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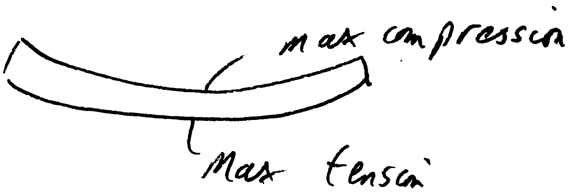
Engineering Tripos Part IIA: Module 4C2
Designing with Composites
CRIB - 2004/5

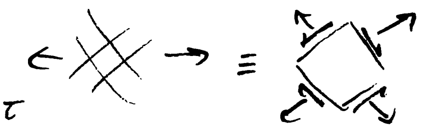
MPFS.
May '05

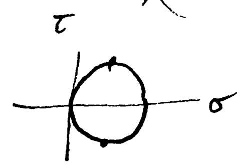
1(a)



- Local loading point may be a problem - eg under central roller

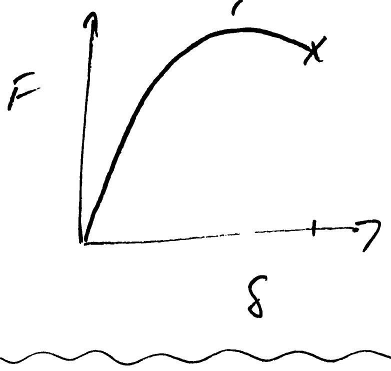
- At bottom of beam in centre we have so expect shear failure of outermost ply exacerbated by tensile stress. This may lead on to interlaminar failure.





To calculate failure due to shear, perform a laminate plate analysis, find the stresses in the plies and compare them with failure stresses.

Failure due to load point - use finite element analysis or empirical tests.

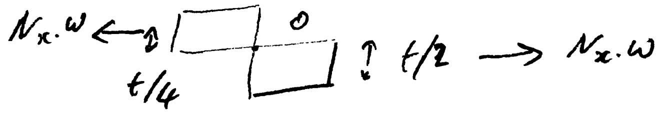


ductility associated with failure of ±40° plies

[30%]

More care on this part (a) scored good marks. Part (c) not so well done - the analysis is easy but it is important to separate out the constraints.

1 (b) $M = \frac{FL}{4}$



Moments about 0:

$$2 . N_x w \, t/4 = FL/4$$

$$\Rightarrow N_x = FL/2tw$$

[10%]

(c) Minimum mass ⇒ minimum thickness.

G_{xy} constraint: Use carpet plots - $G_{xy} \geq 8.3$ GPa

$\Rightarrow \% \pm 45 \geq 45\%$

With this many ±45°, probably not necessary to include 90° plies.

Now check failure. Use ε allowables. Strictly the strain allowables assume plies in all directions, and in fact the critical ply will be the omitted 90° plies. Either ignore this factor, or use an estimate slightly larger ε allowable.

The minimum mass will correspond to maximising % 0° plies.

So choose 55% 0° plies $\Rightarrow E_x = 32$ GPa.

[40%]

Then $\underbrace{e^{+}}_{=0.003} = \sigma/E_x = \frac{FL}{t^2 w \, 32 \text{ GPa}} \Rightarrow t = 20$ mm

critical ↗

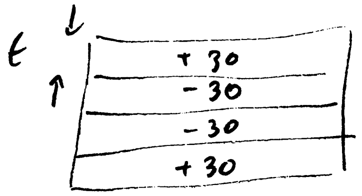
$0° : \pm 45° : 90° = 55\% : 45\% : 0\%$

(d) Verify: - do laminate calculations
- test

[20%]

Improve: Put 0° plies at outside
Add loading point reinforcement
Add tough outer layer if needed.
Consider thicker section towards middle where moment is biggest.
Use sandwich panel.

2 (a)



Method: Find Q
Find $\bar{Q}$
Find A
Use $\underline{N} = \underline{A}\,\underline{\varepsilon}$

$Q_{11} = E_1 / (1-\nu_{12}\nu_{21})$ $\quad \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = \nu_{12}\, E_1/E_2 = 0.4$

$= 1.008E$

$Q_{22} = E_2 / (1-\nu_{12}\nu_{21}) = 0.2016\, E_1$, $\quad Q_{12} = 0.0403 E_1$, $Q_{66} = G_{12} = 0.2 E_1$

Using data ~~book~~/sheet expressions

$$\Rightarrow [Q] = E_1 \begin{bmatrix} 1.008 & 0.0403 & 0 \\ 0.0403 & 0.2016 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

For +30° $\quad c = \cos\theta = \sqrt{3}/2$, $c^2 = 3/4$, $c^4 = 9/16$
$s = \sin\theta = 1/2$, $s^2 = 1/4$, $s^4 = 1/16$, $s^2c^2 = 3/16$

$$\frac{\bar{Q}_{11}}{E_1} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2$$
$$= 1.008\cdot\frac{9}{16} + 0.2016\frac{1}{16} + 2(0.0403 + 0.4)\frac{3}{16} = 0.7447$$

Similarly $Q_{12}/E_1 = 0.10199$, $Q_{22}/E_1 = 0.3415$, $Q_{66}/E_1 = 0.2617$

Q_{16} and Q_{26} not needed as balanced symmetric.

From datasheet $A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k - z_{k-1}) = 4t\,\bar{Q}_{ij}$
↑ for A_{11}, A_{12}, A_{21} A_{66}

$A_{16} = A_{26} = 0$

$$[A] = \begin{bmatrix} 0.7447 & 0.10199 & 0 \\ 0.10199 & 0.3415 & 0 \\ 0 & 0 & 0.2617 \end{bmatrix} 4tE_1$$

$[B] = 0$ as symmetric [65%]

This part well done. Good marks for right method.

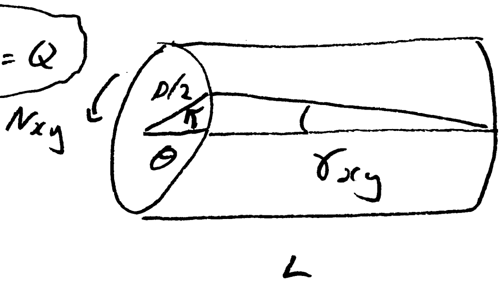
2 (b) (i) $N_x = P/\pi D \quad N_x = N_{xy} = 0$

$\Rightarrow$ only need 2×2 part of $[A]^{-1}$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \frac{1}{4tE_1} \begin{bmatrix} 0.7447 & 0.10199 \\ 0.10199 & 0.3415 \end{bmatrix} \begin{bmatrix} P/\pi D \\ 0 \end{bmatrix}$$

$$\Rightarrow \varepsilon_x = 0.35 \frac{P}{\pi D t E_1} , \quad \varepsilon_y = -0.1045 \frac{P}{\pi D t E_1}$$ [20 %]

(ii) $N_x = N_y = 0$ $\quad N_{xy} \cdot \pi D \cdot \frac{D}{2} = Q$



$$\Rightarrow N_{xy} = 2Q/\pi D^2$$

$$\begin{pmatrix} 0 \\ 0 \\ N_{xy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} \Rightarrow \varepsilon_x = \varepsilon_y = 0$$

$$\gamma_{xy} = N_{xy}/A_{66}$$

$$\Rightarrow \gamma_{xy} = 0.9553 \frac{N_{xy}}{tE_1}$$

From ~~tcde~~ tube diagram

$$\theta \cdot D/2 = \gamma_{xy} \cdot L \Rightarrow \frac{\theta}{L} = \frac{2\gamma_{xy}}{D}$$

$$\Rightarrow \frac{\theta}{L} = \frac{1.216\, Q}{tD^3 E_1}$$ [15 %]

Note that in this part only A_{66} is needed while in part b(ii) only part of the A ~~star~~ matrix is needed. Some difficulties deriving the $\theta/L = \frac{2\gamma_{xy}}{D}$ formula

3 (a) (i) Need σ's in plies

→ calculate A matrix

→ calculate laminate ε's = lamina ε's

→ calculate lamina σ's

→ apply failure criterion

$$Q_{90} = \begin{bmatrix} 9 & 2.7 & 0 \\ 2.7 & 139 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{GPa} \Rightarrow A = t \begin{pmatrix} 287 & 8.1 & 0 \\ 8.1 & 157 & 0 \\ 0 & 0 & 20.7 \end{pmatrix} \text{GPa}$$

where t is ply thickness.

No shear due to symmetry ↙ N

$$\Rightarrow \begin{pmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \end{pmatrix} = \begin{pmatrix} 287 & 8.1 \\ 8.1 & 157 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0.1 \end{pmatrix} \cdot \frac{N_x}{t} \text{GPa}^{-1} = \begin{pmatrix} 157 & -8.1 \\ -8.1 & 287 \end{pmatrix} \begin{pmatrix} 1 \\ 0.1 \end{pmatrix} \frac{N_x}{t\Delta} \text{GPa}$$

where $\Delta = 287.157 - 8.1^2 \text{ GPa}^2$
$= 44993$

$$= \begin{pmatrix} 156.19 \\ 20.6 \end{pmatrix} \frac{N_x}{t\Delta} \text{GPa} = \begin{pmatrix} 0.028 \\ 0.0037 \end{pmatrix} N_x \frac{\text{GPa}}{\text{mm GPa}^2} = \frac{1}{\text{mm GPa}} = \frac{1}{\text{MN/m}}$$

Since both ε's positive, expect failure as transverse tension of 90° (most likely - start with that!) or perhaps axial failure of 0° ply. ← ε_1 along fibre direction

90° ply

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = [Q] \begin{pmatrix} 0.0037 \\ 0.028 \end{pmatrix} N_x = \underset{\text{GPa}}{\begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix}} \begin{pmatrix} 0.0037 \\ 0.028 \end{pmatrix} N_x = \begin{pmatrix} 0.58 \\ 0.263 \end{pmatrix} N_x \frac{\text{GPa}}{\text{MN/m}}$$

$$\text{Failure: } \left(\frac{\overset{580 \leftarrow \text{MPa}}{\cancel{3785}}}{\underset{\underset{S_L^+}{\uparrow}}{1448}} \right)^2 - \frac{580.263}{1448^2} + \left(\frac{263}{\underset{\underset{S_T^+}{\uparrow}}{48.3}} \right)^2 = \left(\frac{1}{N_x} \right)^2 (\text{MN/m})^2$$

MPa

$\Rightarrow N_x = 0.183$ MN/m ← CRITICAL - expected failure mode.

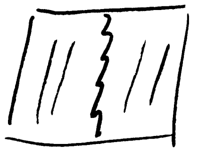
0° ply

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix} \begin{pmatrix} 0.028 \\ 0.0037 \end{pmatrix} = \begin{pmatrix} 3.86 \\ 0.11 \end{pmatrix} N_x \frac{\text{GPa}}{\text{MN/m}}$$

$$\left(\frac{3860}{1448} \right)^2 - \frac{3860.110}{1448^2} + \left(\frac{110}{48.3} \right)^2 = \left(\frac{1}{N_x} \right)^2 (\text{MN/m})^2 \Rightarrow N_x = 0.288 \text{ MN/m}$$

Failure in 90° ply due to transverse cracking. [65%]

3 (a) (ii) Failure of 90° ply due to transverse failure. ⑥



$\Rightarrow$ $Q_{11} = 0$

$\nu_{12} = \nu_{21} = 0$

Q_{22} unchanged

no contribution from 90

New A matrix $= \begin{pmatrix} 139\times2 & 2.7\times2 \\ 2.7\times2 & 9\times2+139 \end{pmatrix}$

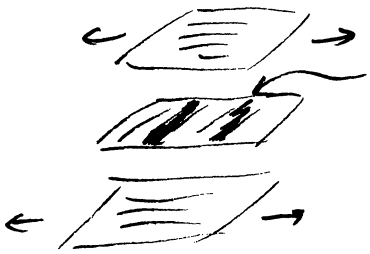
$\Rightarrow$ E_y unchanged

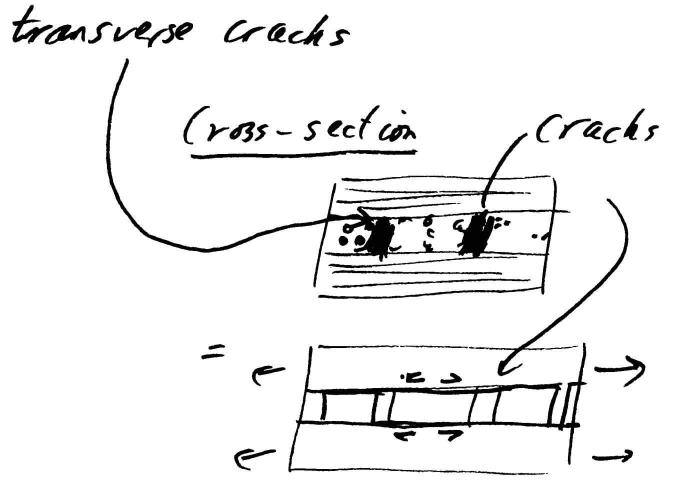
E_x reduced by $9/287 = 3\%$

[15%]

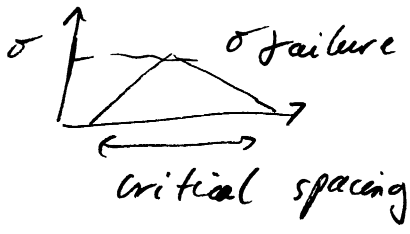
(b) Need to appreciate that the failure mode is transverse cracking in the 90° plies

i.e.





At the crack the ply σ is zero. Stresses built up due to shear from 0° plies. Use shear lag theory.



[In fact a better model should also include a fracture model.]

(b) not well done, as physical picture not identified.
a(i) well done - good marks for correct method.

4 (a) Composites have the ability to be formed into complex shapes, and to be tailored to give strength and stiffness in the required directions. They also suffer from poor through-thickness properties and joining, and have manufacturing limitations (e.g. sharp corners). Hence it is essential to couple design of the structure (e.g. shape, details of joints and cutouts, reinforcements), with the design of the layup to ensure that the shapes can be readily produced by the composite and that the properties achievable by the composite at a particular point match those given by the loading and overall design. Often a metal component with a given geometry can be improved upon when being replaced by composite, by modifying the shape of the component to take advantage of the composite properties and avoid problems with joining or problems with secondary material properties, rather than by replacing the metal directly (CFRP as 'black metal'). [25%]

(b) Through-thickness properties of the laminate are the secondary properties, which are matrix-dominated. Such poor properties can lead to matrix controlled failure processes, like delamination cracking. Cut-outs, holes, surfaces (edges) concentrate shear stresses and tensile stresses at interfaces leading to premature cracking and damage accumulation and losses in material properties, fatigue, impact damage, etc. [25%]

(c) The successful application of structural composites to large integrated automotive structures is dependent on the ability to use rapid and economic fabrication processes. The fabrication process must be capable of close control of composite properties to achieve lightweight, efficient structures. The rate of manufacture is critical to satisfying the economic need. Glass fibre-reinforced composites must be capable of satisfying the three primary criteria: fatigue (durability), energy absorption (crash loads) and ride quality (vehicle dynamics). These functional requirements must be totally satisfied in a cost-effective manner. Appropriate composite fabrication procedures must be applied or developed which satisfy high production rates but still maintain the critical control of fibre placement and distribution. [25%]

Sheet-moulding composites (SMC) (plastic-based composite materials) are in general automotive use, consisting of approximately 25 wt% chopped glass fibres in a polyester matrix. A characteristic moulding time for SMC is of the order of two minutes, which is on the borderline of viability for automotive production rates. Currently, the only commercial process which comes close to satisfying these requirements, is compression moulding of sheet moulding compounds (SMC) or some variant of the process.

The form of the glass fibre will be application-specific and both chopped and continuous glass fibres will find extensive use. It is expected that most of the structural applications involving significant load inputs will utilise a combination of both chopped and continuous glass fibre with the particular proportions of each depending on the component or structure. [25%]

Alternatively or in addition to the above answer, the student could give the following answer:

There is a process, which hold distinct potential in terms of combining high production rates, precise fibre control and high degrees of part integration. High-speed resin-transfer moulding (RTM) offers these potential benefits, provided technical developments can be achieved. For comparison of the two alternate composite construction techniques, consider the body side assembly. In a typical SMC approach the complete body side consists of two mouldings, which would be bonded together. The major difference of the RTM procedure for the same structure is the elimination of the adhesive bonding and the incorporation of a foam core. Note that the degree of integration is higher for RTM reflecting the greater versatility of this procedure. (In terms of reduction in parts, the moulded SMC body structure could consist of some where between 10 and 20 major parts (compared to approximately 300 major steel parts) and the RTM structure could be composed of about 10 major parts).

(d) Construction of this section, which includes stringers, would start with the computerised lay-down of carbon fibre-epoxy pre-preg composite tape onto an appropriately sized mould. The mould would be mounted on a tool that rotated as the tape was applied (known as filament winding). The structure would then be wrapped and placed in an autoclave for curing. One critical issue would be the curing (thermal) cycle and the heating-rate and cooling-rate of the moulding, which would affect residual stresses in the final part. Finally, it would be unwrapped, inspected, and the tool removed. Subsequently, the windows and door would be cut out in the section and tested by non-destructive means, followed by a painting process. It would also be tested for structural integrity. [25%]

(Using a composite material allowed Boeing to create optimised structural designs and develop an efficient production process. By integrating this section into a single composite structure, Boeing was able to cut the number of parts in the section significantly and reduced the weight by almost 20% and a corresponding reduction of 20% less fuel).