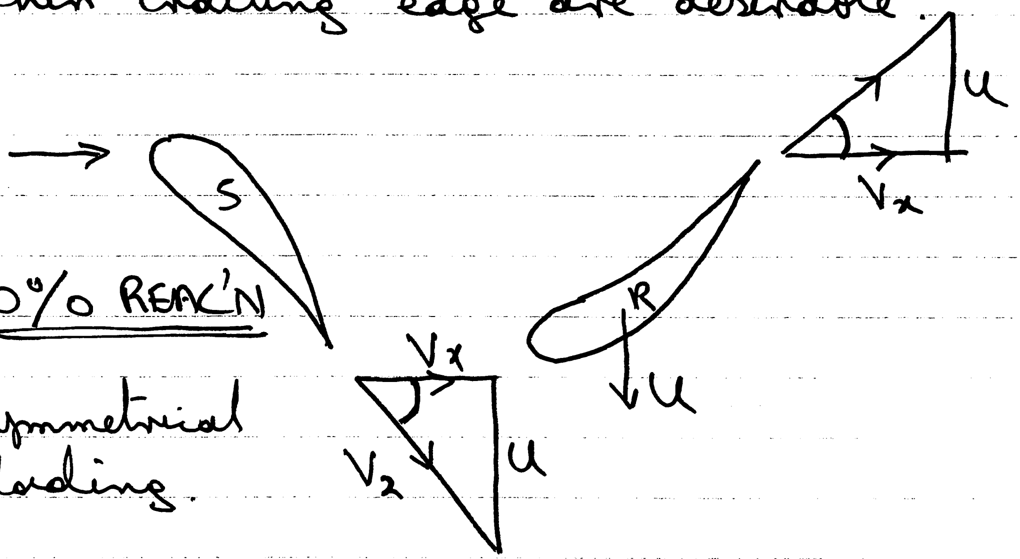
Q1

PAPER 4A3 2005
SOLUTIONS.

MARKS OUT OF (25) PER Q.

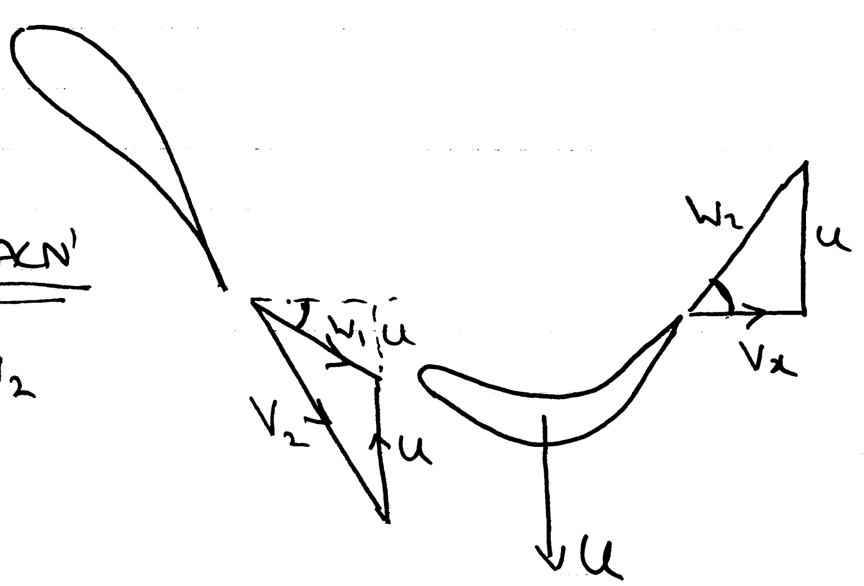
Q1a) The loss coefficient rises rapidly as $M_2 = 1.0$ is approached, hence the efficiency tends to fall. Most of the increase in loss is due to the trailing edge, hence blades with a thin trailing edge are desirable. (2)

b)



50% REAC'N

Symmetrical blading.



0% REAC'N

$W_1 = W_2$

For 50 % reacn $\Delta h_0 / U^2 = 1$

and $U = V_2 \sin \alpha_2$

For zero reacn. $\Delta h_0 / U^2 = 2$

and $U = 0.5 V_2 \sin \alpha_2$

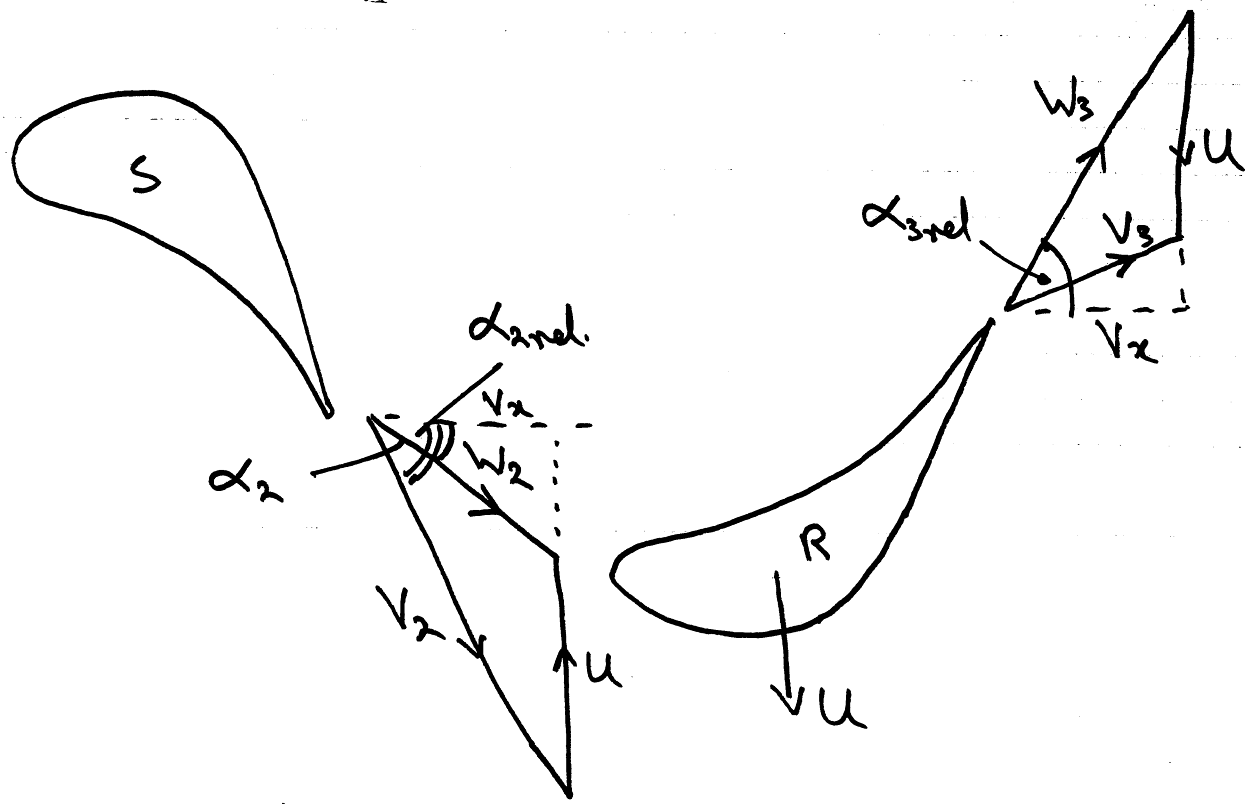
$$\therefore \frac{\Delta h_0}{V_2^2} = \sin^2 \alpha_2 \quad \text{for } \lambda = 50\%$$

Q1b con't

and $\frac{\Delta h_0}{V_2^2} = 0.5 \sin^2\alpha_2$ for 0% Λ.

So for a situation where V_2 is limited to $M_2 = 1$ a 50% reaction design gives twice the loading of a zero reaction design. (4)

1c)



d) $\phi = \frac{V_x}{U} = 0.4$, $\alpha_2 = 75°$

$\tan\alpha_{2rel} = \tan\alpha_2 - \frac{1}{\phi} \rightarrow \alpha_{2rel} = \underline{50.93°}$

$V_x = V_2 \cos 75° = .2588\, V_2$

$W_2 = V_x / \cos\alpha_{2rel} = 0.4106\, V_2$

$M_2 = 1$, $\therefore M_{2rel} = \underline{0.4106}$

$T_2 = \frac{T_{02}}{1 + \frac{\gamma-1}{2} M_2^2} = .8333\, T_{02}$ at $M_2 = 1$

$T_{02rel} = T_2\left(1 + \frac{\gamma-1}{2} M_{2rel}^2\right) = 0.8614\, T_{02}$

Q1 cont'd

Since $M = 1$ at exit from both blade rows

$$\frac{W_3}{V_2} = \sqrt{\frac{T_{02rel}}{T_{02}}} = \sqrt{.8614}$$

$$\rightarrow \quad \underline{\underline{W_3 = 0.9281\ V_2}}$$

$$\frac{W_3}{V_x} = \frac{.9281\ V_2}{V_x} = \frac{.9281}{\cos 75^\circ} = 3.586$$

$$\rightarrow \quad \alpha_{3rel} = \cos^{-1}\frac{1}{3.586} = \underline{\underline{73.8^\circ}}$$

$$V_{\theta 3} = V_x \tan 73.8^\circ - U$$

$$= V_x(3.444 - 2.5) = .9437\ V_x$$

$$V_{\theta 2} = V_x \tan 75^\circ = 3.732\ V_x$$

$$\Delta V_\theta = (3.732 - .9437)V_x = 2.788\ V_x$$

Stage loading coeff $= \dfrac{\Delta h_0}{U^2} = \dfrac{\Delta V_\theta}{U}$

$$= 2.788\frac{V_x}{U} = 2.788 \times 0.4 = \underline{\underline{1.115}} \qquad 8$$

d)
$$Y_P = \frac{P_{01} - P_{02}}{P_{02} - P_2} = \frac{\Delta P_0}{P_{02}(1 - P_2/P_{02})}$$

At $M_2 = 1$, for $\gamma = 1.4$ this gives

$$\Delta P_0 / P_{02} = 0.4717\ Y_P.$$

$$T_0\, dS = C_p\, dT_0 - \frac{1}{\rho_0} dP_0$$

Adiabatic flow - $dT_0 = dT_{0rel} = 0$

Q1d cont'd $\quad \therefore ds = -R \dfrac{dP_o}{P_o}$

For small losses $dP_o / P_o \simeq -\Delta P_o / P_{o2}$ ③

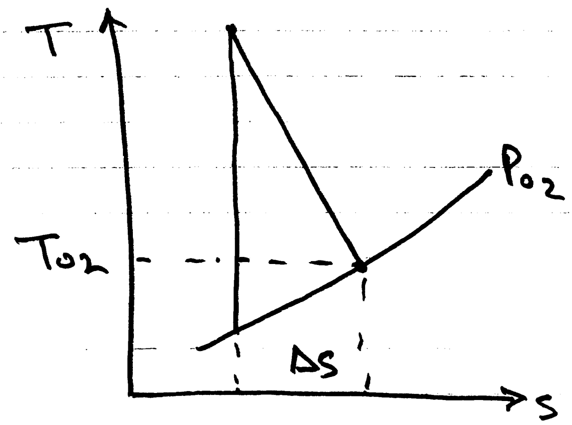
$\rightarrow \quad \Delta S = + R \times \cdot 4717 \, Y_p$

For air $R = 287.5$

$$\underline{\underline{\Delta S = 135.6 \, Y_p}} \quad J/Kg/K$$

Same for both blade rows.

e)



Lost work $= T_{o2} \Delta S$

2 blade rows

$$\text{Lost work} = \left(T_{o1} - \frac{1.115 \, U^2}{C_p}\right) \times 2 \times 135.6 \times Y_p$$

$$\text{Actual work} = 1.115 \, U^2$$

$$\text{At } M_2 = 1, \quad V_2 = \sqrt{C_p T_{o1}} \times 0.5773$$

$$\therefore \ U/\sqrt{C_p T_{o1}} = \cdot 5773 \times 2.5 \times \cos 75^\circ$$

$$= 0.3736$$

$$\therefore \text{Actual work} = 0.1556 \, C_p T_{o1}$$

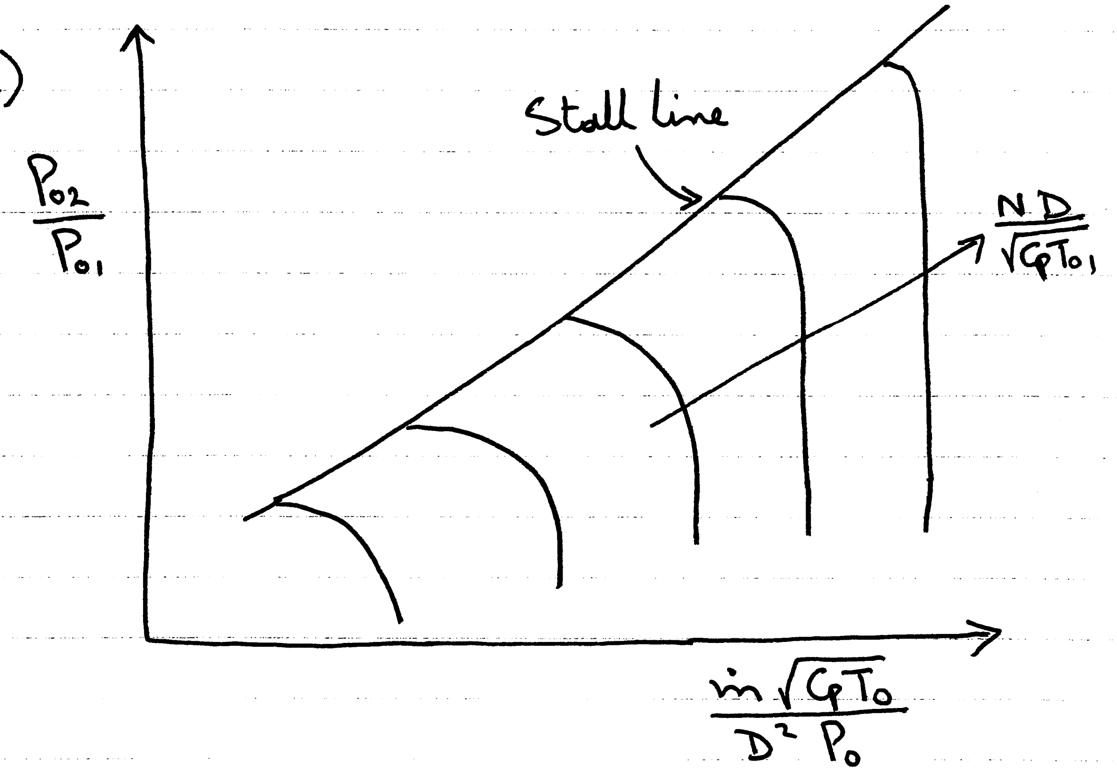
$$\text{Lost work} = T_{o1}(22.9)$$

$$= C_p T_{o1} \times \cdot 02278.$$

$$\therefore \eta_{TT} = \frac{\cdot 1556}{\cdot 1556 + \cdot 02278}$$

$$\eta_{TT} = \underline{\underline{87.2\%}}$$ ⑧

Q2 a)



There are two reasons why the const speed lines are so steep at high speeds.

i) The range of mass flows for the multi-stage is the same as for a single stage but the range of pressure ratios is r^N where r is the pressure ratio per stage and N the number of stages

ii) There is a "snowballing effect". An increase in $\dot{m}$ for stage 1 $\rightarrow$ a decrease in P_0 entering stage 2 $\rightarrow$ a larger change in $\dot{m}\sqrt{T_0}/P_0$ for stage 2 than for stage 1 $\rightarrow$ an even larger decrease in P_0 entering stage 3, etc.

3

Q2 cont'd

b) $\dfrac{\dot{m}\sqrt{C_p T_{01}}}{A\, P_{01}}$ will be the same

$$\therefore \quad \frac{100\sqrt{288}}{1} = \frac{\dot{m}\sqrt{308}}{0.9}$$

$\rightarrow \quad \dot{m} = \underline{\underline{87.03}}$ kg/sec.

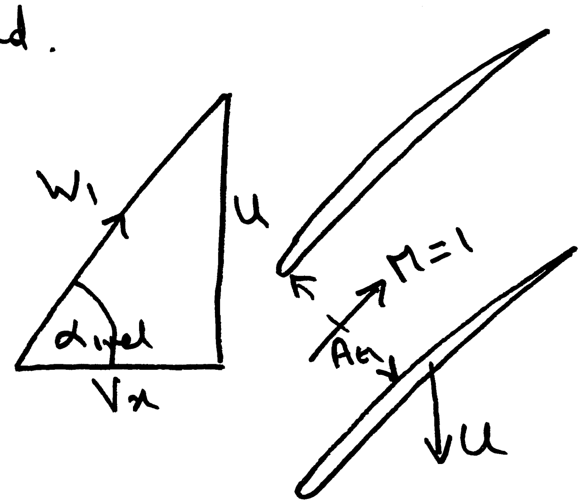
$\dfrac{\Delta T_0}{T_{01}}$ will also be the same

$$\frac{\dot{m}_1\, \Delta T_{01}}{\dot{m}_2\, \Delta T_{02}} = \frac{100 \times 288}{87.03 \times 308}$$

Ratio of powers $= \underline{\underline{0.931}} = \dfrac{\text{new}}{\text{orig}}$ (4)

c) – See end.

d)



$$T_{01rel} = T_1 + \tfrac{1}{2}U^2/C_p + \tfrac{1}{2}V_x^2/C_p.$$
$$= T_{01} + \tfrac{1}{2}U^2/C_p.$$

$$T_{01rel}/T_{01} = 1 + \tfrac{1}{2}U^2/C_pT_{01} = (1+G)$$

Since no losses before throat

$$P_{01rel}/P_{01} = (1+G)^{\frac{\gamma}{\gamma-1}}.$$

The throat is choked so $\dfrac{\dot{m}\sqrt{C_p T_{01rel}}}{A_t\, P_{01rel}} = F(1)$

Q2 d-cont)

ie
$$\frac{\dot{m}\sqrt{C_pT_{01}}}{A_t P_{01}} \times \frac{(1+G)^{1/2}}{(1+G)^{\frac{\gamma}{\gamma-1}}} = F(1)$$

$$\dot{m} = F(1).A_t.\frac{P_{01}}{\sqrt{C_pT_{01}}}(1+G)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (7)$$

For the inlet flow

$$F(M_x) = \frac{\dot{m}\sqrt{C_pT_{01}}}{s\ P_{01}}, \quad s = \text{pitch}$$

$$\therefore \quad F(M_x) = F(1).\frac{A_t}{s}.(1+G)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (4)$$

For $A_t/s = 0.5$, $u/\sqrt{C_pT_{01}} = 0.6$

$$G = \tfrac{1}{2} \times 0.6^2 = .18$$

$$F(M_x) = 1.281 \times 0.5 \times 1.643 = 1.0524$$

Tables $\rightarrow$ $M_x = 0.578$ (taking subsonic solution)

It is tempting to try to calculate M_u from the given value of $u/\sqrt{C_pT_0}$ but this is wrong because u is not the velocity of the flow.

$M_x = 0.578 \rightarrow T_1/T_{01} = 0.9374$

$$M_u = \frac{u}{\sqrt{\gamma R T_1}} = \frac{u}{\sqrt{C_pT_0}} \times \sqrt{\frac{T_0}{T_1}} \times \sqrt{\frac{C_p}{\gamma R}}$$

$$M_u = \frac{u}{\sqrt{C_pT_{01}}} \times 1.6331 = 0.9798 \quad (4)$$

$$\tan\alpha_{1rel} = \frac{M_u}{M_x} = 1.695, \quad \alpha_{1rel} = \mathbf{59.46^\circ}$$

Q2c) The useful operating range is limited by stall at +ve incidence and by either stall or choking at -ve incidence.

The amount of diffusion on a given blade increases with Mach no. according to $dV/V = 1/(1-M^2)\, dA/A$ and so the incidence to produce both +ve and -ve incidence stalling reduces as M increases.

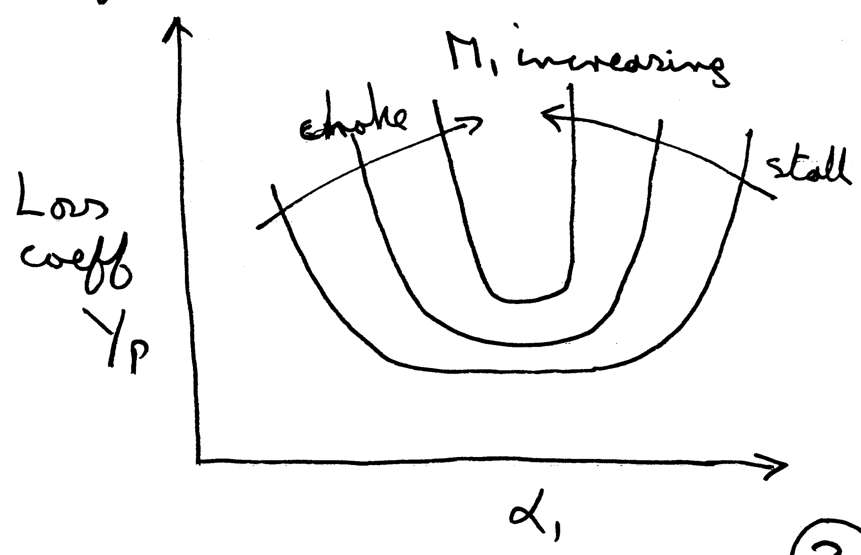
Also, at -ve incidence choking occurs when:

$$F(M_1) = F(1) \times t/s \cos\alpha_1$$

$$\text{ie} \quad \cos\alpha_1 = \frac{F(1)}{F(M_1)} \times \frac{t}{s}$$

so the higher the value of $F(M_1)$ the larger the value of α_1 at which choking occurs and so the less the -ve incidence before choking.

Typically



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Q3

a) At high rotational speed the compressor produces a higher pressure rise and density rise than the annulus was designed for. ∴ The axial velocity at the rear of the compressor is low, blade speed is high and so the blades have +ve incidence and are likely to stall. The blade height is small so this is likely to be full-span stall.

At low rotational speeds the density change is much less than the annulus was designed for, so V_x is high at compressor exit and the blades are likely to choke. This restricts the flow entering the compressor → high incidence on the early stages which are likely to stall. These have long blades → part span rotating stall.

At the design speed all blades should be the same distance from the stall line and all stages should stall around the same point. (3)

b) Direct from lecture notes (9)

c) At design $\frac{T_{02}}{T_{01}} = 10^{\frac{\gamma-1}{\gamma}\frac{1}{0.9}} = 2.077$

At stall $\frac{T_{02}}{T_{01}} = 6^{\frac{\gamma-1}{\gamma}\frac{1}{0.75}} = 1.979$

3c) cont'd

From the eqn proved in part (b)

$$\frac{T_{03\,stall}}{T_{03\,design}} = \frac{1.979-1}{2.077-1} = 0.909$$

The turbine is choked at both conditions

$$\therefore \left.\frac{\dot{m}\sqrt{T_{03}}}{P_{03}}\right)_{design} = \left.\frac{\dot{m}\sqrt{T_{03}}}{P_{03}}\right)_{stall}$$

$$\rightarrow \quad \frac{\dot{m}_{stall}}{\dot{m}_{design}} = \frac{P_{03\,s}}{P_{03\,d.}}\sqrt{\frac{T_{03\,des}}{T_{03\,stall}}}$$

$$= \frac{6}{10}\sqrt{\frac{1}{0.909}}$$

$$\frac{\dot{m}_{stall}}{\dot{m}_{design}} = \underline{\underline{0.629}} \qquad 7$$

d) For the choked test nozzle $\frac{\dot{m}_T\sqrt{C_{pa}T_{02T}}}{A_T\,P_{0T}}$
$= 1.281$

For the choked turbine stator

$$\frac{\dot{m}_e\sqrt{C_{pe}T_{03e}}}{A_N\,P_{03e}} = 1.3468$$

The mass flows and pressure ratios must be the same to have the same stall point.

$$\therefore \frac{A_N}{A_T}\sqrt{\frac{C_{pa}T_{0T}}{C_{pg}T_{03e}}} = \frac{1.281}{1.3468}$$

$T_{0T} = T_{02} = 1.979\,T_{01}$ from part (c).

3d) cont'd.

$$\therefore \quad \frac{A_N}{A_T} = \frac{1.281}{1.3468} \sqrt{\frac{C_{pg}}{C_{pa}}} \sqrt{\frac{1}{1.979}} \sqrt{\frac{T_{03e}}{T_{01}}}$$

From the eqn proved in part (b).

$$\frac{T_{03}}{T_{01}} = \frac{1}{1 - 0.4^{.2533}} \frac{C_{pa}}{C_{pg}} \times 0.979$$

$$\frac{T_{03}}{T_{01}} = \overset{4.726}{\cancel{2.388}} \frac{C_{pa}}{C_{pg}}$$

$$\therefore \quad \frac{A_N}{A_T} = \frac{1.281}{1.3468} \sqrt{\frac{1}{1.979}} \sqrt{4.726}$$

$$A_N/A_T = \underline{\underline{1.47}}$$

$$\text{or} \quad A_T/A_N = \underline{\underline{0.680}} \qquad 6$$