

Q1

(a) From the 4A1 Data Sheet the general neutron diffusion equation is

$$\frac{dn}{dt} = -\nabla \cdot \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

In steady state $\frac{dn}{dt} = 0$; in a non-multiplying medium $\eta = 0$; for a source-free system $S = 0$.

$$\therefore 0 = -\nabla \cdot \underline{j} - \Sigma_a \phi$$

Substituting for \underline{j} using $\underline{j} = -D\nabla\phi$

$$\therefore 0 = \nabla \cdot (D\nabla\phi) - \Sigma_a \phi$$

If D is constant

$$\therefore 0 = D\nabla^2\phi - \Sigma_a \phi$$

Defining $L^2 = \frac{D}{\Sigma_a}$ (L is the diffusion length)

$$\therefore 0 = \nabla^2\phi - \frac{\phi}{L^2}$$

For spherical symmetry, from the 4A1 Data Sheet: $\nabla^2 = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right)$

$$\therefore 0 = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) - \frac{\phi}{L^2}$$

$$\text{Now } \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = \frac{1}{r^2} \left(2r \frac{d\phi}{dr} + r^2 \frac{d^2\phi}{dr^2} \right) = \frac{1}{r} \left(2 \frac{d\phi}{dr} + r \frac{d^2\phi}{dr^2} \right)$$

$$\text{and } \frac{1}{r} \left(\frac{d^2(\phi r)}{dr^2} \right) = \frac{1}{r} \left(\frac{d}{dr} \left[\phi + r \frac{d\phi}{dr} \right] \right) = \frac{1}{r} \left(2 \frac{d\phi}{dr} + r \frac{d^2\phi}{dr^2} \right)$$

$$\therefore 0 = \frac{1}{r} \left(\frac{d^2(\phi r)}{dr^2} \right) - \frac{\phi}{L^2}$$

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(b) The result from (a) can be rewritten as

$$\left(\frac{d^2(\phi r)}{dr^2} \right) - \frac{\phi r}{L^2} = 0$$

By inspection this differential equation has a general solution

$$\phi r = A \exp\left(\frac{r}{L}\right) + B \exp\left(-\frac{r}{L}\right)$$

From physical considerations as $r \rightarrow \infty$, $\phi \rightarrow 0$. As $\exp(r/L)$ increases faster than r , this means $A = 0$

$$\therefore \phi r = B \exp\left(-\frac{r}{L}\right)$$

At $r = R_1$, $\phi = \phi_1$

$$\therefore \phi_1 R_1 = B \exp\left(-\frac{R_1}{L}\right)$$

$$\therefore B = \phi_1 R_1 \exp\left(\frac{R_1}{L}\right)$$

$$\therefore \phi = \frac{B}{r} \exp\left(-\frac{r}{L}\right) = \frac{\phi_1 R_1}{r} \exp\left(\frac{R_1}{L}\right) \exp\left(-\frac{r}{L}\right) = \frac{\phi_1 R_1}{r} \exp\left(\frac{R_1 - r}{L}\right)$$

[30%]

(c) For a multiplying medium $\eta > 0$, therefore the diffusion equation is now

$$\therefore 0 = D\nabla^2\phi + (\eta - 1)\Sigma_a\phi$$

$$\therefore 0 = \nabla^2\phi + B_m^2\phi$$

where $B_m^2 = \frac{(\eta - 1)\Sigma_a}{D}$ is the material buckling

So with spherical symmetry

$$\therefore 0 = \frac{1}{r} \left(\frac{d^2(\phi r)}{dr^2} \right) + B_m^2\phi$$

If $k_\infty > 1$ then $B_m^2 > 0$, therefore $\frac{d^2(\phi r)}{dr^2} + B_m^2\phi r = 0$ has a general solution

$$\phi r = A \sin(B_m r) + C \cos(B_m r)$$

$$\therefore \phi = \frac{A}{r} \sin(B_m r) + \frac{C}{r} \cos(B_m r)$$

When $r = 0$, the flux ϕ must be finite $\therefore C = 0$

$$\therefore \phi = \frac{A}{r} \sin(B_m r)$$

The flux ϕ will not be zero at the physical edge of the reactor, $r = R_2$ say, because of neutron leakage. The standard boundary condition is to assume that the flux falls to zero at a small distance outside the reactor, at $r = R_2 + \delta$ say. Using this boundary condition

$$\therefore 0 = \frac{A}{R_2 + \delta} \sin(B_m [R_2 + \delta])$$

$$\therefore B_m [R_2 + \delta] = \pi$$

$$\therefore R_2 = \frac{\pi}{B_m} - \delta$$

[45%]

Q2

(a) The major simplifications of this model are that it assumes there is no spatial variation in behaviour, whereas in practice the reactor core is highly heterogeneous and the neutron population varies spatially, and it also assumes that there is only one type of precursor, whereas in reality there are a large number of them with widely varying production rates and half-lives.

[10%]

(b) In steady-state operation $\frac{dc}{dt} = 0$

$$\therefore \frac{\beta}{\Lambda} n_0 = \lambda c_0 \Rightarrow \frac{c_0}{n_0} = \frac{\beta}{\lambda \Lambda} = \frac{0.0075}{0.1 \times 10^{-4}} = 750$$

[10%]

(c) For a source-free system the governing equations are

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c \quad (1)$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c \quad (2)$$

Taking Laplace transforms (with p as the transform variable)

$$p\bar{n} - n_0 = \frac{\rho - \beta}{\Lambda} \bar{n} + \lambda \bar{c} \quad (3)$$

$$p\bar{c} - c_0 = \frac{\beta}{\Lambda} \bar{n} - \lambda \bar{c} \quad (4)$$

Now, from (b),

$$c_0 = \frac{\beta}{\lambda \Lambda} n_0$$

Substituting for c_0 in (4)

$$\therefore \bar{c}(p + \lambda) = \frac{\beta}{\lambda \Lambda} (\lambda \bar{n} + n_0)$$

Then for \bar{c} in (3)

$$\therefore \bar{n} \left(p + \frac{\beta - \rho}{\Lambda} \right) = n_0 + \frac{\beta (\lambda \bar{n} + n_0)}{\Lambda (p + \lambda)}$$

$$\therefore \bar{n} \left(p + \frac{\beta - \rho}{\Lambda} \right) (p + \lambda) = n_0 \left(p + \lambda + \frac{\beta}{\Lambda} \right) + \frac{\beta \lambda}{\Lambda} \bar{n}$$

$$\therefore \bar{n} \left(p^2 + \left[\lambda + \frac{\beta - \rho}{\Lambda} \right] p + \frac{\beta \lambda}{\Lambda} - \frac{\rho \lambda}{\Lambda} - \frac{\beta \lambda}{\Lambda} \right) = n_0 \left(p + \lambda + \frac{\beta}{\Lambda} \right)$$

To find the system time constants solve

$$p^2 + \left[\lambda + \frac{\beta - \rho}{\Lambda} \right] p - \frac{\rho \lambda}{\Lambda} = 0$$

$$\therefore p^2 + \left[0.1 + \frac{0.0075 - 0.005}{10^{-4}} \right] p - \frac{0.005 \times 0.1}{10^{-4}} = 0$$

$$\therefore p^2 + 25.1p - 5 = 0 \Rightarrow p = 0.19765 \text{ or } -25.29765 \text{ s}^{-1}$$

Therefore the dominant time constant (the positive one) is $T_+ = \frac{1}{p_+} = \frac{1}{0.19765} = 5.0594 \text{ s}$

In the *prompt jump approximation* the neutron population is assumed to stay in equilibrium with the precursor population.

So if $\frac{dn}{dt} = 0$, (1) gives that $\frac{\rho - \beta}{\Lambda} n + \lambda c = 0 \Rightarrow n = \frac{\Lambda \lambda}{\beta - \rho} c$

Substituting for n in (2)

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} \left(\frac{\Lambda \lambda}{\beta - \rho} \right) c - \lambda c = \frac{\beta \Lambda \lambda - \lambda \Lambda (\beta - \rho)}{\Lambda (\beta - \rho)} c = \frac{\rho \lambda}{\beta - \rho} c$$

By inspection $p = \frac{\rho \lambda}{\beta - \rho} = \frac{0.005 \times 0.1}{0.0075 - 0.005} = 0.2 \text{ s}^{-1}$

$$\therefore T_+ = \frac{1}{p_+} = \frac{1}{0.2} = 5.0 \text{ s}$$

Thus, the prompt jump approximation gives a good estimate of the exact time constant. Importantly, from a safety point of view, the prompt jump approximation underestimates T_+ . Thus, the estimate is *conservative* – the real system will respond slower than predicted.

[60%]

(d) Delayed neutrons play a vital role in determining T_+ , and thus the controllability of a reactor. Delayed neutrons will be present for both the AGR and the FBR. If we assume that the values of β and λ are the same as for the PWR – this is a reasonable assumption for the AGR as it is also a thermal reactor, but less good for the FBR in which fission conditions are significantly different due to the fast neutron spectrum – then the value of T_+ will not vary enormously between reactors and any variation will depend on the value of Λ , the prompt neutron reproduction time. Λ is determined by the effectiveness of the moderator.

In the AGR moderation is less effective than in a PWR (water is a better moderator than graphite) and therefore Λ will be longer, and in consequence T_+ will be slightly longer.

In the FBR there is no moderation (by design) and therefore Λ is much (three orders of magnitude) shorter. This will result in T_+ being slightly shorter than for the PWR.

Note that the prompt jump approximation predicts no variation in T_+ with Λ .

[20%]

Q3

- (a) Assume that the effect of a mixture of batches of different burnups τ_i can be modelled by the *partial reactivity model*, so that the overall reactivity ρ is given by

$$\rho = \sum_i \rho'_i(\tau_i)$$

where the partial reactivity of an individual batch is given by

$$\rho'_i(\tau_i) = \frac{1}{M} \rho_0 \left(1 - \frac{\tau_i}{T_1} \right)$$

Assume that fuel is replaced on a first-in, first-out basis, and that the end-of-cycle (EOC) condition is that $\rho = 0$.

At the end of the first cycle all M batches have accumulated burnup τ_1 , therefore the EOC condition gives

$$\begin{aligned} \sum_{i=1}^M \frac{1}{M} \rho_0 \left(1 - \frac{\tau_1}{T_1} \right) &= 0 \\ \therefore \frac{M}{M} \rho_0 \left(1 - \frac{\tau_1}{T_1} \right) &= 0 \Rightarrow \tau_1 = T_1 \end{aligned}$$

At the end of the second cycle $(M-1)$ batches have accumulated burnup $(\tau_1 + \tau_2)$ and one batch has accumulated burnup τ_2 , therefore the EOC condition gives

$$\begin{aligned} \frac{M-1}{M} \rho_0 \left(1 - \frac{\tau_1 + \tau_2}{T_1} \right) + \frac{1}{M} \rho_0 \left(1 - \frac{\tau_2}{T_1} \right) &= 0 \\ \therefore M - \frac{(M-1)\tau_1 + M\tau_2}{T_1} &= 0 \\ \therefore M\tau_2 &= MT_1 - (M-1)\tau_1 \end{aligned}$$

Substituting for τ_1 $\therefore M\tau_2 = MT_1 - (M-1)T_1 = T_1 \Rightarrow \tau_2 = \frac{T_1}{M}$

At the end of the M -th cycle the oldest batch has accumulated burnup $(\tau_1 + \tau_2 + \dots + \tau_M)$, the second oldest $(\tau_2 + \tau_3 + \dots + \tau_M)$ etc. and the most recently added batch has accumulated burnup τ_M , therefore the EOC condition gives

$$\begin{aligned} \frac{1}{M} \rho_0 \left(1 - \frac{\tau_1 + \tau_2 + \dots + \tau_M}{T_1} \right) + \frac{1}{M} \rho_0 \left(1 - \frac{\tau_2 + \tau_3 + \dots + \tau_M}{T_1} \right) + \dots + \frac{1}{M} \rho_0 \left(1 - \frac{\tau_M}{T_1} \right) &= 0 \\ \therefore 1 - \frac{\tau_1 + 2\tau_2 + \dots + M\tau_M}{MT_1} &= 0 \\ \therefore \tau_M &= T_1 - \frac{1}{M} \sum_{i=1}^{M-1} i\tau_i \end{aligned} \quad (1)$$

[25%]

(b) In steady-state operation all cycles are of length τ_∞ , so setting $\tau_i = \tau_M = \tau_\infty$ in equation (1)

$$\tau_\infty + \frac{1}{M} \sum_{i=1}^{M-1} i\tau_\infty = T_1$$

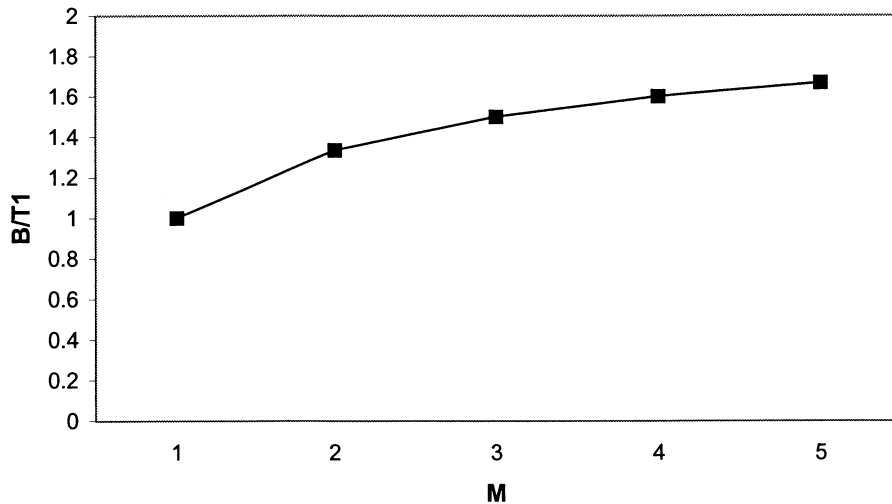
$$\therefore M\tau_\infty + \sum_{i=1}^{M-1} i\tau_\infty = \sum_{i=1}^M i\tau_\infty = MT_1$$

$$\therefore \frac{1}{2}M(M+1)\tau_\infty = MT_1 \Rightarrow \frac{\tau_\infty}{T_1} = \frac{2}{M+1}$$

Each batch resides M cycles, so the accumulated burnup B is given by

$$B = M\tau_\infty = \frac{2MT_1}{M+1}$$

It is convenient to plot $\frac{B}{T_1} = \frac{2M}{M+1}$ against M



This plot shows that to increase fuel utilisation (B_M) it is desirable to use a large number of batches. In the limit as $M \rightarrow \infty$, $B_M \rightarrow 2T_1$. Large numbers of batches will be operationally acceptable if the reactor can be refuelled at power; if it has to be shutdown to be refuelled then frequent refuelling outages will lead to unacceptable loss of availability.

[30%]

(c) Under the refuelling conditions now specified the oldest batch in the reactor at each refuelling has accumulated burnup $(m+1)\tau_\infty$ and is of size $N_0 - mN = fN$ assemblies. Younger batches are all of size N . The steady-state EOC condition is now

$$\frac{fN}{N_0} \rho_0 \left(1 - \frac{(m+1)\tau_\infty}{T_1}\right) + \frac{N}{N_0} \rho_0 \left(1 - \frac{m\tau_\infty}{T_1}\right) + \frac{N}{N_0} \rho_0 \left(1 - \frac{(m-1)\tau_\infty}{T_1}\right) + \dots + \frac{N}{N_0} \rho_0 \left(1 - \frac{\tau_\infty}{T_1}\right) = 0$$

$$\therefore f \left(1 - \frac{(m+1)\tau_\infty}{T_1}\right) + \left(1 - \frac{m\tau_\infty}{T_1}\right) + \left(1 - \frac{(m-1)\tau_\infty}{T_1}\right) + \dots + \left(1 - \frac{\tau_\infty}{T_1}\right) = 0$$

$$\therefore m + f - \frac{\tau_{\infty}}{T_1} \left(f(m+1) + \sum_{i=1}^m i \right) = 0$$

$$\therefore m + f = \frac{\tau_{\infty}}{T_1} \left(f(m+1) + \frac{1}{2} m(m+1) \right)$$

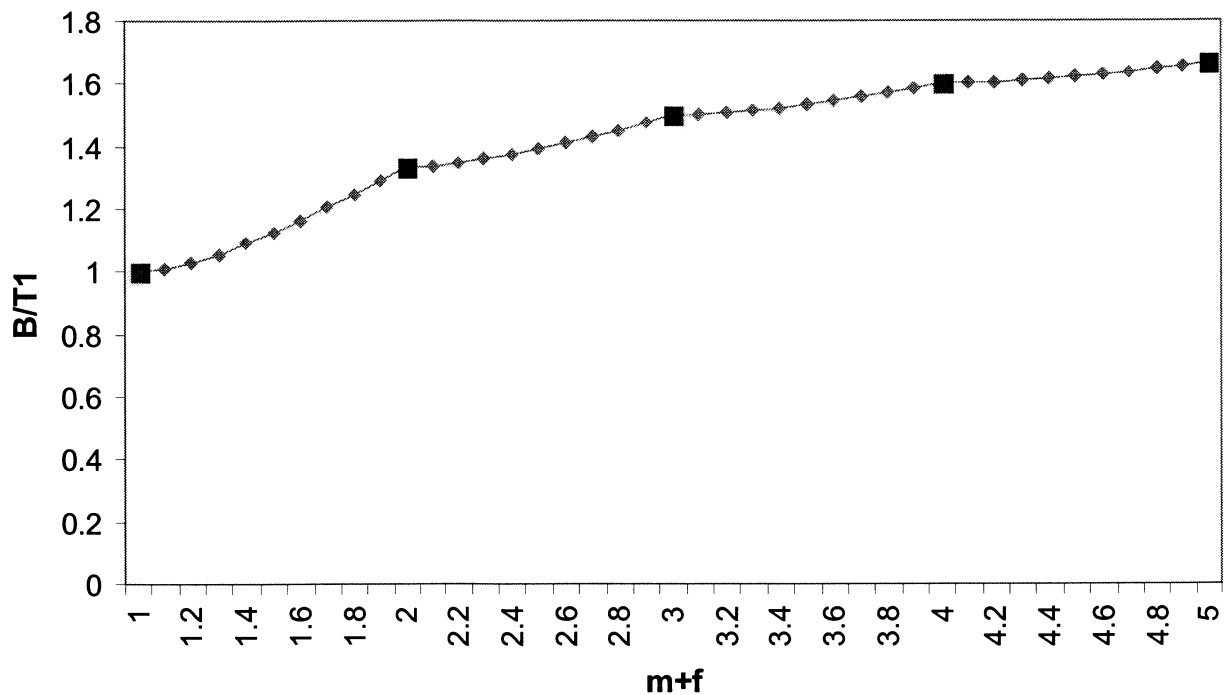
$$\therefore \frac{\tau_{\infty}}{T_1} = \frac{2(m+f)}{(m+1)(m+2f)} \Rightarrow \tau_{\infty} = \frac{2(m+f)T_1}{(m+1)(m+2f)}$$

The average dwell time of a batch is $N_0/N = m + f$, hence

$$B = (m+f)\tau_{\infty} = \frac{2(m+f)^2 T_1}{(m+1)(m+2f)}$$

[30%]

(d) The resulting plot of B against $(m+f)$ is as shown below (done with more accuracy than required). The key feature is that the line “drapes” over the points corresponding to integer batch strategies (as found in (b)). B still increases monotonically with $(m+f)$ but the relative performance improvement is less for non-integer strategies.



[15%]

Q4

(a) The three basic methods are:

- 1) Hold up and decay
- 2) Ion exchange
- 3) Evaporation

Hold up and decay involves simply storing the wastes in tanks etc. until the nuclides decay down to an acceptable level. It is particularly suited to short half life nuclides and has the advantage of not requiring complex plant and equipment and not simply concentrating wastes, thus giving rise to another disposal/storage problem. It is obviously not suited to very long half life nuclides unless they are first concentrated by one of the other processes to reduce storage space. It is also, in general, the cheapest method and gives rise to the lowest operator dose uptake. It can be used for either gases or liquids, but in the case of gases some form of compression or adsorption is required, again to reduce volumes.

Ion exchange is only suited to liquid wastes and is really a form of concentration in which the effluent stream is passed through a bed of material (the media) which selectively adsorbs the active nuclides. By careful selection of ion exchange media it is possible to bring about very significant reductions in the activity of liquid streams. Both organic and inorganic media are used, and, unlike most ion exchange processes, no attempt is made to regenerate the spent ion exchange media. Current practice is to encapsulate the spent media in cement and place the encapsulated material in a long-term storage repository, though some work is in progress on alternative methods of treatment of organic media. Ion exchange is relatively cheap, having low energy and labour costs, the main problem (and cost) being the disposal of the spent ion exchange media. Operator dose levels are generally low, though the spent media can be very active.

Evaporation is, again, only suited to liquid wastes and can achieve very high degrees of decontamination. The design of the evaporators must always be a compromise between energy efficiency and operational and maintenance problems. Most nuclear waste evaporators are kept relatively simple, which reduces operator dose uptake at the expense of energy costs. Thus they tend to be very expensive to operate and are normally only used when there is no alternative. The concentrate stream must be encapsulated for long-term storage. The actual waste volumes are much lower than those arising from ion exchange, but the energy costs and operator dose uptakes are much higher.

Other methods of treatment include filtration and chemical treatment. Most gaseous and liquid streams are filtered before treatment and the spent filter cartridges are disposed of by encapsulation. Chemical treatment may be simple neutralisation to control the pH before discharge or the next stage of treatment, or it may include complex chemical precipitation techniques followed by filtration and the encapsulation of the resulting precipitate.

Most solid wastes are encapsulated in concrete to immobilise the radioactivity, though the very highly active wastes arising from reprocessing are converted into glass by a process of vitrification. Very low activity wastes can be disposed of in suitable containers in controlled, engineered landfill. At the moment most solid wastes are stored on the surface in purpose-built repositories until long-term disposal strategies are developed.

[60%]

(b)
$$P = \frac{A_I Q \rho}{\lambda_I}$$

where A_I is the inlet activity = $24 \text{ Bq g}^{-1} = 24 \times 10^3 \times 3600$
 $= 8.64 \times 10^7 \text{ disintegrations kg}^{-1} \text{ hr}^{-1}$

Q is the flow rate ($0.063 \text{ m}^3 \text{ hr}^{-1}$)

ρ is the effluent density (1000 kg m^{-3})

λ_I is the decay constant = $\frac{\ln 2}{T_{1/2}} = \frac{0.693}{5.26 \times 8760} = 1.504 \times 10^{-5} \text{ hr}^{-1}$

$$\therefore P = \frac{8.64 \times 10^7 \times 0.063 \times 10^3}{1.504 \times 10^{-5}} = 3.619 \times 10^{14} \text{ atoms hr}^{-1}$$

Atoms accumulated
$$N_I = \frac{P[1 - \exp(-\lambda T)]}{\lambda}$$

where T is the filling time = $\frac{V}{Q} = \frac{15}{0.063} = 238 \text{ hr}$

V being the hold-up tank volume

$$\therefore N_I = \frac{3.619 \times 10^{14} (1 - \exp(-1.504 \times 10^{-5} \times 238))}{1.504 \times 10^{-5}} = 8.60 \times 10^{16} \text{ atoms}$$

The outlet activity is given by

$$A_O = \frac{\lambda_I N_I}{\rho V} = \frac{1.504 \times 10^{-5} \times 8.60 \times 10^{16}}{10^3 \times 15} = 8.62 \times 10^7 \text{ disintegrations kg}^{-1} \text{ hr}^{-1}$$

$$= \frac{8.62 \times 10^7}{10^3 \times 3600} = 23.94 \text{ Bq g}^{-1}$$

Note how the hold-up process has hardly any effect due to the very long half life of Ni-60 – substantial partial credit available for observing this and skipping the above calculations.

The final treatment has a decontamination factor of 10, so the final activity of the effluent is

$$A_F = \frac{A_O}{DF} = \frac{23.94}{10} = 2.39 \text{ Bq g}^{-1}$$

[40%]