infinity in the direction $[0\ 1\ 1]^T$. Using the basic Phong reflection model given in (a), with $I_a = I_p = k_s = c_b = 1$, $k_a = k_d = 0.5$, $c_r = c_g = 0$ and $n = 1000$, and assuming Phong shading, calculate the rendered intensity at the triangle's circumcentre. Speculate on the likely outcome if Gouraud shading were used instead. [40%]

(c) Explain how the computational expense of rendering the triangle depends on the choice of shading algorithm and the orientation of the triangle with respect to the pixel array. [20%]

6 (a) Explain what is meant by the computer graphics term *rasterisation*. [15%]

(b) A rectangle of area four pixels has its vertices at pixel coordinates (0,0), (0,4), (1,4) and (1,0). The rectangle is part of a mesh which is to be processed by a standard surface rendering pipeline. At one stage of the pipeline, non-horizontal edges of the rectangle are rasterised using the following algorithm:

$$
\begin{aligned}
& m = (x_2 - x_1)/(y_2 - y_1); \\
& \texttt{for } y = y_1 \texttt{ to } y_2 \texttt{ do } \{ \\
& \quad x = x_1 + m(y - y_1); \\
& \quad \mathit{output}(\mathit{round}(x),\ y); \\
& \}
\end{aligned}
$$

where (x_1, y_1) and (x_2, y_2) are the edge's integer end points, $y_2 > y_1$ and the function *round()* rounds a real number to the nearest integer.

(i) Sketch the rectangle and the local pixel array on graph paper, and indicate which pixels would be output by the edge rasterisation algorithm. [10%]

(ii) On a separate sketch, indicate which pixels are subsequently shaded to render the rectangle. Verify that the rendered area is four pixels. [15%]

(iii) The rendering is part of an animated sequence, in which the rectangle subsequently appears rotated 45° clockwise around pixel (0,0), but otherwise undistorted. Find the new locations of the rectangle's vertices, round them to the nearest pixels and hence sketch the rendered appearance of the rotated rectangle. [30%]

(c) With reference to (b), discuss the problem of aliasing in computer graphics. Suggest how aliasing artifacts might be suppressed using (i) an intermediate framebuffer with resolution several times higher than the final framebuffer, or (ii) a floating point rendering pipeline, which calculates sub-pixel edge representations. [30%]

END OF PAPER