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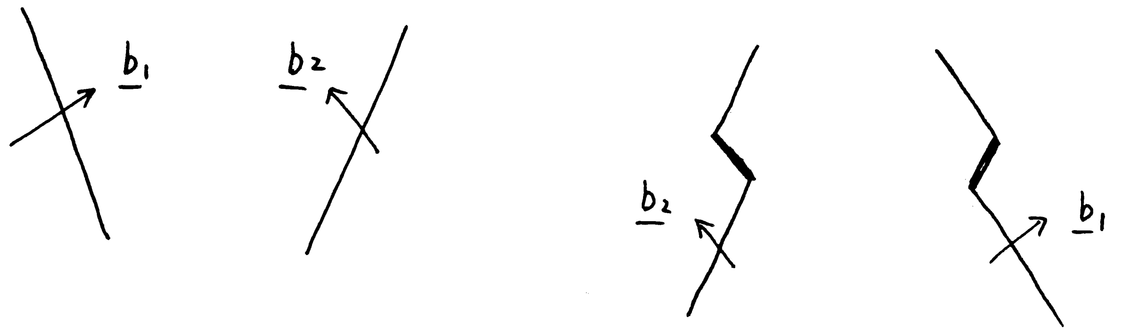
4C1: Crib for Tripos 2005

1. a) Dislocations travelling on different slip planes run into each other to form jogs:



before

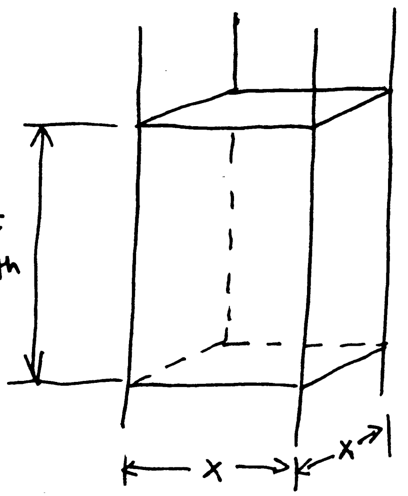
after

The jogs are sections of dislocation with Burgers vector which do not lie in the slip plane of the rest of the dislocation. Jogs therefore cannot move by glide. (20%)

b) Dislocation density ρ is defined as the length of dislocation line per unit volume. It increases with strain, (10%) as

$$\rho \propto \varepsilon^{\alpha}, \quad \alpha \text{ typically has value of } 0.5 \sim 1.5$$

c) Assume dislocation spacing $x \approx$ jog spacing



square array of dislocations

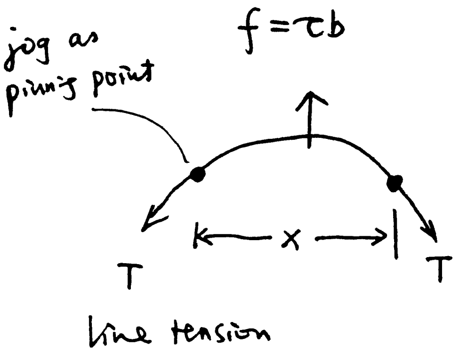
dislocation density

$$\rho = \frac{\text{length of disl.}}{\text{Volume}} = \frac{1}{x^2}$$

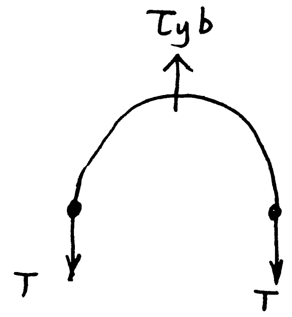
$$\Rightarrow \boxed{x \cong \rho^{-1/2}}$$

(25%)

1. d) Frank-Read Source



At maximum $\tau = \tau_y$



Force equilibrium

$$2T = \tau_y b x$$

But T = energy / unit length $= Gb^2/2$ (30%)

$\Rightarrow$ yield stress $\boxed{\tau_y = Gb/x = Gb\rho^{1/2}}$ (1)

from c)

e) Equation (1) implies ordinary work-hardening curve.

Other hardening mechanisms:

- substitutional solid solution hardening
- intersitial solid solution hardening
- precipitate and dispersion hardening

(15%)

2. (a)

Intergranular creep failure:

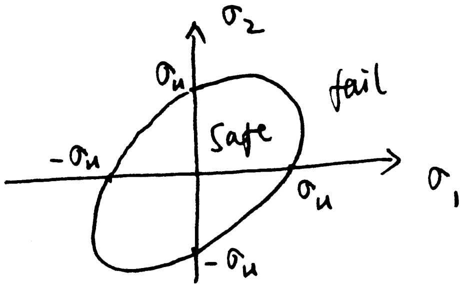
At low stresses and high temperatures, voids tend to nucleate and grow on grain boundaries typically lying normal to the applied tensile stress. As with void nucleation on a particle, a critical strain is required for nucleation of a grain boundary void. Voids grow by a combination of diffusion along grain boundaries, diffusion through the bulk, and power law-creep (depending on stress and temp.). At high temperatures, voids maintain a roughly spherical shape by surface diffusion. Failure is by void coalescence, leading to a fracture path along g.b.

In an effective stress material, the lifetime is dominated by the time to nucleate voids on g.b.. If it fails in uniaxial tension at a stress σ_u, then under multiaxial stress it will fail in the same time when $\sigma_e = \sigma_u$.

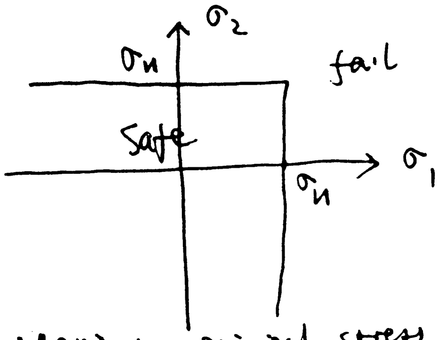
In an maximum principal stress material, the lifetime is dominated by the time to grow voids on g.b. In this case, the maximum principal stress σ_I determines the time to failure, so under multiaxial stress it will fail when $\sigma_I = \sigma_u$.

(30%)

(b) Isochronous failure surface:



Effective stress material



Maximum principal stress material

(20%)

2. (c) Hoop stress $\sigma_h = \frac{pR}{t} = \frac{2\times10^6 \times 150\times10^{-3}}{t} = \frac{0.3}{t}$ MPa

Shear stress $\tau = \frac{T}{2\pi R^2 t} = \frac{20\times10^3}{2\times3.14\times(150\times10^{-3})^2 t} = \frac{0.141}{t}$ MPa

<u>Alloy A</u>

Uniaxial stress required to give failure time of 100,000 hours is

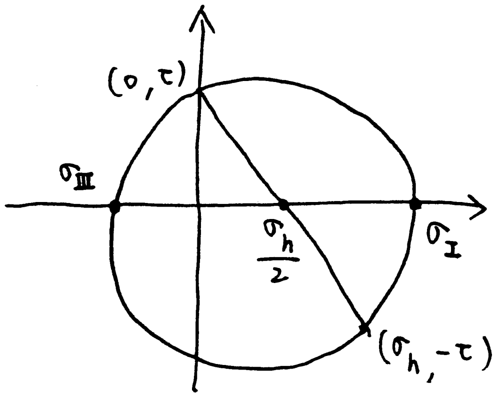
$$\sigma_u = \frac{100\times10^6}{(100{,}000/10{,}000)^{1/5}} = 63 \text{ MPa}$$

Alloy A is a maximum principal stress material, so for design loading

$$\sigma_I = \frac{\sigma_h}{2} + \frac{1}{2}\sqrt{\sigma_h^2 + 4\tau^2} = \frac{0.356}{t} \text{ MPa}$$

$$\left(\sigma_{III} = \frac{\sigma_h}{2} - \frac{1}{2}\sqrt{\sigma_h^2+4\tau^2}\right)$$

Mohr's circle



Hence

$$\sigma_I = \sigma_u$$

$$\Rightarrow \frac{0.356}{t} = 63 \Rightarrow t = 5.65 \text{ mm}$$

Since density is same for both alloys,

mass $\propto t$, hence total cost $\propto$ Cost$\times t$

$\Rightarrow$ total cost $\propto 0.35 \times 5.65 = 1.978$

<u>Alloy B</u>

Uniaxial stress required to give failure time of 100,000 hours is

$$\sigma_u = \frac{120\times10^6}{(100{,}000/10{,}000)^{1/5}} = 75.7 \text{ MPa}$$

It is an effective stress material, so $\sigma_e = \sqrt{\sigma_h^2 + 3\tau^2} = \frac{0.386}{t}$ MPa

Hence $\sigma_e = \sigma_u \Rightarrow \frac{0.386}{t} = 75.7 \Rightarrow t = 5.11$ mm

Total cost $\propto 0.4 \times 5.11 = 2.044$

(50%)

$\therefore$ <u>Alloy A is the most economical.</u>

3 (a)

In SST the plastic zone size $\sim \dfrac{K^2}{\pi \sigma_Y^2}$ is much less than leading dimensions like plate size, crack length etc with the non-linear zone completely embedded in an outer elastic K-field. Thus the K-field determines the state in the non-linear zone & thus K is an adequate parameter to correlate fracture (15%)

(b)

i) $$a_m = \left(\frac{K_{IC}}{1.13\,\sigma_h\sqrt{\pi}}\right)^2 = \left(\frac{60}{1.13\times 540\sqrt{\pi}}\right)^2 = 3.08\times 10^{-3}\ \text{m}$$

(15%)

(ii) $$\frac{da}{dt} = 6\times 10^{-6}\,K$$

$$= 6\times10^{-6}\times 1.13\times\sigma\sqrt{\pi a}$$

$$= 1.2\times10^{-5}\,\sigma\sqrt{a}$$

$$\Rightarrow 2\left[\sqrt{a_m}-\sqrt{a_i}\right] = 1.2\times10^{-5}\,\sigma\, t_f$$ (25%)

$t_f = 1$ hour, $\sigma = 540$ MPa

$$a_i = 2.73\times10^{-3}\ \text{m}$$

(iii) The proof test must cause cracks greater ~~that~~ than a_i in length to propagate immediately. Hence

$$\sigma_P = \frac{K_{IC}}{1.13\sqrt{\pi a_i}} = 573\ \text{MPa}.$$ (30%)

(iv) Condition for LEFM

$$a_i > 2.5\left(\frac{K}{\sigma_Y}\right)^2$$

at fracture $K = K_{IC}$

$$\Rightarrow \quad 2.5\left(\frac{K_{IC}}{\sigma_Y}\right)^2 = 2.7\,mm$$

$a_i = 2.73\,mm$ so LEFM condition just satisfied.

(15%)

4

(a) G is the rate of change of potential energy with crack length $G = -\frac{\partial u}{\partial a}$

where u is the potential energy & a the crack length

Fracture occurs when $G = G_c$, where G_c is a material parameter

K is a measure of the magnitude of the stress singularity in the vicinity of a sharp crack in a nominally elastic material. Fracture occurs when $K = K_c$ where K_c is a material parameter. (30%)

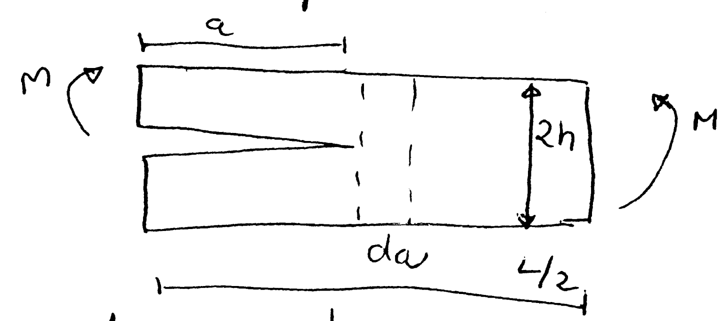
(b) Consider a section of beam of depth d, length l & ~~width~~ depth B

a constant moment M produces curvature $\kappa = \frac{M}{EI}$, so stored energy

$$U = \frac{1}{2} M \kappa l$$

$$= \frac{6 M^2 l}{B E d^3}$$

Consider ½ specimen



when crack grows da

$$dU = \frac{6M^2}{BE(2h)^3}\left(\frac{L}{2} - a\right) - \left[\frac{6M^2}{BE(2h)^3}\left(\frac{L}{2} - a - da\right) + \frac{6M^2}{BEh^3} da\right]$$

$$dU = -\frac{21}{4}\frac{M^2}{Bh^3E} da$$

$$G = -\frac{1}{B}\frac{\partial u}{\partial a}$$

$$\boxed{G = \frac{21}{4}\frac{M^2}{E h^3 B^2}}$$ (50%)

(c) $$G_c = \frac{21}{4}\frac{65^2}{(25\times10^{-3})^2\times 70\times10^{9}\times(5\times10^{-3})^3}$$

(20%)

$$G_c = 4056 \ \mathrm{J\,m^{-2}}$$