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1(a) 20%

$M_1 = 0.24 \qquad \frac{\dot{m}\sqrt{C_pT_0}}{S\,P_{01}} = 0.5133$

$M_2 = 0.85 \qquad \frac{\dot{m}\sqrt{C_pT_0}}{S\cdot\cos 65^\circ P_{02}} = 1.2551$

$\frac{P_{02}}{P_{01}} = \frac{0.5133}{1.2551\cdot\cos 65^\circ} = 0.9677$

$P_{02} - P_2 = P_{02}(1 - 0.6235)$

$Y_p = \frac{P_{01} - P_{02}}{P_{02} - P_2} = \frac{\frac{1}{0.9677} - 1}{1 - 0.6235} = \underline{\underline{0.0886}}$

1b 20%

$V_2/\sqrt{C_pT_0} = 0.5025$

$V_{\theta 2} = V_2\cdot\sin 65^\circ = 0.5025\cdot\sin 65^\circ\cdot\sqrt{C_pT_0} = 0.4554\sqrt{C_pT_0}$

$\dot{m} = 0.5133\cdot\frac{S P_{01}}{\sqrt{C_pT_0}}$

$\dot{m}V_{\theta 2} = 0.5133\cdot 0.4554\cdot S P_{01} = 0.2338\, S\cdot P_{01} = F_\theta$

$Z_w = \frac{F_\theta}{(P_{01} - P_2)C_x} = \frac{0.2338}{1 - P_2/P_{01}}\cdot\frac{S}{C_x} = 0.9$

$S/C_x = 0.9\left(1 - \frac{P_2}{P_{02}}\cdot\frac{P_{02}}{P_{01}}\right)/0.2338 = \underline{1.527}$

1C 30%

At $M_{2,is} = 1.2$, $Y_p = 0.0886 \qquad P_2/P_{01} = 0.4124$

$\frac{P_{01} - P_{02}}{P_{02} - P_2} = Y_p \Rightarrow P_{02}/P_{01} = \frac{1 + 0.0886\cdot 0.4124}{1 + 0.0886} = 0.9522$

$P_2/P_{02} = 0.4124/0.9522 = 0.4331 \Rightarrow M_2 = 1.165$

$V_2/\sqrt{C_pT_0} = 0.6539 \qquad V_{\theta 2} = 0.6539\cdot\sin 60^\circ\sqrt{C_pT_0} = 0.5663\sqrt{C_pT_0}$

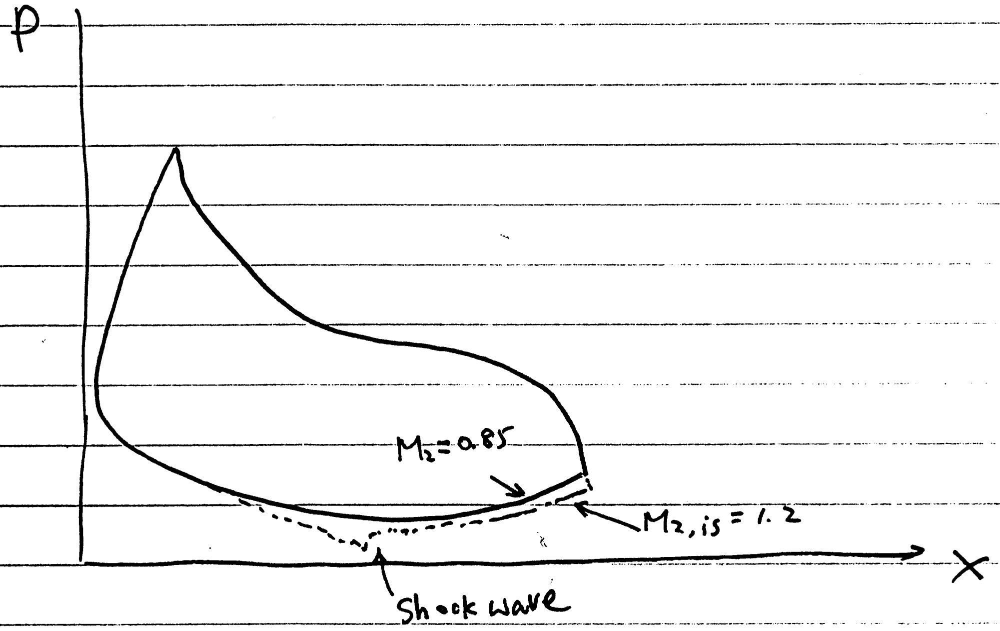
$\dot{m}V_{\theta 2} = 0.5663\sqrt{C_pT_0}\cdot 0.5133\frac{S P_{01}}{\sqrt{C_pT_0}} = 0.2907\, S P_{01} = F_\theta$

$Z_w = \frac{F_\theta}{(P_{01} - P_2)C_x} = \frac{0.2907\cdot P_{01}}{P_{01}(1 - P_2/P_{01})}\cdot\frac{S}{C_x} = \frac{0.2907}{1 - 0.4124}\cdot 1.527$

$= \underline{0.7554}$

1d.

1.5%



15% 1. As shown in sketch in d. after the flow is choked.

the loading upstream of the throat cannot be changed therefore only the part of the suction surface downstream of the throat can contribute to the loading increase as back pressure lowers. So relatively for a subsonic flow, the pressure profile is "fuller" thus closer to the ideal loading – high Zweifel coeff. but at supersonic flow the pressure profile is relatively less "full" so Zweifel coeff. reduces.

20% 2. The unique incidence condition in a transonic compressor cascade is governed by the continuity as well as the Prandtl-Meyer relation as the flow cannot turn freely without changing Mach number whileas at subsonic flow, the Mach number is independent from the flow angle thus the flow is only governed by the continuity. Although both relate inlet Mach number with flow angle. the physics, as discussed above, are different.

40% 2b. the throat in a transonic compressor cascade is defined as the first swallown Mach wave from the suction surface. It varies with flow condition. the definition is necessary because this is the only simple wave across the passage where flow property is uniform. at the geometrical throat where the minimum flow area locates, the flow is not uniform and usually not possible to define.

Prandtl-Meyer : $\alpha_1 + \nu_1 = \alpha_E + \nu_E$ $\quad 65 + 8.99 = 61.2 + \nu_E$

$\Delta\nu = 3.8°$. $\quad \nu_E = 12.79$. $\quad \underline{M_E = 1.53.}$

$f(M_1)\cos\alpha_1 = f(M_E)(\cos\alpha_E - t/s)$

$1.149 \cdot 0.4226 = 1.0702(0.4818 - t/s)$ $\quad \underline{t/s = 0.028}$

40% 2C. A/A^* is defined as the ratio of upstream stream tube width to the throat width. for high subsonic blade sections A/A^* should be close to but larger than 1. however too close to 1 will leave the section with small choking margin thus narrows its operable range, but too much larger than 1 indicates the flow will have to accelerate to throat unnecessarily and incur extra loss.

for choked flow.

$$\frac{S}{A^*}\cdot\cos\alpha_1 = \frac{F(1)}{F(M_\infty)}\cdot\frac{P_0^*}{P_{0_1}}$$ assuming no loss

up to the throat,

$$\frac{\cos\alpha_1}{\cos\alpha_1'} = \frac{F(M_\infty')}{F(M_\infty)} \qquad \cos\alpha_1' = \cos\alpha_1\cdot\frac{F(M_\infty)}{F(M_\infty')}$$

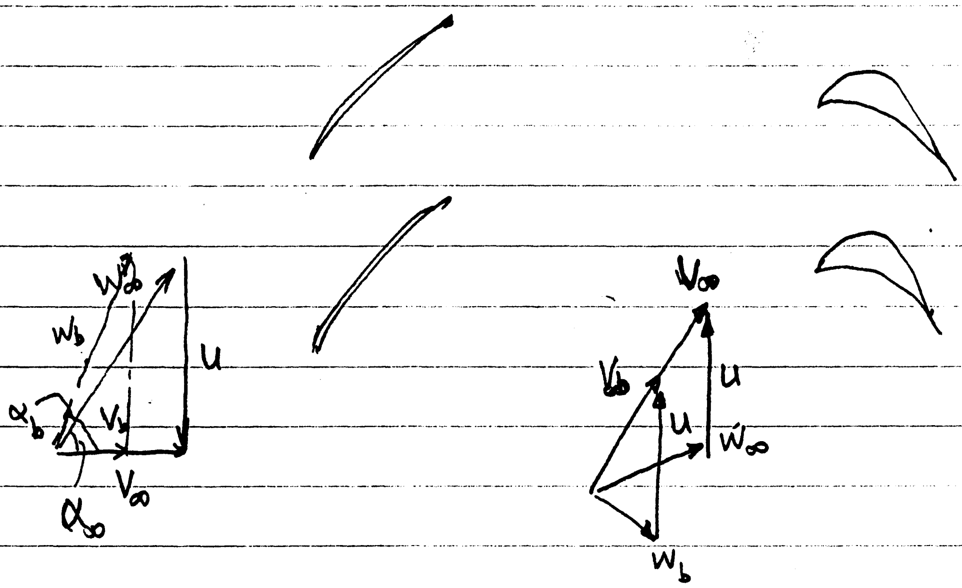
$$= \cos 28^\circ\cdot\frac{F(0.85)}{F(0.90)} = 0.8829\cdot\frac{1.2551}{1.2698} \qquad \alpha_1' = 29.22^\circ$$

$$\Delta\alpha = 29.22^\circ - 28^\circ = \underline{\underline{1.22^\circ}}$$ incidence will <u>increase by 1.22°</u>

20% 3a) Secondary flow is caused by the cross-blade passage (pitchwise) pressure gradient acting on the lower kinetic energy endwall boundary layer flows. The pressure gradient provides the force to turn the flow in mainstream. however with lower speed in endwall boundary layer the same pressure will cause flow to overturn, promoting flow migration from pressure surface to suction surface. In highly loaded compressor the transverse pressure gradient is high and the secondary flow will be stronger, the accumulation of the secondary flow on the endwall-suction surface corner will promote flow to separate, resulting in large flow blockage, high loss and high deviation. The thicker the incoming boundary layer, the more mass flow is involved in the secondary flow thus the stronger its adverse effects on the cascade performance. as the result, a thicker inlet boundary layer is more likely to cause the corner stall.

3b | compressor. turbine

20%



~~for the compress.~~ The relative motion of the rotor to the upstream stator causes the flow angle to change across the boundary layer. thus the boundary layer is "skewed" relative to the blade row. for the compressor. the skewness of the boundary layer is towards increased flow angle as the speed reduces close to the wall (W_b vs W_∞ and $\alpha_b > \alpha_\infty$), which will result in higher loss and stronger secondary flow. ② for turbine blade. the skewness is opposite to the blade moving direction. and results in smaller flow angle towards the end wall. this will some how relieve the secondary flow by reducing the blade loading in the fore-part of the blade.

20% 3c. The small tip gap in a cantilever blade will allow some ~~leage~~ leakage flow to pass over the blade tip from pressure surface to suction surface. this will some how relieve the pressure gradient which is driving the secondary flow. more importantly, the interaction of such leakage flow with the main passage will form a leakage vortex which is of opposite direction to the secondary flow vortex thus acts to

2/2 3c cont.

weaken the latter. for highly loaded blade where the corner stall is likely to appear. the leakage flow can effectively "blow" away the corner separation thus improve the flow in that region.

2/2 3d1. For internal flows. the conservation of mass is very important near sonic condition where the flow function is flat. small error in mass flow could cause large error in Mach number thus all other flow properties. using conservative form of equations will help ~~to~~ conserve the mass flow and other ~~flow~~ conservative properties and ~~less~~ has ~~less~~ smaller numerical errors. The finite volume methods solve the integrated form of the flow equations instead of the differential ~~[illegible]~~ equations. By ~~[illegible]~~ using integration the requirement for the smoothness of the flow field is significantly weaker than that is required for a differential equation so the algorithms will cope better in the cases of where flow has discontinuity / rapid variation such as shockwaves and slip lines.

2/2 3e. Subsonic flow four conditions required at inlet and one at exit, usually the stagnation pressure and temperature distributions are specified. together with two flow angles, at inlet and static pressure specified at the exit, inside the computational domain. periodic condition is specified on the boundaries which are not solid surface and away from the blade. on the solid surface. no flow is permitted to cross the surface so the velocity perpendicular to the surface is set to zero. if the inlet and exit flows are both supersonic, only inlet flow is ~~sepa~~ specified and no exit boundary condition is required as no information can travel upstream in supersonic flow. all five conditions are specified at the inlet. The periodical and solid wall condition remain unchanged.