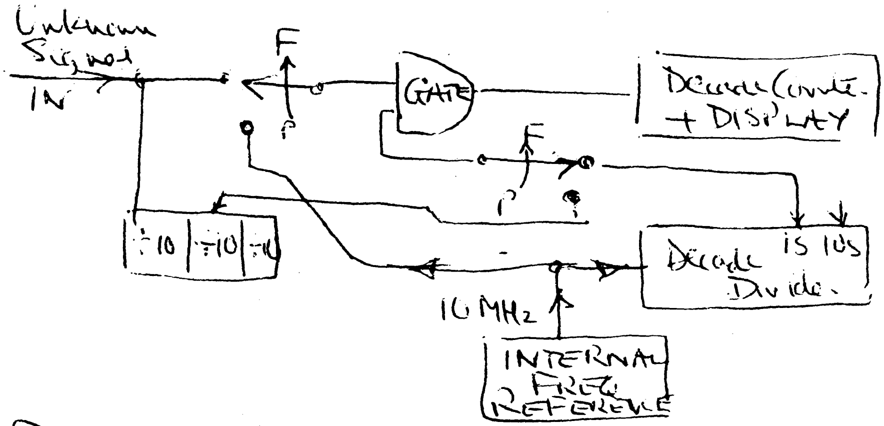
Dr P A Robertson par10@eng.cam.ac.uk

4B13 2005

1 (a)



The Input Signal Frequency, 2 switches in the F position, measures the no of pulses of the unknown in 1 sec (or 10 sec if allowed). This is an accurate time got by dividing down the Internal Reference

So 100 kHz gives a count of 100,000 (or 99999 & resolution is 1 in 10^5 or 10 ppm

For Low Frequencies, extra precision with switches in P position, the Period is measured. The gate is now opened by the Unknown ÷ factors of 10 & the no of 10 MHz pulses are amassed in the counter ∝ period of unknown.

So 400 Hz signal, 2.5 ms period × 1000 gives a 2.5 sec period and count of 25,000,000 ±1 (1 pulse per 0.1 μS) & resolution (is 0.4 ppm)

~~Also~~ Period measurement best for inputs < xtal Freq

35%]

Maintaining a Group of Standards.

(a) Intercompare them regularly (each day/week). Which one has least temp coeff & least affected by vibration or environment

(Contd

(b) Compare "best" unit carefully against others on the day before calibration.

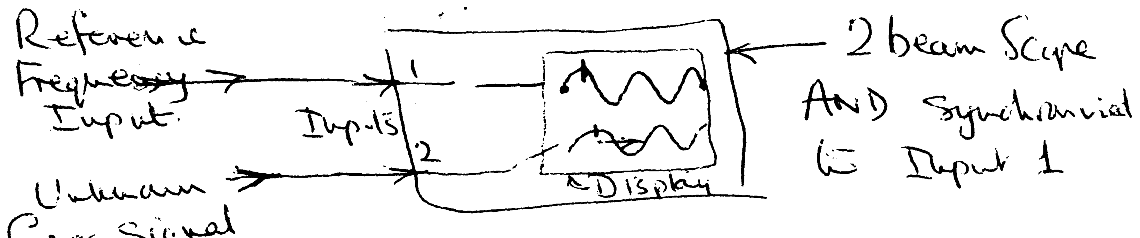
(c) Get calibration of "best" unit. Its errors will be reported to you as well as uncertainty of readings found & of uncertainty of system used.

(d). On return of "best" unit, compare it with others. Are the differences the same showing it has not been affected by travel & vibration.

(e) Use present differences to get errors of all units including those not sent for calibration. Use mean of all measurements in future for important

3(ii)] ~~calibn~~ measurement work.

ii



The Time T, measured by stopwatch, for the lower frequency lower display to drift to the ~~right~~ left (fewer cycles in given time) (or vice-versa for higher frequency), by a no of whole cycles, say 10.

In T secs, $T \times 10^7$ and $T \times f$ waves appear on 2 traces for an unknown of freq f.

25%] But $T.10^7 = Tf + 10$, so $f = 10^7 - \left(\frac{10}{T}\right)$ ← Error in Freq.

∴ Here ·025ppm error is when

$\frac{T}{10} - 0.25$ or $T \simeq 40$ secs (measured with stopwatch ± 1/5 sec ~ ±1/2 %)

So if T is measured with stopwatch to ± 1/5 sec, this is 1 in 200 resolution or ± 1/2 % which is sufficient

—— // ——

The 6 readings are entered, 22, 24, 22, 23, 27, 22, into a calculator in its statistics mode to give (parts in 10^9 or ppb used)

Mean = 23.33' ppb

Standard Deviation (n-1) = 1.96 ppb

So Uncertainty of Measurement = $2 \times 1.96 / \sqrt{6}$ — no of measurements.

= 1.60 ppb (or parts in 10^9)

So Instrument Error is − 23.3 ± 1.6 ppb

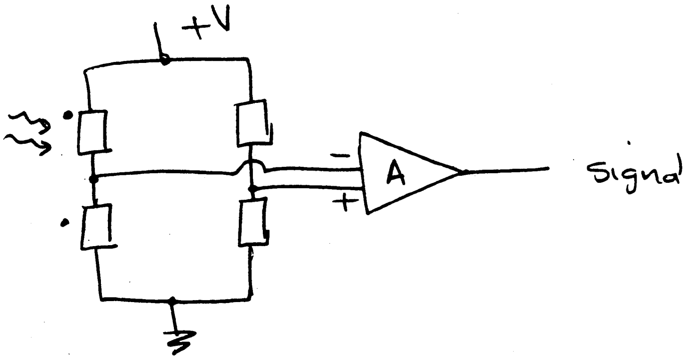
~~20~~ 15%

where the uncertainty is expressed with a ×2 multiplier denoting a 95% confidence that the errors will be in the range ± 1.6 ppb.

(ie between −24.9 & −21.7 so researcher giving −25 ppb is just on the extreme).

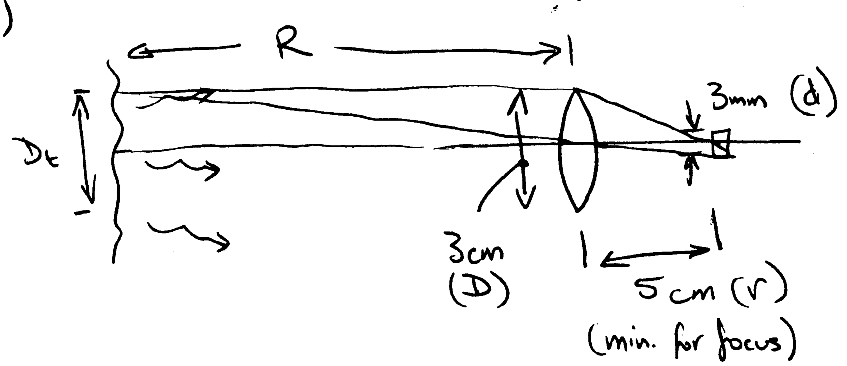
Important to add this

2(a) Connect platinum resistors into a wheatstone bridge circuit with a pair of precision resistors:



A pair of sensors compensates for ambient temperature changes – it would be practically impossible with a single resistor to detect small temperature changes. Also, the compensated pair cancels the effect of self heating due to the bridge excitation.

(b)



Lambert's Law: $\delta W = \frac{W\cos\theta}{\pi} . A . \delta\omega = \frac{W}{\pi} . \frac{\pi D_t^2}{4} . \frac{\frac{\pi D^2}{4} \times 4\pi}{4\pi R^2}$

Stephan's Law: $W = \epsilon \sigma_{SB} T^4$

and $\frac{D_t}{R} = \frac{d}{r}$ $\therefore$ $\delta W = \epsilon \sigma_{SB} T^4 \frac{\pi}{16} \frac{d^2 D^2}{r^2}$

$\therefore \delta W = 0.7 . 5.6\times10^{-8} . (273+850)^4 . \frac{\pi}{16} . \frac{0.3^2 . (5\times10^{-2})^2}{5^2}$

$= 0.11\ W$

@ 200°C/W $\therefore$ temp. rise of resistor = 22°C

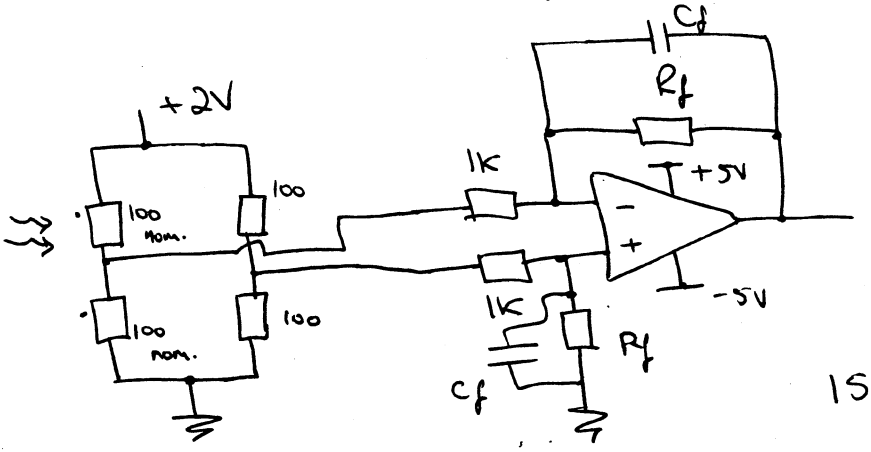
$\therefore$ resistance change = 0.0847 = 8.47% or 8.47 Ω

2 (c) At 1000°C, the resistance change $= \frac{8.47}{100} \times \left(\frac{1000+273}{850+273}\right)^4 \%$

$= 14.0\,\%$

So, if we drive the bridge with 2V, the output voltage from the bridge $= 1 - \frac{100}{(100+114)} = 65\text{ mV}$

Hence we need a gain of ×15.3 for an op-amp circuit.



1V @ 1000°C

$15.3 = \frac{R_f}{1000+100}$

$\therefore R_f = 16.8\text{ k}\Omega$ (15kΩ + 1.8kΩ)

$\therefore T = 2s = 2.2cR \Rightarrow C = 54\mu F$ (say 47µF std. value)

If the emissivity changes by ±0.1 about 0.7, then the voltage error will be $x \pm \sqrt[4]{\frac{0.1+0.7}{0.7}} = x \pm 1.034$

or ±3.4% of abs. temp. = ±43°C

(d) Electrical noise $V_n = \sqrt{4kTRB}$ with $B \simeq \frac{1}{\pi T} = 0.16\text{Hz}$

for 50Ω (resistors in parallel) $= 0.36\,nV_{rms}$.

$\therefore$ Noise $= \sqrt{(0.36^2 + 0.36^2 + (6 \times \sqrt{0.16})^2} = 1.63\,nV_{rms}$ at input

$\therefore$ at output $= 25\,nV_{rms}$.

$\equiv \sim 25\,\mu°C$ at $1V = 1000°C$

however, relationship is not linear and this will represent a greater error at lower temperatures.

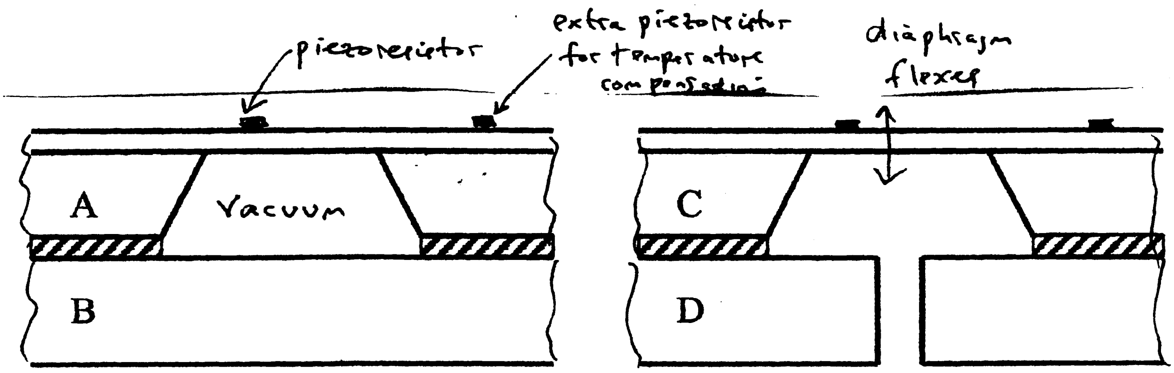
Even so, electrical noise is not significant compared to emissivity uncertainty.

4B13 Q3 Dfm 9.2.2005

a) Silicon can be precisely micromachined using a combination of lithography and etching techniques. Taking advantage of the crystalline structure wet etchants are used which terminate on (111) related crystal planes in the silicon, allowing precisely defined structures in the horizontal plane to be transferred into the vertical dimension.

Pressure sensors are made in silicon technology by etching through a bulk silicon wafer to leave a membrane

b)



Absolute pressure sensor

Differential pressure sensor

To make the absolute pressure sensor wafer A is bulk micromachined to form the membrane and bonded in vacuum to wafer B.

Doped silicon is piezoresistive with a much larger gauge factor than sputter deposited metals. However the large temperature coefficient of resistivity necessitates a second compensation piezoresistor on a part of the wafer which does not flex. This easily integrates in the fabrication technology without additional steps.

A differential pressure sensor is made by etching through wafer D.

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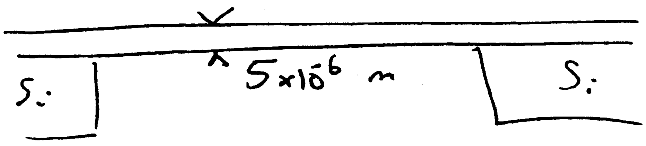
Q3 (continued)

The range of the sensor can be adjusted by choice of membrane thickness and dimension. Hence a very similar silicon micromachining technology can be used for a range of sensors.

c)



$$\frac{\sigma}{y} = \frac{M}{I} = 2\times10^{13} \quad N m^{-3}$$

for a 5 µm membrane $\quad y = \frac{1}{2}\times 5\times10^{-6} = 25\times10^{-7}$ m.

$$\therefore \sigma = 2\times10^{13}\times 25\times10^{-7} = 5\times10^{7} \quad N m^{-2}$$

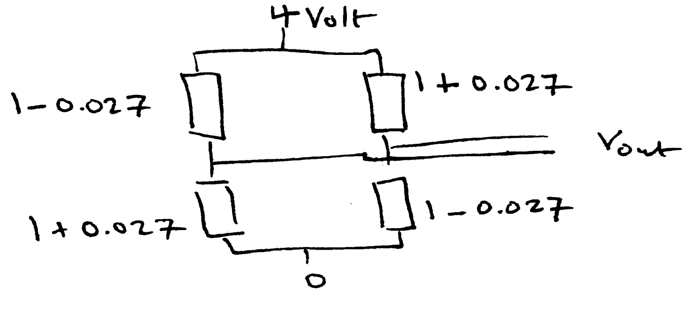
But $\sigma = E\varepsilon$

$$\therefore \varepsilon = \frac{\sigma}{E} = \frac{5\times10^{7}}{110\times10^{9}} = \frac{5}{11\times10^{3}} = 4.5\times10^{-4}$$

For a gauge factor 60

$$\frac{\Delta R}{R} = 60\,\varepsilon = 60\times4.5\times10^{-4} = 0.027$$

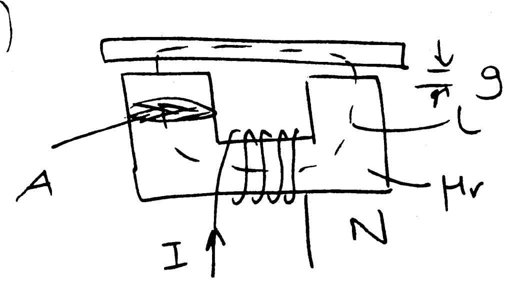
Bridge



$$V_{output} = 4\times0.027 = \underline{0.11 \text{ volt}}$$

Piezoresistive sensors are widespread in automotive, aerospace and medical applications.
For higher accuracy resonant based sensors or capacitance sensors are used but the cost is higher.

4 (a)



Ampere's Law:

$NI = \int H.dl \quad , \quad B = \mu H$

$\therefore \; NI = 2g \, H_{air} + L \, H_{core}$

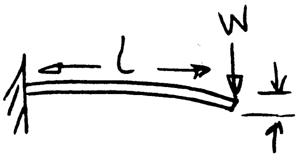
Flux conservation: $B_{air} = B_{core} \quad \therefore \; H_{core} = H_{air} / \mu_r$

$\therefore \; NI = 2g \, H_{air} + L \dfrac{H_{air}}{\mu_r} = H_0 \left(2g + L/\mu_r\right)$

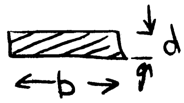
$\therefore \; B = \mu_0 N I / 2g + L/\mu_r$ and $L = NBA / I$

$$\therefore \; L = \frac{N^2 \mu_0 A}{(2g + L/\mu_r)} = \frac{N^2 . \mu_0 . A}{2g + L/1200}$$

(b)



$\delta = \dfrac{WL^3}{3EI} \quad , \quad I = \dfrac{1}{12} bd^3$



$d = 3 \times 10^{-3}$ m

$W = 150$ N

$\delta = 10^{-3}$ m

To allow room for the pole pieces of 0.5 cm^2 each, we shall choose a width, b, of say 20 mm.

$$\therefore \; 10^{-3} = \frac{150 . L^3}{3 . 200 \times 10^9 . 4.5 \times 10^{-11}}$$

$E = 200 \text{ GN/m}^2$ for steel

$I = 4.5 \times 10^{-11} \text{ m}^4$

$\Rightarrow \; L = 56.5$ mm

These dimensions seem reasonable.

(c) $$L = \frac{N^2 . \mu_0 . A .}{(2g + L/1200)} = \frac{400^2 . 4\pi \times 10^{-7} . 0.5 \times 10^{-4}}{\left(2 \times 10^{-3} + \frac{35 \times 10^{-3}}{1200}\right)} = 4.95 \text{ mH}$$

and $f_0 = \dfrac{1}{2\pi \sqrt{LC}} = 1000 \text{ Hz} \quad \therefore \; C = 5.1 \, \mu F$

($4.7 \, \mu F$ std)
// 390 nF

4 (c) contd.

For 5kg = 50N load, $g = 1 \pm 0.333$ mm

10kg = 100N load, $g = 1 \pm 0.667$ mm

g (mm)	L (mH)	f_{res} (Hz)	Δf (Hz)
1	4.95	1000	0
0.667	7.33	823	334
1.333	3.71	1157	
0.333	14.4	588	706
1.667	2.97	1294	
2	2.48	1416	1296
0	345	120	

$$L = \frac{10^{-5}}{(2g + 2.9 \times 10^{-5})}$$

$$f = \frac{1}{2\pi\sqrt{L} \cdot 2.26\times10^{-3}} = \frac{70.5}{\sqrt{L}}$$

Hence, non-linearity over full range is approx:-

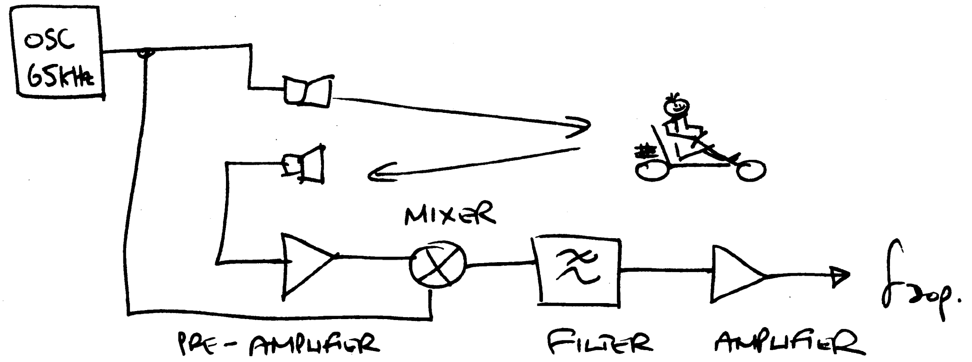
$$\frac{(1296 - 3\times 334)}{3\times 334} \times 100\% = 29\%$$ over 0-15kg optional

or

$$\frac{(706 - 2\times 334)}{2\times 334} \times 100\% = 5.7\%.$$ over 0-10kg

Actually, not too bad considering the non-linear g vs L and L vs f relationships. These would present no problems for a microcontroller to compensate.

5 (a)



50 km/hr = 13.9 m/s

@ 65 kHz, 340 m/s $\Rightarrow$ λ = 5.23 mm

For a reflection from an object moving at 13.9 m/s, the path length change is 27.8 m/s. With λ = 5.23 mm we get a Doppler frequency of $\frac{27.8}{5.23 \times 10^{-3}}$ = $\underbrace{5.31 \text{ kHz}}$.

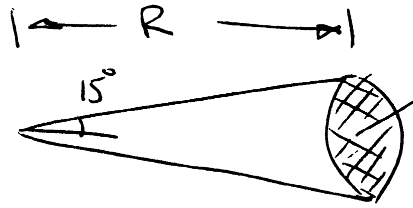
(b) For 24Vp-p drive, input power to transducer = $\left(\frac{24}{2\sqrt{2}}\right)^2 / 1000$ = 72 mW

$\therefore$ emitted power coupled into air = $72 \times 0.1 \times \underbrace{\left[1 - \left(\frac{1800 - 439}{1800 + 439}\right)^2\right]}_{0.63}$ = 4.54 mW

(acoustic impedance of air = 340×1.29 kg m^{-2}s^{-1})

Attenuation for 10m round-trip = 2dB $\equiv$ $\times$ 0.63
30m —"— = 6dB $\equiv$ $\times$ 0.25

For 15° cone ½ angle:



$A = \pi (R \tan 15°)^2$
= 5.64 m^2 @ 5m
= 50.8 m^2 @ 15m

Assume go-kart ~~cross-sectional~~ area = 0.4 m^2 and incident ultrasound is 100% back-scattered into a hemi-sphere (ignore ground reflections)

$\therefore$ For 5m range: reflected intensity back at transducer = $4.54 \times 10^{-3} \cdot 0.63 \cdot \frac{0.4}{5.64} \cdot \frac{\cancel{0.63}}{2\pi \cdot 5^2}$

= 1.29 μW/m2

5 (b) cont<u>d</u>.

$\therefore$ Power output from receiving transducer into matched load $= 1.29\times10^{-6} \cdot \frac{\pi\, 0.05^2}{4} \cdot 0.63 \cdot 0.1$

$\therefore P_r = 1.60\times10^{-10}$ W

$P_r = V_r^2/R$ with $R = 1000\,\Omega$ $\therefore V_r = 0.40$ mV loaded

$\Rightarrow V_r = 0.80$ mV open cct. @ 5m

For 15 m range : vs. 5m

power incident on target $\div 3^2 \Rightarrow \div 9$

rec<u>d</u> power $\div 3^2 \Rightarrow \div 9$

correct for attenuation $\div 0.63 \times 0.25 \Rightarrow \times 0.40$

$$\therefore P_r = \frac{1.60\times10^{-10}}{9\times9\times 1/0.40} = 7.90\times10^{-13}\text{ W} = V_r^2/R$$

$\therefore V_r = 56\,\mu$V open cct. @ 15m

(c) $R = R_o e^{\beta'/T} = 2000 = R_o e^{3280/273}$ $\therefore R_o = 0.0121$

Hence @ 20°C, T = 293 K $\quad R = 0.0121\, e^{3280/293} = 880\,\Omega$

@ 30°C, T = 303 K $\quad R = 0.0121\, e^{3280/303} = 608\,\Omega$

@ 10°C, T = 283 K $\quad R = 0.0121\, e^{3280/283} = 1307\,\Omega$

$$\frac{dR}{dT} = \frac{-\beta'}{T^2} R_o e^{\beta'/T} = \frac{-\beta'}{T^2} R \quad \therefore \frac{dR}{R} = -\frac{\beta'}{T^2}\, dT$$

$\therefore$ Taking linear slope at 10°C, to 30°C $\quad \frac{dR}{R} = \frac{-3280}{283^2} \cdot 20 = -0.82$

So $R_{lin} = 1307 - (0.82\times1307) = 235\,\Omega$

$\Rightarrow$ Voltage @ 20°C $= 880\times5\times10^{-3} = 4.40$ V

$$\text{Non-linearity} = \left|\left(\frac{\text{lin. o/p} - \text{actual o/p}}{\text{lin. o/p change}}\right)\right| \times100\% = \left|\frac{235-608}{235-1307}\right| \times100\%$$

$= 34.8\,\%$

ie: it is not v. linear at all!!