ENGINEERING TRIPOS PART IIB 2005

Paper 4C4

DESIGN METHODS

1 (a) The safety of a system is dependent upon minimising the chances of unexpected failure and/or minimising the consequences of failure.

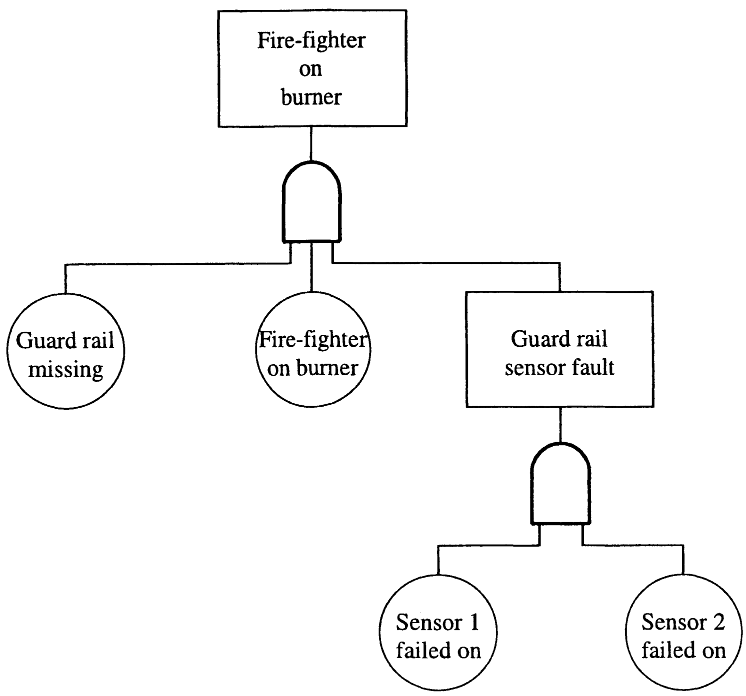
Failure can be minimised by designing for safe life or by designing with redundancy. Monitoring systems can help to predict an increased likelihood of system failure which may then be corrected through planned maintenance.

The consequences of failure can be minimised by designing for fail safe operation or by use of protective systems. [30%]

(b) A fault-tree analysis can be simply described as an analytical technique, whereby an undesired state of a system is specified (usually a state that is critical from a safety standpoint), and the system is then analysed in the context of its environment and operation to find all credible ways in which the undesired event can occur. The fault-tree itself is a graphical model of the various parallel and sequential combinations of faults that will result in the occurrence of the predefined undesirable event.

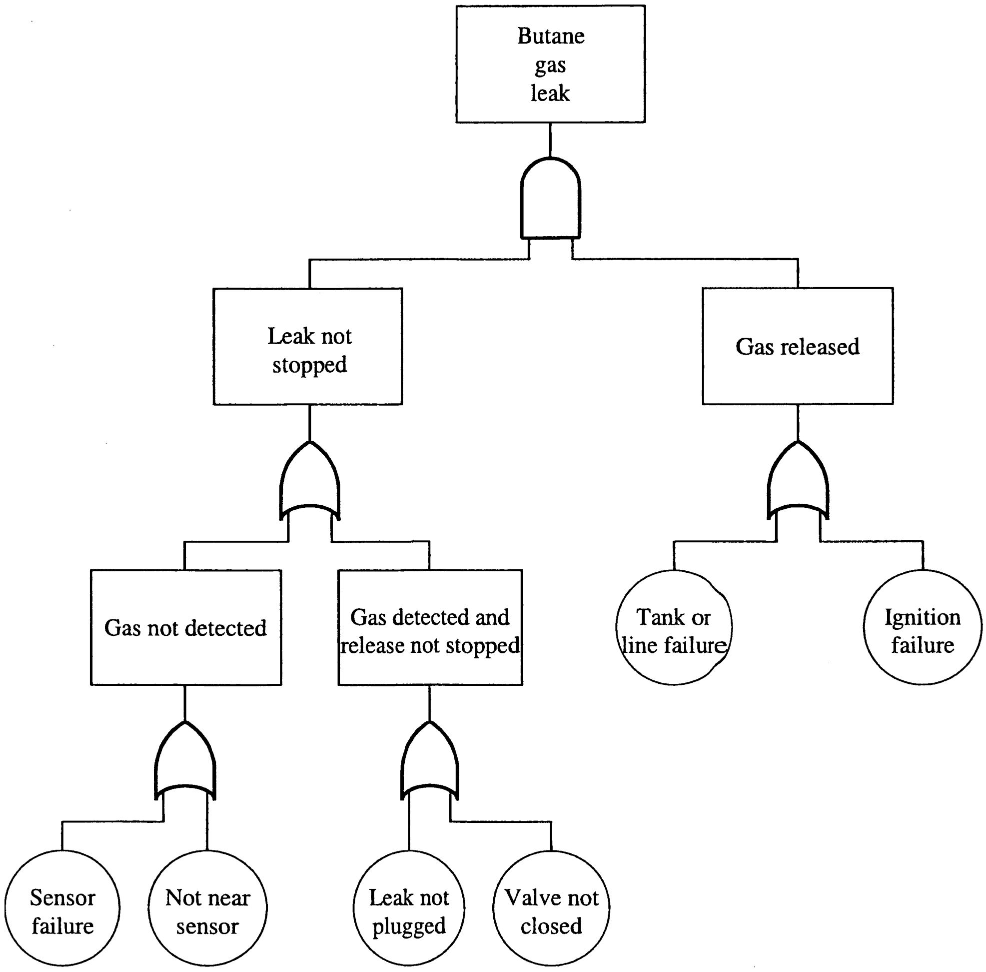
The faults can be events that are associated with component hardware failures, human errors, or any other pertinent events which can lead to the undesired event. A fault-tree thus depicts the logical inter-relationships of basic events that lead to the undesired event - which is at the top of the fault tree. [20%]

(c) (i) A fire-fighter can only fall on the burner if the *guard rail* is not in place and the two guard rail sensors have failed on.



[15%]

(ii) There are a number of events that can contribute to a gas leak. The figure below shows one of a number of fault tree layouts.



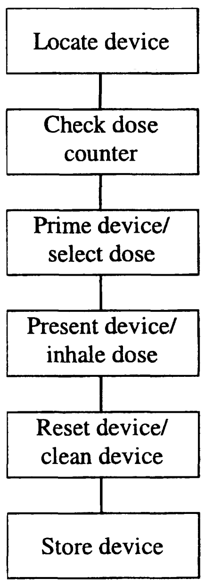
[35%]

2 (a) Deliver a unit dose of dry powder inhalant to the user on demand. [10%]

(b) Key requirements include:

- Accurate dosing
- Ensuring particulate size
- Providing moisture barrier
- Enabling appropriate inhalation flow
- Dose counter
- Easy priming/triggering
- Sensible physical size for transport/use [20%]

(c) Typical diagram may include the following steps:



[10%]

(d) Compressed pellet formulation will require some process of *carving/grating* a dose from the bulk and ensuring the subsequent powder has the correct granularity. Moisture exclusion will be a particular challenge. Location of powder is well known.

Loose uncompressed powder formulation will require *grabbing* of dose, again through a moisture seal. Granularity should not be such a problem. Location of powder must be carefully controlled.

Dose counters are generally troublesome, but useful. Ease of priming/firing is important. [50%]

(e) Relative merits are as in (d). Note that this is a challenging design problem and very few successful products have made it to the market despite many attempts by some of the worlds best designers. [10%]

3 (a) (i) The mechanical power P extracted from the wind by the windmill is given by

$$P = \frac{1}{2}\rho U^3 C_P A$$

where the swept area of the blades is $A = \pi r^2$

given that $\frac{U(h)}{U_r} = \left(\frac{h}{h_r}\right)^{\frac{1}{3}}$ we can substitute for U and A to obtain:

$$P = \left[\frac{\pi\rho U_r{}^3 C_P}{2h_r}\right] h r^2$$

If no part of the windmill must exceed h_{max} then one constraint is $r + h \leq h_{\max}$. An additional physical constraint is that the length of the blades r can be no longer than the hub height h (otherwise they would contact the ground), therefore $r \leq h$.

A formal optimisation statement for *maximising* the mechanical power P is:

Minimise $\quad f = -P = -\left[\frac{\pi\rho U_r{}^3 C_P}{2h_r}\right] h r^2$

Subject to: $\quad r + h - h_{\max} \leq 0$

$r - h \leq 0$ (inequalities in negative-null form)

Constants: ρ is the density of air

Parameters: C_P coefficient of performance

U_r wind velocity at a reference height

h_r reference height

Variables: h hub height

r blade tip radius [30%]

(ii) f is a minimum, and the power P is a maximum, when both constraints are active.

Therefore $r + h - h_{\max} = 0$ and $r - h = 0$. Hence $r = h = \frac{1}{2}h_{\max}$. This is a boundary optimum. [20%]

(b) (i) The net cost f of the windmill is its cost less its current value. Therefore

$$f = 800r^3 + 3h^3r^2 - kP = 800r^3 + 3h^3r^2 - 900hr^2$$ [15%]

(ii) $$\nabla f = \left(\frac{\partial f}{\partial r} \quad \frac{\partial f}{\partial h}\right)^T = \left(2400r^2 + 6h^3r - 1800hr \quad 9h^2r^2 - 900r^2\right)^T$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial r^2} & \frac{\partial^2 f}{\partial r \partial h} \\ \frac{\partial^2 f}{\partial h \partial r} & \frac{\partial^2 f}{\partial h^2} \end{bmatrix} = \begin{bmatrix} 4800r + 6h^3 - 1800h & 18h^2r - 1800r \\ 18h^2r - 1800r & 18hr^2 \end{bmatrix}$$

Starting at r = 7 m and h = 16 m. $\mathbf{x_0} = (7,16)$ $f(\mathbf{x_0})$ = £ 170912 and therefore this windmill design is uneconomic.

$$\nabla f(\mathbf{x_0}) = (88032 \quad 68796) \text{ and } H(\mathbf{x_0}) = \begin{bmatrix} 29376 & 19656 \\ 19656 & 14112 \end{bmatrix}$$

$$\mathbf{s_0} = -\nabla f(\mathbf{x_0}) = (-88032 \quad -68796)^T$$

$$\alpha_0 = \frac{-\mathbf{s_0}^\mathbf{T} \nabla f(\mathbf{x_0})}{\mathbf{s_0}^\mathbf{T} \mathbf{H} \mathbf{s_0}}$$

$$= -\frac{88032^2 + 68796^2}{(-88032 \quad -68796)\begin{bmatrix} 29376 & 19656 \\ 19656 & 14112 \end{bmatrix}\begin{pmatrix} -88032 \\ -68796 \end{pmatrix}} =$$

$$\frac{1.248\times10^{10}}{88032\times3.938\times10^9 + 68796\times2.7012\times10^9} = 2.344\times10^{-5}$$

$$\mathbf{x_1} = \mathbf{x_0} + \alpha_0 \mathbf{s_0} = (4.937\ ,\ 14.39)$$

$f(\mathbf{x_1})$ = £ -1514 and this improved design has a negative net cost and is therefore economic

If we continued with the gradient method we would eventually find a minimum at r = 5m and h = 10m where the net cost is f = £ -50000. [35%]

4 (a) (i) *Safety factor* = (Pitch difference) / (Jump limit)

For mean case:

$$Safety\ factor = \frac{P_0(1+\alpha t) - P_0(1+\beta t)}{0.5} = \frac{P_0(\alpha-\beta)t}{0.5} = \frac{0.24t}{0.5} = 1$$

Hence for P_0=12 mm, σ_0=0.01 mm, α=0.04 and β=0.02:

$t = 2.08$ years [20%]

(ii) *Safety factor* = (Pitch difference) / (Jump limit)

For worst case:

$$Safety\ factor = \frac{(P_0(1+\alpha t)+\sigma_0(1+10\alpha t)) - (P_0(1+\beta t)-\sigma_0(1+10\beta t))}{0.5-0.1}$$

$$= \frac{P_0(\alpha-\beta)t + 10\sigma_0(\alpha+\beta)t + 2\sigma_0}{0.4} = \frac{0.246t+0.02}{0.4} = 1$$

Hence for P_0=12 mm, σ_0=0.01 mm, α=0.04 and β=0.02:

$t = 1.54$ years [30%]

(b) *Safety margin* = (Jump limit) - (Pitch difference)

$$= (\mu_m, \sigma_m) = (\mu_l, \sigma_l) - (\mu_d, \sigma_d)$$

$$= (\mu_l, \sigma_l) - (P_c, \sigma_c) + (P_r, \sigma_r)$$

$$= \left(\mu_l - P_0(\alpha-\beta)t,\ \sqrt{\sigma_l^2 + \sigma_o^2(1+10\alpha t)^2 + \sigma_o^2(1+10\beta t)^2}\right)$$

From tables, 5% failure rate (assuming failures only occur when chain pitch is longer than chainring pitch): $\mu_m / \sigma_m = 1.645$, hence

$$\frac{\mu_l - P_0(\alpha-\beta)t}{\sqrt{\sigma_l^2 + \sigma_o^2(1+10\alpha t)^2 + \sigma_o^2(1+10\beta t)^2}} \approx \frac{\mu_l - P_0(\alpha-\beta)t}{\sigma_l} = \frac{0.5-0.24t}{0.1} = 1.645$$

Hence for P_0=12 mm, σ_0=0.01 mm, μ_l=0.5 mm, σ_l=0.1 mm, α=0.04 and β=0.02:

$t \approx 1.4$ years [40%]

(c) If α=0.02 and β=0.01, time scales with wear:

$t \approx 2.8$ years (double the previous time) [10%]