

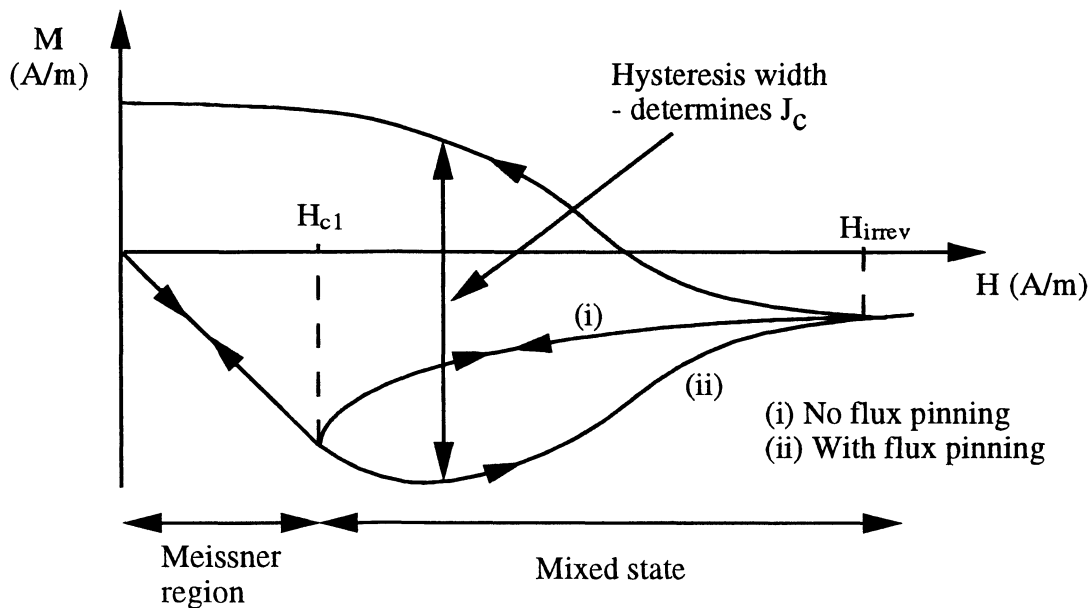
ENGINEERING TRIPOS PART IIB**Thursday 5th May 2005 2.30 to 4****Module 4C3 : ELECTRICAL AND NANO MATERIALS SOLUTIONS**

1. (a) (i) Magnetic flux density is generated in bulk superconductors by the Faraday effect. This involves applying a changing magnetic field to the superconductor, which induces an e.m.f., which causes an Eddy current to flow throughout the bulk material, which, in turn, generates magnetic flux. As a result, bulk superconductors generate fields in a way analogous to a wire-wound solenoid. The magnitude of the net magnetic moment depends on the magnitude of these induced currents and the area over which they flow. Hence the volume magnetisation (defined as the net magnetic moment per unit volume) increases in direct proportion to the cross sectional area of the sample, normal to the direction of applied magnetic field (i.e. in proportion to r^2).

(ii) Magnetic flux density is generated in permanent magnets by the alignment of unpaired electron spins associated with specific ions. An ordered array of spins may be approximated by a surface current although this does not flow throughout the material, as is the case for a homogeneous superconductor which carries current everywhere. The magnitude of the net magnetic moment is directly proportional to the sample volume. Hence the volume magnetisation does not vary with increasing sample volume.

[30%]

(b)



Increased flux pinning in bulk type II superconductors makes it more difficult to move individual flux quanta through the bulk microstructure. The Lorentz force, \mathbf{F}_L , acts on flux lines in a bulk superconductor carrying a current density \mathbf{J} ;

$$\mathbf{F}_L = \mathbf{B} \times \mathbf{J}$$

As a result, a bulk superconductor can support higher J and a given flux density before the Lorentz force is overcome and J_c increases.

[40%]

(c) Variation of flux density with thickness in a sample of slab geometry is given by;

$$\frac{dB}{dx} = \mu_0 J_c$$

$$\text{For } J_c = \frac{K}{B}, \int_0^{B_{\max}} B dB = K \mu_0 \int_0^{d/2} dx \quad \text{i.e.} \quad \frac{B_{\max}^2}{2} = K \mu_0 \frac{d}{2} \rightarrow B_{\max} \propto \sqrt{d}$$

Field trapped in a slab in which $J_c = \frac{K}{B}$ is proportional to \sqrt{d} , i.e.;

$$\frac{B(d_2)}{B(d_1)} = \frac{\sqrt{d_2}}{\sqrt{d_1}} = 2$$

$$\text{Hence; } \frac{d_2}{d_1} = 4 \quad \text{Thus} \quad d_2 = 4 \times d_1 = 4 \times 5 = \mathbf{2 \text{ cm}}$$

[30%]

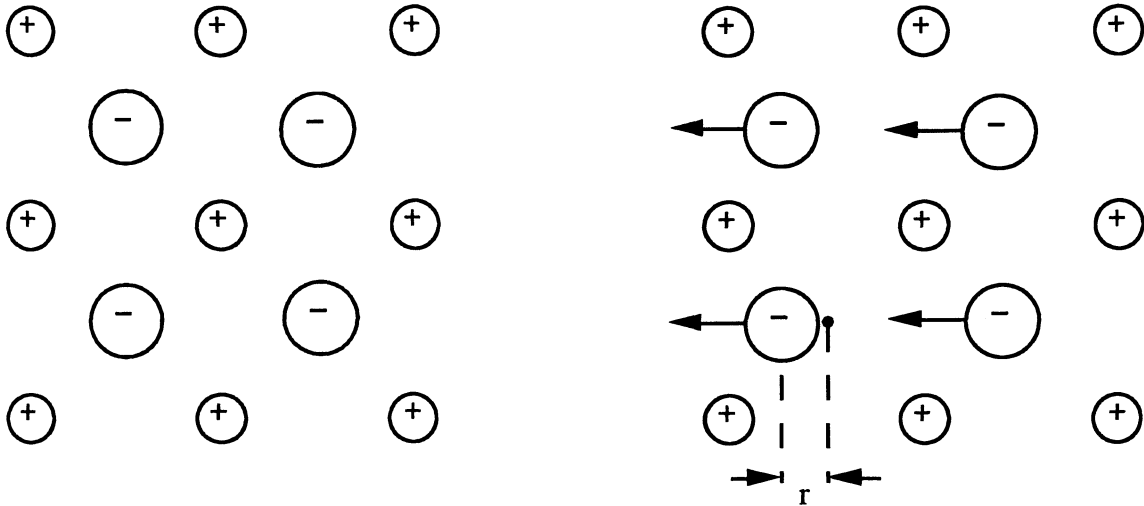
2. (a) Pyroelectric and piezoelectric materials are sub-classes of dielectrics, and are characterised by an asymmetry in their crystallographic structure, which leads to polar properties. The crystal structure of piezoelectric materials is characterised by a lack of centre of symmetry (i.e. they exhibit point or axial asymmetry). As a result, 20 of the known 21 dielectric structures that exhibit such a lack of centre of symmetry are piezoelectric.

Pyroelectricity, on the other hand, occurs in polar dielectric materials whose structure contains at least one *axis* along which an electric dipole moment exists. Only 10 of the 21 dielectric structures, therefore, exhibit pyroelectric properties (orthorhombic, tetragonal and triclinic, for example). A pyroelectric material is necessarily piezoelectric, although the converse is not true.

Both piezo and pyroelectric materials exhibit an intrinsic spontaneous polarisation, \mathbf{P}_s , which is given by the net electric dipole moment per unit volume, i.e. ;

$$\mathbf{P}_s = \frac{\sum_{\text{All unit cells}} q \mathbf{r}}{V} \quad \text{Cm}^{-2}$$

Microscopically, \mathbf{P}_s originates from a structural transition which results in a displacement of the positive and negative centres of charge symmetry, i.e. ;



Non-polar lattice

Polar lattice

Pyroelectricity is the change in polarisation that occurs in pyroelectric materials as their temperature changes. Quantitatively this is described in terms of the pyroelectric coefficient, \mathbf{p} , which is given by the rate of change of \mathbf{P}_s , with temperature, i.e. the gradient of the \mathbf{P}_s (T) curve;

$$\mathbf{p} = \frac{\Delta \mathbf{P}_s}{\Delta T} = \frac{d \mathbf{P}_s}{dT}$$

The piezoelectric effect can be described microscopically in a similar way to pyroelectricity, but driven by different extrinsic variables.

The majority of useful piezoelectrics and pyroelectrics are ferroelectric, since these have a number of polar axes and are therefore relatively easy to pole.

The direct piezoelectric effect is the production of a change in dielectric polarisation by the application of a mechanical stress. Conversely, application of an electric field to a piezoelectric will cause it to strain mechanically (indirect effect).

[40%]

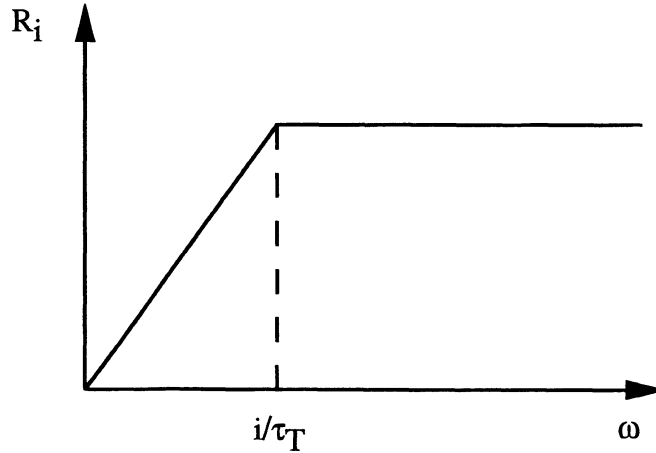
(b) Current responsivity is defined as; $R_i = \frac{i_p}{W_0}$ where i_p is the current which flows in an external circuit due to charge release associated with the temperature change.

$$\Delta T = \frac{\eta W_0 e^{i\omega t}}{G_T + i\omega H} \quad i_p = A p \frac{dT}{dt} = \frac{\eta p A i \omega W_0 e^{i\omega t}}{G_T + i \omega H}$$

$$R_i = \frac{i_p}{W_0} = \frac{\eta p A i \omega e^{i\omega t}}{G_T (1 + i \omega \tau_T)} = \frac{\eta p A i \omega e^{i\omega t} (1 - i \omega \tau_T)}{G_T (1 + \omega^2 \tau_T^2)} \quad (\text{Since } H = \tau_T G_T)$$

$$|R_i| = \left| \frac{\eta p A \omega e^{i\omega t} (i + \omega \tau_T)}{G_T (1 + \omega^2 \tau_T^2)} \right| = \frac{\eta p A \omega \sqrt{1 + \omega^2 \tau_T^2}}{G_T (1 + \omega^2 \tau_T^2)} = \frac{\eta p A \omega}{G_T \sqrt{1 + \omega^2 \tau_T^2}}$$

Sketch of variation of R_i with ω ;



R_i is directly proportional to frequency for $\omega \ll 1/\tau_T$, i.e.;

$$\omega^2 \tau_T^2 \ll 1, \text{ so } \sqrt{1 + \omega^2 \tau_T^2} \sim 1 \quad \rightarrow \quad |R_i| = \frac{\eta p A \omega}{G_T}$$

R_i is constant for frequencies $\gg 1/\tau_T$, i.e.;

$$\omega^2 \tau_T^2 \gg 1, \text{ so } \sqrt{1 + \omega^2 \tau_T^2} \sim \sqrt{\omega^2 \tau_T^2} = \omega \tau_T \quad \rightarrow \quad |R_i| = \frac{\eta p A}{G_T \tau_T} = \frac{\eta p}{c d}$$

$$(\tau_T = H/G_T \text{ and } H = c d A)$$

[40%]

$$(c) \text{ In the constant frequency regime; } |R_i| = \frac{\eta p A}{G_T \tau_T} = \frac{\eta p}{c d}$$

For $t = 30 \mu\text{m}$ and $\eta = 0.95$, $p = 27 \mu\text{Cm}^{-2}\text{K}^{-1}$ and $c = 2.6 \text{ MJm}^{-3}\text{K}^{-1}$;

$$|R_i| = \frac{0.9 \times 27 \times 10^{-6}}{2.6 \times 10^6 \times 30 \times 10^{-6}} \sim 0.31 \times 10^{-6} \text{ A W}^{-1}$$

[20%]

3 (a) The atomic number is the charge on the nucleus and therefore equal to the orbiting charge. The atomic weight is the ratio of the mass of the atom to that of 1/12th that of the carbon atom, or approximately that of the hydrogen atom.

The atomic weight is approximately twice the atomic number since nuclei contain about the same numbers of protons and neutrons. [20%]

(b) Equating the electrostatic and centrifugal forces;

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

For a whole number of wavelengths;

$$2\pi r = n\lambda = \frac{nh}{mv}$$

Eliminating v;

$$r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

In the ground state, $n = 1$;

$$r = \frac{h^2 \epsilon_0}{\pi m e^2}$$

[30%]

(c) The charge on the nucleus is screened by the orbiting electrons, so that if there is only one electron in an outer orbit it effectively sees a charge of one electron. As a result the radius of most atoms is very similar to that of the hydrogen atom.

The ionisation energy determines atomic bonding energies and hence the following materials parameters; heats of reaction, elastic moduli, boiling points, electrode potentials and band gaps in semiconductors. [20%]

(d) Molecular beam epitaxy (MBE) is the slow and carefully controlled evaporation of (usually) semiconductors. The elements to be deposited are heated electrically in separate cells called Knudsen cells that contain a small hole through which the atoms emerge in a beam towards the substrate. The rate of deposition is controlled by the heater and position of shutters in front of the cell. A mass spectrometer monitors the rate of evaporation of each element and feeds-back the appropriate heating power to produce the required ratio of elements in the film.

The main advantage of MBE is that can be used to put down compounds layer-by-layer, with the appropriate control of composition in each layer. As a result, however, the process is slow and expensive. A very high vacuum is also needed because of the slow rate of deposition and to avoid contamination.

The usual analysis techniques are RHEED (reflection high energy electron diffraction) and Auger spectroscopy. RHEED is very low angle electron diffraction which gives the structure of the first few layers by glancing a beam from the surface of the film. A soft x-ray is emitted in Auger spectroscopy as a secondary effect of electron emission, which characterises the composition of the layers. [30%]

4 (a) In substitutional doping, the dopant atom substitutes at the lattice site of the host (e.g. Si) so it forms bonds to the host. A substitutional donor element has a valency of 1 higher than the host atom it replaces and therefore supplies 1 electron more than is required for bonding. This electron becomes the donor electron. A substitutional acceptor element has a valency of 1 lower than the host atom it replaces and therefore supplies 1 electron less than is required for bonding. This creates a hole in the valence band.

An interstitial dopant enters an interstice or 'hole' in the lattice, and so is not bonded to the host lattice atoms. If it is a metal, it ionises and donates its electrons as donors. If it is a halogen, it forms a negative ion and leaves a hole in the valence band.

For group IV semiconductors, group V elements are donors and group III elements are acceptors.

For group III-V semiconductors, group VI elements at group V element sites are donors, group II elements at group III element sites are acceptors. Group IV elements at group V sites are acceptors and group IV elements at group III element sites are donors.

[30%]

(b) If a donor atom ionises, the electron is released and leaves a positively charged donor (e.g. P⁺). The electron around the donor ion experiences a coulombic field, as in the case of a hydrogen ion. In this case, however, the dielectric constant of the materials is ϵ and the electron has an effective mass m^* . As a result, the binding energy is;

$$R = R_0 \frac{m^*}{m \epsilon_r^2}$$

Here R_0 is the energy level of the hydrogen ion, 13.6 eV.

[30%]

(c) Doping is difficult in semiconductors with wide band gaps, which causes m^* to increase and ϵ to decrease.

Three reasons that make doping difficult are;

1. The dopant levels become un-ionisable or deep, due to large values of m^* and small values of ϵ_r .
2. The dopant becomes compensated by the creation of defects in the case of wide bandgap semiconductors
3. The dopant is not soluble in the host lattice due to a mismatch between the atomic Radii of the dopant and host.

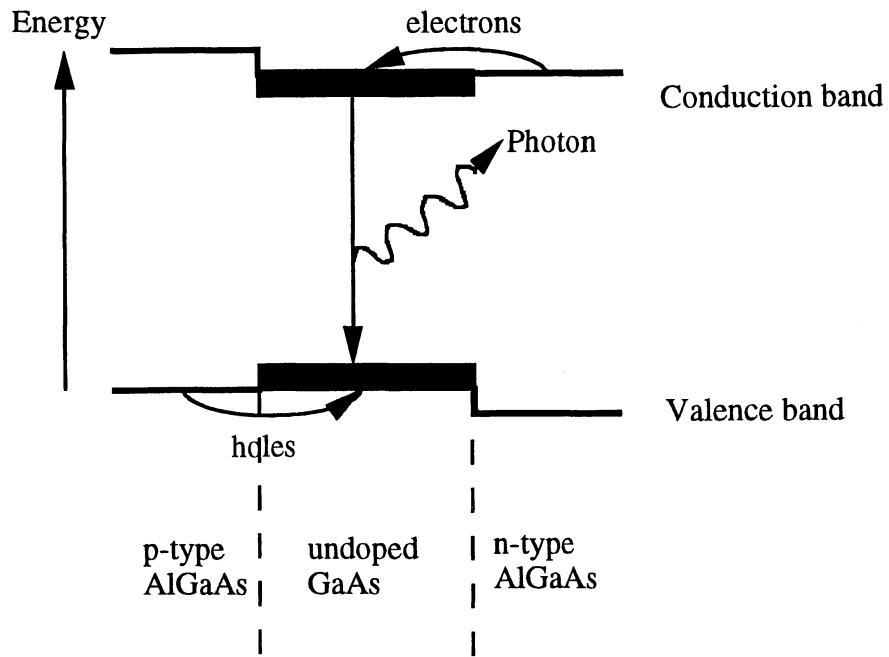
(i) Si has low m^* and high ϵ , and so has shallow donor energies, $e = 0.024$ eV, which are lower than kT at room temperature (0.025 eV). hence there are no difficulties doping this semiconductor.

(ii) Diamond has higher m^* and high ϵ , and so has deeper donor energies, $e = 0.6$ eV, which are higher than kT at room temperature (0.025 eV). This leads to doping difficulties.

(iii) GaAs has very low m^* and reasonable high ϵ , and so has very shallow donor level energies, so there are no difficulties doping this semiconductor.

(iv) SiO₂ has low ϵ , and so has deep levels. A high hole mass leads to very deep holes (greater than the band gap, if that is possible). This leads to doping difficulties.

[40%]



The interfaces should be lattice-matched to avoid dangling bonds which cause recombination.

Alloying is used to control the band gaps, and at the same time to control the lattice constants to give lattice matching.

[40%]